**CSC8-151 Discrete Mathematics for Computer Science**
**Exam 1 April 25, 2018**

Solution
name

For credit on these problems, you must show your work. On this exam, take the natural numbers to be $N = \{0,1,2,3, \ldots\}$. This exam is closed book and closed notes. Calculators are allowed. 100 pts. possible.

1. (8 pts.) State one of DeMorgan's Laws for symbolic logic. Prove this, using a truth table.

see sample exam solutions

2. (10 pts.) Give a deduction (2 column proof) for the following hypothesis and conclusion, justifying each step. Use these predicates: B(x): x is beautiful, M(x): x is by Matisse, S(x): The museum has x, domain: All paintings.

Hypothesis: The museum has a painting by Matisse. All paintings by Matisse are beautiful. Conclusion: The museum has a beautiful painting.

Text 1.13.1.c

3. (9 pts.) The domain of discourse is all Cornell students. The predicate C(x) means that x is a member of Chess and Games, B(x, y) means that person x has beaten person y at some point in time. Give a logical expression equivalent to the following English statements. You can assume that it is possible for a person to beat himself or herself.

See text 1.10.3

a. Every member of chess and games has been beaten by someone.

b. Every member of chess and games has beaten Chad..

c. There is exactly one Cornell student who has beaten Dimo.

d. Express the negation of the logical expression in **part b** with the negation moved as far inside as possible.

4. (2 pts. each) Determine the truth value of each expression below. The domain is the set of all real numbers. No justification required.

See text 1.9.3

(a) $\forall x \exists y\ (xy > 0)$

(b) $\exists x \forall y\ (xy = 0)$

(c) $\forall x \forall y \exists z\ (z = (x - y)/3)$

(d) $\forall x \forall y\ (xy = yx)$

(e) $\forall x \exists y\ y^2 = x$

(f) $\forall x \exists y\ (x < 0 \vee y^2 = x)$

5. (8 pts.) Let $g: N \rightarrow N$ be given by the rule $g(n) = n^2$.

similar to
sample exam.

   a. Carefully prove or disprove g is 1-1.

   b. Is g is onto? Justify your answer.

6. (9 pts.) For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(a) f: $\{0, 1\}^4 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and dropping the first bit. For example f(1011) = 011.

see text 4.3.4

(b) f: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example f(011) = 110.

(c) f: $\{0, 1\}^3 \rightarrow \{0, 1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, f(100) = 1001.

7. (6 pts.) Suppose we are using a bit string representation of sets where the universal set is $\{1, 2, 3, 4, 5\}$.

a. Represent $\emptyset$.

see sample exam

b. Let $A = \{1, 3, 5\}$. Represent A.

8. (4 pts.) a.. Suppose you are going to prove the theorem: If $x^3$ is irrational, then x is irrational. If you choose to prove this using a proof by contraposition, what is the first sentence of your proof? (Do not complete this proof.)

See sample exam

b. Suppose you are going to prove the theorem: For any sets A and B, $A \cup (B - A) \subseteq A \cup B$. What is the first sentence of your proof? (Do not complete this proof.)

9. (2 pts. each) True or false—circle one. No need to justify your answers.

True False a. For any real numbers x and y, $\lceil x \rceil + \lceil y \rceil = \lceil x + y \rceil$

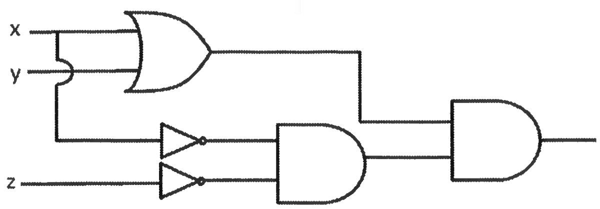
True False b. $\lfloor \log_2 1000 \rfloor = 10$

True False c. For any set A, $\emptyset \subset A$.

True False d. For any sets A and B, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

True False e. $\{\wedge, \vee\}$ is a functionally complete set of logical operators.

10. (4pts. each) a. Give the Boolean algebra expression for the following circuit:




see text 5.6.2

b. Draw a circuit for the following Boolean algebra expression: $(\bar{x} + y)(\bar{y} + z)$

see sample exam

11. (6 pts.) Produce the disjunctive normal form for the Boolean function f given here:

| x | y | z | f(x, y, z) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

see sample exam

← (This is 5.6.1.b)

12. (10 pts.) Let A = { 3, 4, { 3, 4 }, { 1, 2, 3 } } and B = { 3, 4 }

a. What is |A|? $= 4$

See sample exam

b. Give P(B), the power set of B.

$\{\emptyset, \{3\}, \{4\}, \{3,4\}\}$

c. Is B $\in$ A?

yes

d. Is B $\subset$ A?

yes

e. What is | P(A × B) |?

$|A \times B| = 4 \cdot 2 = 8$

so

$|P(A \times B)| = 2^8 = 256$