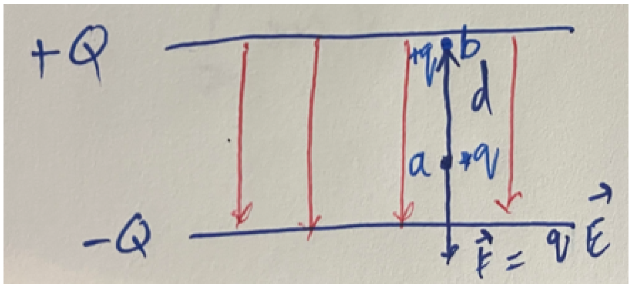
Chapter 24 - Electric Potential

Electric Potential Energy

Let's first visit the gravitational potential energy. We know that when an object move in gravitational field from point a to point b then the change in the potential energy is:

$$\Delta U = -\int_a^b \vec{F} \cdot d\vec{r}$$

where $\vec{F}$ is gravitational force, and the negative sign indicate that potential energy will increase if $d\vec{r}$ is opposite to the direction of gravitational force. This general expression can be applied to the change in electric potential energy inside an electric field as well, but then the force in the equation will be the electric force. To see it in more detail, let's consider two parallel plates that have charges +Q and -Q on them and it create a uniform electric field between the plates.



Let say that a charge q move from point a to point b in this uniform electric field, then the change in electric potential energy of the charge in the field according to the above equation will be

$$\Delta U = -\int_a^b q\vec{E} \cdot d\vec{r}$$

Since E is uniform it is not changing, then we can take it out of the integration. Also $\vec{E}$ and $d\vec{r}$ are in opposite direction so the above equation will become:

$$\Delta U = qE \int_a^b dr = qEd$$

so change in electric potential energy is qEd which is similar to change in gravitational potential energy mgh.

Potential Energy Between Two Charges

Electric potential energy of any charge configuration is equal to the work required to bring these charges together to the current configuration from the configuration where the electric potential energy was zero. The electric potential is zero when the charges are infinitely far away from each other. So the equation for change in electric PE become

$$\Delta U = -\int_\infty^r \vec{F} \cdot d\vec{r}$$

where r is the distance between the charges in the current configuration. If there are only two charges q_1 and q_2 then

$$\Delta U = -\int_\infty^r \frac{kq_1q_2}{r^2} dr$$

$$U_f - U_i = +kq_1q_2 \left[\frac{1}{r}\right]$$

Since $U_i = 0$ then $U_f = U$ will become

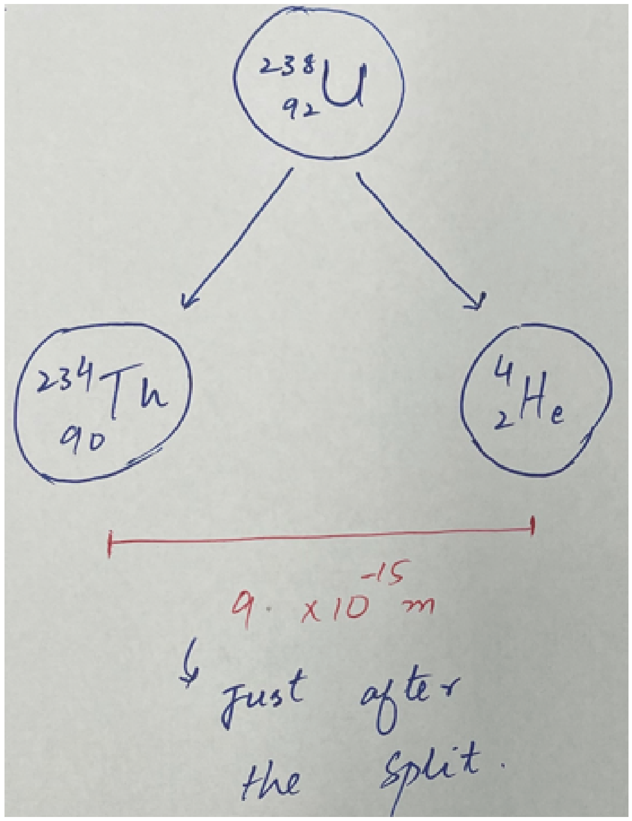
$$U = \frac{kq_1q_2}{r}$$

This is the electric potential energy between two point charges separated by a distance r. If there are three charges then the total potential energy of the three charges configuration will be

$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

We can extend the formula for any number of charges.

