CSC4-151 Discrete Mathematics for Computer Science
Exam 1 December 2017

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name

For credit on these problems, you must show your work. On this exam, take the natural numbers to be $\boldsymbol{N} = \{0,1,2,3, \ldots\}$. This exam is closed book and closed notes. Calculators are allowed. 100 pts. possible.

1. (8 pts.) State one of DeMorgan's Laws for symbolic logic. Prove this, using a truth table.

2. (10 pts.) Give a deduction for the following hypothesis and conclusion, justifying each step. Use these predicates: W(x) : x is on the webteam, C(x): x is a coder, D(x) : x is a designer, M(x): x drinks Mountain Dew, domain: All people.

Hypothesis: Everyone on the webteam either is a coder or a designer. Carolina, who is on the webteam, isn't a designer. Everyone who is a coder drinks Mountain Dew. Conclusion: Carolina drinks Mountain Dew.

3. (9 pts.) Let L(x,y) be the predicate: “x likes y”, where the domains are given by: x is a CS student and y is a kind of food. Let D(x) be the predicate: x is a student in this discrete class, where the domain is all CS students

Express the following statements using those predicates and any required quantifiers; you may not use the “there exists unique” quantifier.

a. Every CS student likes some kind of food.

b. Every student in this discrete class likes grilled cheese sandwiches.

c. There is exactly one CS student who likes tofu. (You may **not** use there exist unique quantifier.)

d. Express the negation of the logical expression in **part b** with the negation moved as far inside as possible.

4. (2 pts. each) Let P(m,n) be the predicate “m + n is even” where the domain for each is the set **N.**

a. What is the truth value of $\forall m \exists n$ (P(m,n))? Justify.

b. What is the truth value of $\exists m \forall n$ (P(m,n))? Justify.

c. What is the truth value of $\forall m \forall n$ ((m = n) → P(m,n))? Justify.

5. (8 pts.) Let $g: N \rightarrow O$, where O is the set of odd natural numbers, be given by the rule $g(n) = 2n+1$.
   a. Carefully prove or disprove g is 1-1.

   b. Is g is onto? Justify your answer.

6. (6 pts.) Let B be the set of all bit strings and $f: B \dashrightarrow \mathbb{Z}$ be defined by the number of 0's in the bit string minus the numbers of 1's.
   a. Is f 1-1? Briefly justify your answer

   b. Is f onto? Briefly justify your answer.

7. (3 pts. each) Give an example of the following. Carefully justify that your example satisfies the given conditions. If no example exists, state this and briefly explain why.
   a. A set A such that $|\mathbb{P}(A)| = 5$.

   b. A set that satisfies the predicate $A \notin A$.

   c. A pair of sets A and B with $A \subset B$ and a function $f: A \rightarrow B$ that is 1-1 and onto.

8. (6 pts.) Suppose we are using a bit string representation of sets where the universal set is {1, 2, 3, 4}.

a. Represent ∅.

b. Let A = {1, 2}. Represent A.

9. (6 pts.) a. In the 5th century BCE Hippasus of Metapontum discovered his famous proof that the square root of 2 is irrational. What is the first sentence of his proof?

b. Suppose you are going to prove the theorem: If x^3 is irrational, then x is irrational. If you choose to prove this using an indirect proof, what is the first sentence of your proof? (Do not complete this proof.)

c. Suppose you are going to prove the theorem: For any sets A and B, $A \cup (B - A) \subseteq A \cup B$. What is the first sentence of your proof? (Do not complete this proof.)

10. (2 pts. each) True or false—circle one. No need to justify your answers.

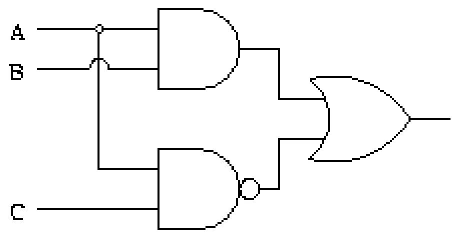
True False a. For any real numbers x and y, $\lceil x \rceil + \lceil y \rceil = \lceil x + y \rceil$

True False b. Given any two sets A and B, |A - B| = |B - A|.

True False c. For any set A, $\emptyset \in A$.

True False d. $\{\wedge, \neg\}$ is a logically complete set of logical operators.

11. (4pts. each) a. Give the Boolean algebra expression for the following circuit:



b. Draw a circuit for the following Boolean algebra expression: $(x + y)(\bar{z} + \bar{y})$

12. (6 pts.) Produce the disjunctive normal form for the Boolean function f given here:

p	q	r	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

13. (8 pts.) Let A = { 3, 4, { 3, 4 }, { 1, 2, 3 }, 5 } and B = { 3, 4 }

a. What is |A|?

b. Is $B \subset A$?

c. Is $B \in A$?

d. What is $| B \times B |$?