CSC4-151 Discrete Mathematics for Computer Science
Exam 1 December 6, 2017

Solution
name

For credit on these problems, you must show your work. On this exam, take the natural numbers to be $N = \{0,1,2,3, \ldots\}$. This exam is closed book and closed notes. Calculators are allowed. 100 pts. possible.

1. (8 pts.) State one of DeMorgan's Laws for symbolic logic. Prove this, using a truth table.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

p q	$\neg$	$(p \wedge q)$	$\leftrightarrow$	$\neg p$	$\vee$	$\neg q$
T T	F	T	T	F	F	F
T F	T	F	T	F	T	T
F T	T	F	T	T	T	F
F F	T	F	T	T	T	T

tautology so this gives $\equiv$

2. (10 pts.) Give a deduction for the following hypothesis and conclusion, justifying each step. Use these predicates: W(x) : x is on the webteam, C(x): x is a coder, D(x) : x is a designer, M(x): x drinks Mountain Dew, domain: All people.

Hypothesis: Everyone on the webteam either is a coder or a designer. Carolina, who is on the webteam, isn't a designer. Everyone who is a coder drinks Mountain Dew. Conclusion: Carolina drinks Mountain Dew.

1. $\forall x (W(x) \rightarrow (C(x) \vee D(x)))$	given
2. $W(c) \wedge \neg D(c)$	given c is Carolina
3. $\forall x (C(x) \rightarrow M(x))$	given
4. $W(c) \rightarrow C(c) \vee D(c)$	1, UI
5. $W(c)$	2, simp
6. $C(c) \vee D(c)$	4,5, MP
7. $\neg D(c)$	2, simp
8. $C(c)$	7,6, D.S.
9. $C(c) \rightarrow M(c)$	3, UI
10. $M(c)$	9,8, MP

3. (9 pts.) Let L(x,y) be the predicate: "x likes y", where the domains are given by: x is a CS student and y is a kind of food. Let D(x) be the predicate: x is a student in this discrete class, where the domain is all CS students

Express the following statements using those predicates and any required quantifiers; you may not use the "there exists unique" quantifier.

a. Every CS student likes some kind of food.

$\forall x \exists y (L(x, y))$

b. Every student in this discrete class likes grilled cheese sandwiches.

$\forall x (D(x) \rightarrow L(x, GCS))$

c. There is exactly one CS student who likes tofu. (You may **not** use there exist unique quantifier.)

$\exists x((L(x, tofu) \land \forall y (x \neq y \rightarrow \neg L(y, tofu))$

d. Express the negation of the logical expression in **part b** with the negation moved as far inside as possible.

$\exists x (D(x) \land \neg L(x, GCS))$

4. (2 pts. each) Let P(m,n) be the predicate "m + n is even" where the domain for each is the set **N**.

a. What is the truth value of $\forall m \exists n (P(m,n))$? Justify.

True if m even choose n even
" " odd " " odd.

b. What is the truth value of $\exists m \forall n (P(m,n))$? Justify.

false There is no number that is you add any number to it, the result is even.

c. What is the truth value of $\forall m \forall n ((m = n) \rightarrow P(m,n))$? Justify.

True any number added to itself is even.

5. (8 pts.) Let g:N→ O, where O is the set of odd natural numbers, be given by the rule g(n) = 2n+1.

   a. Carefully prove or disprove g is 1-1.

   Suppose $n_1, n_2 \in \mathbb{N}$ and $g(n_1) = g(n_2)$. Then $2n_1+1 = 2n_2+1$. So $2n_1 = 2n_2$ and $n_1 = n_2$. Thus g is 1-1.

   b. Is g is onto? Justify your answer.

   yes, every odd number is hit with this formula.

