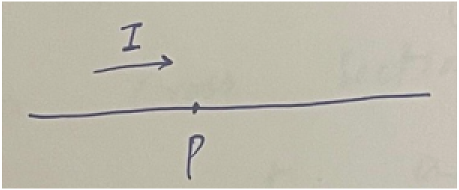
Chapter 26 - Current and Resistance

Current

Current is the flow of charge per unit time.



Then the current is defined as

$$I = \frac{\#\ of\ \ charges\ \ that\ \ flow\ \ past\ \ the\ \ P}{\text{time it takes for charges to flow past}}$$

$$I = \frac{dQ}{dt}$$

Current

- Conventional current is the direction of the flow of positive charges. But, in the real world, the moving charges are often electrons.
- Electrons move to the left, conventional current flows to the right in the above figure.

Current Density

Define

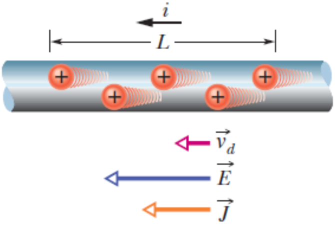
$$J = \frac{i}{A} = \frac{\text{current}}{\text{area}}$$

This is useful if we are interested in the flow of charge through a cross sectional area of the conductor at a particular point.

$$i = \int \vec{J} \cdot d\vec{A}$$

Drift Speed

- The net average speed of positive charges in a conductor in the direction of the applied electric field is called drift speed.



or

- The net average speed of free electrons in a conductor in the direction opposite to the applied electric field is called drift speed.

Drift Speed

Metals like copper have a few free electrons (electrons that can easily move from one atom to the next).

This is usually only 1 or 2 free electrons per atom. Most of the electrons are tightly bound to each atom.

Drift Speed

We can define:

$$\text{Carrier density } = n = \frac{\# \text{ free electrons}}{\text{volume}}$$

If $N =$ number of free electrons, then

$$n = \frac{N}{AL} \quad \Rightarrow \quad N = nAL$$

Since $q = ne$.
The total charge of the carriers in length L each with charge e, is then

$$q = (nAL)e$$

Drift Speed

Then the drift speed v_d is the length L the charge covers in time interval t

$$v_d = \frac{L}{t} \Rightarrow t = \frac{L}{v_d}$$

$$i = \frac{q}{t} = \frac{qv_d}{L} = (nAL)ev_d$$

$$v_d = \frac{i}{nAe}, \quad j = \frac{i}{A}$$

$$\boxed{v_d = \frac{j}{ne}}$$

Example: Drift Speed in Copper Wire

What is the drift speed of electrons in copper if $i = 10$ Amps, $r = 10^{-3}$ m ?

$$A = \pi r^2 = 3.14 \times 10^{-6} \text{m}^2$$

$$n = 8.49 \times 10^{28} \text{ electrons/m}^3, \quad e = 1.6 \times 10^{-19} \text{ C}$$

$$v_d = \frac{i}{neA} = 2.3 \times 10^{-4} \text{ m/s}$$

Very small.

Resistance and Resistivity

Resistance is a measure of the opposition that a material or components offers to the flow of charge. If we apply potential difference V across the ends of a conductor, and it results in a current i in the conductor then resistance is

$$R = \frac{V}{i}$$
$$V = iR$$

The unit of resistance is ohm and

$$1 \text{ ohm} = \frac{1V}{1A} = 1 \ \frac{V}{A}$$

Ohm is represented by Ω

Resistance and Resistivity

Now if we replace V by E and i by J then we also get a constant.

$$\rho = \frac{E}{J}$$

Where ρ is resistivity. The unit of resistivity is

$$\rho \text{ unit} = \frac{1\ V/m}{1\ A/m^2} = 1\ \frac{V}{A \cdot m} = 1\ \Omega \cdot m$$

$$\vec{E} = \rho \vec{J}$$

This is another form of $V = iR$

Resistance and Resistivity

Conductivity is the reciprocal of resistivity

$$\delta = \frac{1}{\rho}, \quad \vec{J} = \delta \vec{E}$$

resistivity of Silver and Copper are given below.

