MAT 1-221 Linear Algebra
Exam 1 September 8, 2008

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name

Put your answers on the paper provided. Please work neatly and staple your solutions, in problem order, to these two pages. Calculators are allowed; you must show your work for credit.

1. (8 pts.) What conditions must b_1, b_2, and b_3 satisfy in order for the following system to be consistent? Justify your answer.

$$\begin{aligned} x + 2y + 3z &= b_1 \\ 2x + 5y + 3z &= b_2 \\ x + 8z &= b_3 \end{aligned}$$

2. (6 pts.) Set up a linear system to find a polynomial p(t) of degree 3 such that p(1) = 1, p(2) = 5, p'(1) = 2 and p'(2) = 9. Show your work.

3. (10 pts.) Suppose you are a spy watching the French coast guard (which is using a linear transformation to encode boat positions, as in our text.) You collect the following data: When the actual position is [5 42] they radio [89 52] and when the actual position is [6 41] they radio [88 53]. You may use a calculator for the computations on this problem, but explain your answers.

a. What is the coding matrix?
b. Suppose they radio [90 50]? What is the actual position?

4. (4 pts. each) Give the matrix that produces the following linear transformations from $\mathbf{R}^2 \to \mathbf{R}^2$. For each part, also give the matrix that produces the inverse transformation, or explain why no such inverse exists.

a. Reflection about the origin.

b. Counterclockwise rotation of $\pi/6$ radians.

c. $y_1 = x_1$
$y_2 = 0$

d. Any shear parallel to the x axis.

5. (6 pts.) For an n by n matrix **A**, we call the linear system $\mathbf{A}\,\mathbf{x} = \mathbf{0}$ a homogeneous system. Prove: If $\mathbf{x_1}$ and $\mathbf{x_2}$ are solutions of a homogeneous linear system, then any linear combination of $\mathbf{x_1}$ and $\mathbf{x_2}$ is also a solution of the homogeneous system.

6. (8 pts.) Prove or disprove:

a. The transformation S: $\mathbf{R}^2 \to \mathbf{R}^2$ given by $y_1 = 2x_1$ and $y_2 = 1$ is linear.

b. The transformation T: $\mathbf{R}^2 \to \mathbf{R}^2$ given by $y_1 = x_1 + x_2$ and $y_2 = 0$ is linear.

7. True or false? Briefly justify each answer. (3 pts. each)

___________ a. If A and B are invertible n by n matrices, then AB=BA.

___________ b. $\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ is an elementary matrix.

___________ c. For A and B invertible matrices, $(ABA^{-1})^4 = A B^4 A^{-1}$

___________ d. Every elementary matrix is invertible.

___________ e. Let $\text{proj}_L(\mathbf{v})$ be the projection of vector **v** onto the line L and $\text{ref}_L(\mathbf{v})$ be the reflection of **v** about L. Then $\text{ref}_L(\mathbf{v}) = 2\,\text{proj}_L(\mathbf{v}) - \mathbf{v}$.

___________ f.. If **A** is an invertible 2 by 2 matrix such that $\mathbf{A}^2 = \mathbf{A}$, then **A** must be $\mathbf{I}_2$.

___________ g. A linear system with fewer unknowns than equations must have infinitely many solutions or none.

___________ h. If vector **u** is a linear combination of vectors **v** and **w**, then **w** must be a linear combination of **u** and **v**.

8. (6 pts.) Suppose A is an invertible matrix with $A^{-1} = \begin{matrix} a & b \\ c & d \end{matrix}$

and B is an invertible matrix with $B^{-1} = \begin{matrix} e & f \\ g & h. \end{matrix}$

Give the matrix for $(AB)^{-1}$

9. (4 pts. each) Give an example that satisfies the following conditions. Explicitly demonstrate that your examples satisfy the given conditions. If the given conditions cannot be satisfied, say so and briefly explain why.

a. A non-homogeneous system of 3 equations and 3 unknowns that has an infinite number of solutions. (Give the solutions.)
b. An invertible 3 by 3 matrix A with two identical rows.
c. A 4 by 4 system with rank 2.
d. A matrix which maps the unit square in $\mathbf{R}^2$ to the square with corners (1 1), (2 1), (2 2) and (1 2).