6. (6 pts.) Let B be the set of all bit strings and f: B --> $\mathbb{Z}$ be defined by the number of 0's in the bit string minus the numbers of 1's.

   a. Is f 1-1? Briefly justify your answer

   no $0 \rightarrow 1$, $010 \rightarrow 1$

   b. Is f onto? Briefly justify your answer.

   yes. To hit $n \geq 0$, choose n 0's. To hit $n < 0$, choose n 1's.

7. (3 pts. each) Give an example of the following. Carefully justify that your example satisfies the given conditions. If no example exists, state this and briefly explain why.

   a. A set A such that $|\mathcal{P}(A)| = 5$.

   No example exist. Card. of powerset is a power of 2.

   b. A set that satisfies the predicate A ∉ A.

   $\{1\} \notin \{1\}$.

   c. A pair of sets A and B with A ⊂ B and a function f: A → B that is 1-1 and onto.

   $\{1,2,3,\ldots\} \subset \{0,1,2,\ldots\}$
   $f(n) = n-1$ is 1-1 & onto

8. (6 pts.) Suppose we are using a bit string representation of sets where the universal set is $\{1, 2, 3, 4\}$.

a. Represent $\emptyset$.

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b. Let $A = \{1, 2\}$. Represent A.

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9. (6 pts.) a. In the 5th century BCE Hippasus of Metapontum discovered his famous proof that the square root of 2 is irrational. What is the first sentence of his proof?

Suppose $\sqrt{2}$ is rational.

b. Suppose you are going to prove the theorem: If x^3 is irrational, then x is irrational. If you choose to prove this using an indirect proof, what is the first sentence of your proof? (Do not complete this proof.)

Suppose x is rational.

c. Suppose you are going to prove the theorem: For any sets A and B, $A \cup (B - A) \subseteq A \cup B$. What is the first sentence of your proof? (Do not complete this proof.)

Suppose $x \in A \cup (B - A)$

10. (2 pts. each) True or false—circle one. No need to justify your answers.

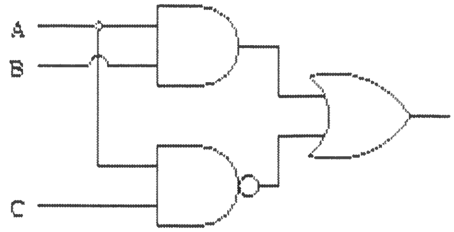
True False a. For any real numbers x and y, $\lceil x \rceil + \lceil y \rceil = \lceil x + y \rceil$

True False b. Given any two sets A and B, $|A - B| = |B - A|$.

True False c. For any set A, $\emptyset \in A$.

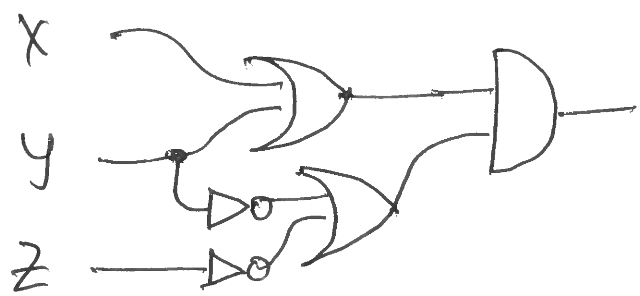
True False d. $\{\wedge, \neg\}$ is a logically complete set of logical operators.

11. (4pts. each) a. Give the Boolean algebra expression for the following circuit:



$AB + \overline{AC}$

b. Draw a circuit for the following Boolean algebra expression: $(x + y)(\bar{z} + \bar{y})$



12. (6 pts.) Produce the disjunctive normal form for the Boolean function f given here:

p	q	r	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$\bar{p}\bar{q}\bar{r} + \bar{p}q\bar{r} + \bar{p}qr$

13. (8 pts.) Let A = { 3, 4, { 3, 4 }, { 1, 2, 3 }, 5 } and B = { 3, 4 }

a. What is |A|? 5

b. Is B ⊂ A? yes

c. Is B ∈ A? yes

d. What is | B × B |? 4