Material	**Resistivity** ρ
Silver	$1.62 \times 10^{-8}\ \Omega \cdot m$
Copper	$1.69 \times 10^{-8}\ \Omega \cdot m$

Resistance and Resistivity

- Important point to note that Resistance is a property of an object. Resistivity is a property of a material.
- If we know the resistivity of a material we can find the resistance of a length of wire made of that material.

$$\rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{VA}{iL} \quad \rho = \frac{RA}{L}$$

$$\boxed{R = \rho\frac{L}{A}}$$

Does it make sense?

Variation of ρ with temperature

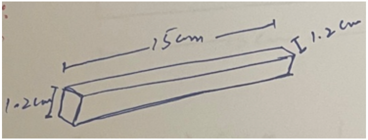
- The value of most physical properties vary with temperature, and resistivity is no exception. Resistivity vary with temperature as

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

- Here T_0 is room temperature which is $T_0 = 293\ K$ ($20\,^{\circ}\mathrm{C}$). The quantity α is called the temperature coefficient of resistivity and ρ_0 is resistivity at room temperature.
- The values of α is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range.
- For Copper

$$\alpha = 4.3 \times 10^{-3}\ (K^{-1})$$

Example



What is the resistance of the Iron block, $R = ?$ if potential difference is applied to the
(1) Square sides (1.2 cm $\times$ 1.2 cm)
(2) Rectangular sides (1.2 cm $\times$ 15 cm)

Example

$$\rho = 9.68 \times 10^{-8}\ \Omega \cdot m$$

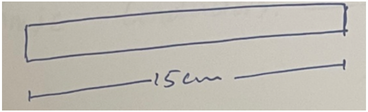
$$L = 15\ \text{cm} = 15 \times 10^{-2}\ \text{m}, \quad A = (1.2 \times 1.2) = 1.44 \times 10^{-4}\ \text{m}^2$$

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8})(15 \times 10^{-2})}{1.44 \times 10^{-4}}$$

$$R = 100 \times 10^{-3}\ \Omega$$

$$\boxed{R = 100\ m\Omega}$$

Similarly for Rectangular side



$$A = 15 \text{ cm} \times 1.2 \text{ cm}, \quad L = 1.2 \text{ cm}$$

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8})(1.2 \times 10^{-2})}{1.8 \times 10^{-3}} = 6.5 \times 10^{-7} \ \Omega$$

$$\boxed{R = 0.65 \ \mu\Omega}$$

Ohm's Law

- The current through a conductor is directly proportional to the potential difference applied to that conductor.

$$i \propto V \quad \text{Ohm's Law}$$

$$iR = V$$

$$\boxed{V = iR}$$

- This equation applies to all materials.
- If R is constant, then those conductors are called ohmic conductors.

Ohm's Law

Important points to note are:

- A conducting device obeys ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference (Ohmic Conductors).
- If not, then it means that R will change with the applied PD and as a result we will not get a straight line (Non ohmic materials)
- A semiconducting PN junction diode is an example of a non-ohmic material.

A Microscopic View of ohm's Law

- To find out why certain materials obey ohm's law, let's look into the details of the conduction process at the atomic level. Since the conduction electrons are completely free they collide with atoms of the metal when moving around.
- If there is an external E-field applied to the conductor then the conduction electrons will feel a force

$$F = eE$$

$$F = ma \Rightarrow eE = ma \Rightarrow a = \frac{eE}{m}$$

This is the acceleration of the electron in the E-field.

