MAT 1-221 Linear Algebra
Old Sample Exam 2

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name

Put your answers on the paper provided. Please work neatly and staple your solutions, in problem order, to these two pages. **You must show your work and justify your answers for credit.** You are free to use your calculators to compute rref, inverses, and matrix arithmetic; you may not store and use information such as formulas or definitions in your calculator. When showing your work, simply indicate where you have used a calculator.

1. (3 pts. each) Definitions:
   a. A basis of a linear space V is a linear independent spanning set. Complete the definition: The set of vectors $v_1, v_2, ..., v_n$ in V is **linearly independent if**

   b. Complete the definition: The matrix A is **similar** to the matrix B if

2. (10 pts) Determine a basis for the kernel and a basis for the image of the following matrix:

1	2	0	3	0
2	4	1	9	5

3. (8 pts) Prove that there is a non-trivial relation among the vectors $v_1, v_2, ..., v_n$ if and only if at least one of the vectors v_i is a linear combination of the other vectors $v_1, v_2, ..., v_{i-1}, v_{i+1}, \ldots, v_n$.

4. (4 pts. each) True or false? Justify your answers.
   a. The set of all invertible 3 by 3 matrices is a subspace of $\mathbf{R}^{3x3}$

   b. The set of all f(x) such that f(1) = 0 is a subspace of $\mathbf{C}^{\infty}$.

5. (4 pts. each) True or false? Justify your answers.

a. The transformation S:V→ V given by S(x0,x1,x2,x3, …) = (x1, x3, x5, …) (drop all even terms) is a linear transformation, where V is the space of infinite sequences of real numbers.

b. The transformation T: $\mathbf{P_2} \rightarrow \mathbf{R}$ given by T(p) = $\int_0^1 p(t)dt$ is a linear transformation.

6. (4 pts. each) The derivative operator Df = f' is a linear operator from $\mathbf{P_2}$ to $\mathbf{P_2}$.

a. Describe the kernel of D and give a basis for the kernel.

b. Describe the image of D and give a basis for the image.

The following diagram characterizes change of basis and the associated matrices of a linear transformation:

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                 A
[x] -----------------------------------> [T(x)]
 ↑                                          ↑
 | S                                        | S
                 B
[x]_B ---------------------------------> [T(x)]_B
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Problems 7 and 8 refer to this diagram.

7 (10 pts.) a. Given the vector [2 3] in $\mathbf{R}^2$ (top left) and the basis [1, 1] and [-1,1] of $\mathbf{R}^2$ find $[\mathbf{x}]_B$.
b. With the same basis as in part a, suppose you are given $[\mathbf{x}]_B = [\mathbf{4}\ \ \mathbf{5}]_B$. What is the vector with respect to the standard basis? Show your work.

c. Describe the matrix S. Is it always an invertible matrix? Justify your answer.

8 (9 pts.) Let T be the linear transformation given by multiplication by the matrix. Give the matrix, B, of this transformation relative to the basis [1 1 1], [0 1 2] and [1 2 4]. Compute B in two ways, by determining the effect of T on the basis elements, and using matrix multiplication with the diagram above.

0	2	-1
2	-1	0
4	-4	1

9. (3 pts. each) True or false? Briefly justify each answer.

a. There exists a 3 by 3 matrix A with ker (A) = im (A).
b. If 2 **u** + 3 **v** + 4 **w** = 5 **u** + 6 **v** + 7 **w** then vectors **u**, **v** and **w** must be linearly dependent.
c. If vectors v_1, v_2, v_3, and v_4 are linearly independent then so are vectors v_1, v_2, and v_3.
d. $\{ 1 - t, t - t^2, t^2 - 1\}$ is a basis for $\mathbf{P_2}$.
e. If matrix **A** is similar to matrix **B**, and matrix **B** is similar to matrix **C**, then matrix **A** is similar to matrix **C**.
f. If **A** is an invertible n by n matrix then the kernels of A and A^{-1} must be the same.
g. If A is a matrix with m rows and n columns then rank A + nullity A = n.

10. (3 pts. each) Give an example that satisfies the following conditions. Explicitly demonstrate that your examples satisfy the given conditions. If the given conditions cannot be satisfied, say so and briefly explain why.

a. An infinite dimensional linear space.
b. A subspace of $\mathbf{R}^{2\mathrm{x}3}$ of dimension 2.
c. A basis of $\mathbf{P_3}$ that includes $1 + t + t^2$
d. A basis for the, the set of all vectors in $\mathbf{R}^3$ that are perpendicular to (1,2,3).