CSC8-151 Sample problems for final exam

May 2018

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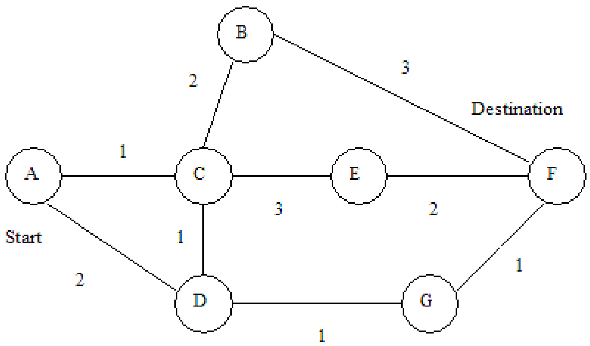
name

125 pts possible. Closed book and closed notes. Calculators are allowed, but no other electronic devices. There is a strict 3 hour time limit.

1. (6 pts.) Describe how Dijkstra's algorithm works by finding the shortest path between A and F.



2. (4 pts.) How many people must be in the same room to guarantee at least two of them share birth months? What is the name of the principle you apply to solve this problem?

For problems 3 – 6 you may leave your answers in combination, permutation, or power format.

3. (5 pts.) In the Maryland Cash In Hand game, the contestant chooses seven distinct numbers among the number 1 to 31. The contestant wins a modest amount ($40) if exactly five numbers, in any order, match those among the seven distinct numbers randomly drawn by a lottery representative. What is the probability of winning this $40?

4. (4 pts.) Find the probability of getting 4 aces in a 5 card poker hand.

5. (6 pts.) a. How many symmetric relations are there on a set with n elements?

b. How many anti-symmetric relations are there on a set with n elements?

6. (4 pts.) How many different (non-isomorphic) simple graphs are there with 3 nodes?

7. (18 pts) For this problem choose among the following growth functions g(n): n, n^2, n^3, log n, n log n, 1, 2^n, n!. For which one of the above growth functions, g(n), is f(n) = $\Theta(g(n))$? No need to justify your answer.

a. Let f(n) represent the complexity of the standard algorithm to check if a relation, represented by an n by n matrix, is transitive.

b. Let f(n) represent how many times the statement ExecuteThis is encountered in the following code, from the algorithm for exponentiation by repeated squaring:

```
result = 1
x = a
while (n > 0) {
        if (n mod 2 == 1)
                result = result * x
        x = x * x
        ExecuteThis;
        n = ⌊n/2⌋
        }
```

c. Average case behavior of Mergesort of a collection of length n.

d. Dijkstra's algorithm on a graph with n nodes (disregard edges)?

e. Worst case of Travelling Salesperson Problem.

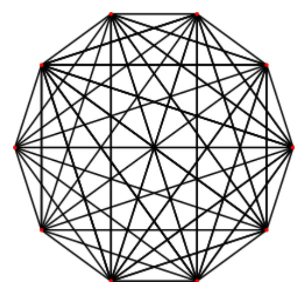
8. (12 pts.) Give a recursive definition of each of the following.

a. Give a recursive specification of "c choose k"-- the binomial coefficient-- which we write as $C(n,k)$ or ${}_nC_k$

b. The set/language of all bit strings.

c. The Euclidean algorithm for GCD(a,b).

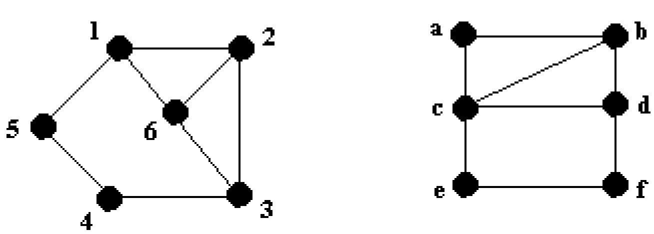
9. (6 pts.) At the right is graphic of K_{10}--the complete graph with 10 vertices. This question is about K_{100}



a. How many edges does K_{100} have? Briefly justify your answer.

b. Does K_{100} have an Euler cycle? Briefly justify your answer.

10. (4 pts.) Are the following graphs isomorphic? Briefly justify your answer.



11. (3 pts.) Give an algorithm, by name or very brief label, which is in each of the following classes:

a. P

b. NP but not NP-complete

c. NP-complete

12. (8 pts.) For what values of n is $2^n < n!$? Prove this inequality, using the Principle of Mathematical Induction, and starting at the appropriate basis value.

13. (2 pts. each) True or false? No need to justify your answer.

__________a. $-3 \equiv 10 \pmod{7}$

__________b. $\Phi(35) = 24$ where Φ is the Euler phi function.

__________c. 25 has an inverse mod 42.

__________d. If a = bq + r, where a, b, q, and r are natural numbers and $0 \leq r < b$, then gcd(a,b)=gcd(b,r).

___________e. The number 150! ends in exactly 37 zeroes.

14. a. Give the first sentence of Euclid's proof that there are infinitely many prime numbers.

b. What is a "collision" when using a hashing function and how might it be resolved? (We discussed three, choose one of these to briefly describe).

15. (10 pts.)

a. Show that gcd(45, 14) = 1 using the Euclidean algorithm.

b. Find the inverse of 14 modulo 45 using your work from part a.

c. What part does this algorithm play in RSA/public key encryption?

16. (12 pts.)

a. Find 5^{12} mod 11 using repeated squaring. Show your work.

b. Exactly how many multiplications would be required to evaluate 177^{642}mod 481 using repeated squaring? Justify your answer—do not calculate the value of the expression.

c. What part does this algorithm play in RSA/public key encryption?