A Microscopic View of ohm's Law

After a time "τ" (mean time duration between collisions), the electron hits an atom then bounces off in a random direction. On average the velocity after the collision is zero. So electrons will have an average drift of

$$\vec{v_d} = \left(\frac{e\vec{E}}{m} \right) \tau$$

A Microscopic View of ohm's Law

Since

$$V_d = \frac{j}{ne} \Rightarrow \frac{j}{ne} = \left(\frac{eE}{m}\right)\tau \Rightarrow E = \frac{mj}{e^2 n\tau}$$

$$E = \left(\frac{m}{e^2 n\tau}\right) j \quad \textit{Comparing this with} \quad E = \rho j$$

$$\boxed{\rho = \frac{m}{e^2 n\tau}} \quad \text{Microscopic} \quad \boxed{\rho = \frac{RA}{L}} \quad \text{Macroscopic}$$

- From the microscopic expression of the resistivity, we can see that for these type of metals, the resistivity is constant, it does not depend on applied $\vec{E}$ field, hence R stays constant and then $i \propto V$.

Example

What is the mean free time τ between collisions for conduction electrons in Copper?
Since

$$\tau = \frac{m}{ne^2\rho}$$

For Copper:

$$n = 8.49 \times 10^{28}\ \mathrm{m}^{-3}$$

$$\rho = 1.69 \times 10^{-8}\ \Omega \cdot m$$

$$m = 9.1 \times 10^{-31}\ \mathrm{kg}, \quad e = 1.6 \times 10^{-19}\ \mathrm{C}$$

$$\tau = \frac{9.1 \times 10^{-31}}{(8.49 \times 10^{28})(1.6 \times 10^{-19})^2(1.69 \times 10^{-8})} = 2.5 \times 10^{-14}\ s$$

Power in Electric Circuits

- When current flows through any device and there is a voltage across that device, then the device converts electrical energy into other forms of energy.
- We saw in Physics-I that power is defined as

$$P = \frac{\text{Energy}}{\text{time}} = \frac{\text{Joules}}{s}$$

The unit is $\frac{\text{J}}{s} = \text{watts}$

Power in Electric Circuits

$$U = qV \Rightarrow P = \frac{U}{t} = \frac{qV}{t} = iV$$

$$\boxed{P = iV}$$

i is current through a device V voltage across the device.
As

$$V = iR$$

$$\boxed{P = i^2 R}$$

R is the value for resistance and i is current through resistor. This equation gives us power or energy dissipation in the resistor. This means that a resistors convert electrical energy into thermal energy (heat)device.

Power in Electric Circuits

- For example: In Incandescent light bulbs, filament gets so hot, it glows white.
- Filaments are usually made from tungsten, it has very high melting point.

Example

What will be the resistance of a 60 watt light bulb, if the voltage is 120 volts?

$$P = 60 \text{ watt}, \quad V = 120 \text{ volts} \Rightarrow i = \frac{P}{V} = \frac{1}{2}\ A$$

$$P = i^2 R \Rightarrow R = \frac{P}{i^2} = \frac{60}{(1/2)^2} = 240\ \Omega$$

$$\boxed{R = 240\ \Omega}$$

What if $P = 40$ watt

$$\boxed{R = 360\ \Omega}$$

Example

FYI: The bulb doesn't know what power it's supposed to be. It only knows its own resistance. So 60 watt in incandescent light bulb really means 240 Ω light bulb.

Now what if voltage is increased to 125 volts, then what will be the power?

$$V = 125 \text{ V} \Rightarrow I = \frac{125 \text{ V}}{240 \ \Omega} = 0.52 \text{ A}$$

$$P = IV = (0.52 \text{ A})(125) = 65 \text{ watts}$$

Example

- So if the voltage goes up a bit, the bulb will get brighter, or more commonly, if the voltage drops a bit, the bulb will get dimmer.
- As the bulb ages some of the tungsten evaporates off the bulb filament and A decreases, that means R increases. then from

$$P = \frac{V^2}{R}$$

  As R increases then P gets smaller.
- So incandescent bulbs slowly grow dimmer as they age.