Example: Uranium Split (Applying conservation of energy)



Uranium nucleus splits into two pieces. Assume both nuclei are at rest right after the uranium split. Just after the split the distance between the product nuclei (Thorium and Helium) is 9×10^{-15} m. How fast is the Helium nucleus moving when it is 2×10^{-14} m away from the Thorium nucleus. Assume that the Thorium nucleus remains at rest. (It does not but the assumption is a good approximation)

Solution

$$U = \frac{kq_1q_2}{r},$$

$$U_i = \frac{(9 \times 10^9)(90e)(2e)}{9 \times 10^{-15}} = 4.6 \times 10^{-12} \text{ J}$$

Initial Energy is $= 4.6 \times 10^{-12} J$

$$U_b = \frac{(9 \times 10^9)(90e)(2e)}{2 \times 10^{-14}} = 2.1 \times 10^{-12} \text{ J}$$

$$K_f = \frac{1}{2} m_{He} v^2$$

$$\frac{1}{2} m_{He} v^2 + 2.1 \times 10^{-12} = 4.6 \times 10^{-12}$$

$$m_{He} = 4m_p = 4 \times 1.67 \times 10^{-27} \text{ kg} = 6.68 \times 10^{-27} \text{ kg}$$

$$v^2 = \frac{2(4.6 \times 10^{-12} - 2.1 \times 10^{-12})}{6.68 \times 10^{-27}} \Rightarrow v = 2.7 \times 10^7 \text{ m/s}$$

very fast.

about 20% the speed of light.

Electrostatic Potential

When we move a test charge in a uniform electric field, then change potential energy qEd. This quantity is dependent upon the magnitude of the test charge. We can define a quantity that will be independent of the test charge, and will be the property of source charge or the electric field. That quantity is Electrostatic potential V.

Electrostatic potential is defined as PE per unit charge.

$$V = \frac{U}{q}$$

It is a scalar quantity. V is measured in volts $= \frac{\text{joules}}{\text{Coulombs}}$

$$1 \text{ volt} = \frac{1J}{1C}$$

V is a function of position (just like U)

$$U = qV$$

if we know $V(\vec{r})$ & q then we can find

$$U(\vec{r}) = qV(\vec{r})$$

Also

$$\Delta U = q\Delta V$$

Potential from E-field

As

$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta U = -\int_i^f \vec{F}\cdot d\vec{r}$$

$$\Delta V = -\frac{1}{q}\int_i^f \vec{F}\cdot d\vec{r} = -\int_i^f \vec{E}\cdot d\vec{r}$$

$$\boxed{\Delta V = -\int_i^f \vec{E}\cdot d\vec{r}}$$

If the test charge is initially at infinity then

$$\boxed{V = -\int_\infty^f \vec{E}\cdot d\vec{r}}$$

Advantage of $V(\vec{r})$ over $\vec{E}(\vec{r})$?. $V(\vec{r})$ is a scalar: no components.

Voltage

Voltage: is the difference in potential between two points.

Often in circuits we ask what is the voltage across a particular circuit element.

A ________________ B

circuit element

$$\text{voltage} = \Delta V = V_B - V_A = \text{potential at point B} - \text{potential at point A}$$

Now if we know the voltage, ΔV, between two points, and we know the size of the charge q that moves between the two points, then we know the change in PE.

$$\Delta U = q\Delta V$$

Very often the charge that are being moved around are electrons.

This means if we move one electron through a change of potential equal to 1 volt

(that is $\Delta V = V_B - V_A = 1$ volt)

then the electron gain/loses (depends which way we go, move from lower potential to higher / from higher potential to lower) One Electron Volt.

one electron volt of energy.

$$\Delta U = (1e)(1\text{Volt}) = 1eV$$

$$1eV = (1.6 \times 10^{-19}C)(1\frac{J}{C})$$

$$\boxed{1eV = 1.6 \times 10^{-19} \text{ J}}$$

Now from this we can say that if an electron is accelerated between parallel plates with 10,000 volts between them, it gains 10,000 eV of energy.

Problem: *An electron is accelerated between a pair of parallel plates with a voltage 20,000 volts. How fast is the electron moving*

is the electron moving (assume it starts from rest) when it has finished accelerating?

$$U = qV$$
$$U = (1.6 \times 10^{-19}C)(20,000 \text{ volts})$$
$$U = 3.2 \times 10^{-15}J$$

$E = U + K = 3.2 \times 10^{-15}$ J

Final is all K

$$\frac{1}{2}m_e v^2 = 3.2 \times 10^{-15}, \quad m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\boxed{v = 8.4 \times 10^7 \text{ m/s}}$$