CSC 321 Computer Graphics

Ray-Object Intersection

Ray-Object Intersection

- General approach
  - Represent ray in *parametric* form

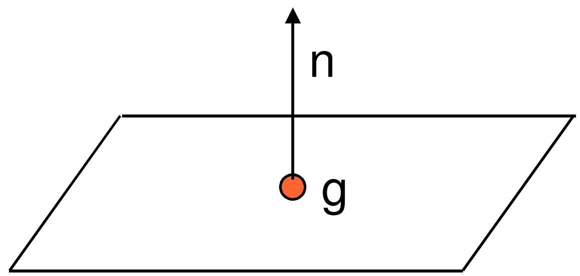
    q = P + t d
  - Represent surface in *implicit* form

    f[q] = 0
  - Substitute ray into surface, and solve for *t* (P, d are known)

    f[P + t d] = 0
  - Substitute *t* back into ray equation, find intersection point q
    - Use the *smallest positive t* (to find nearest intersection point)

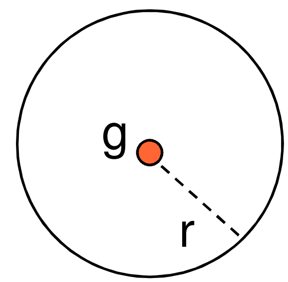
Implicit Functions

Plane



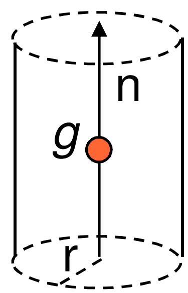
$$f[q] = (q - g) \cdot n$$

Sphere



$$f[q] = (q - g)^2 - r^2$$

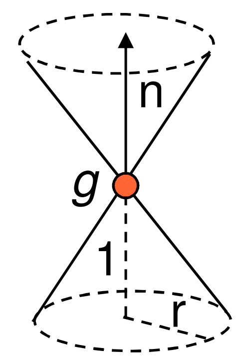
Cylinder (no cap)



$$v = q - g$$

$$f[q] = (v - (v \cdot n)\, n)^2 - r^2$$

Cone (no cap)



$$v = q - g$$

$$f[q] = (v - (v \cdot n)\, n)^2 - r^2\, (v \cdot n)^2$$

Example: Ray-Plane Intersects

Ray equation:
$\mathbf{q} = \mathbf{P} + \mathbf{t}\,\mathbf{d}$

- Plane definition

$$\mathbf{f}[\mathbf{q}] = (\mathbf{q} - \mathbf{g}) \cdot \mathbf{n} = 0$$

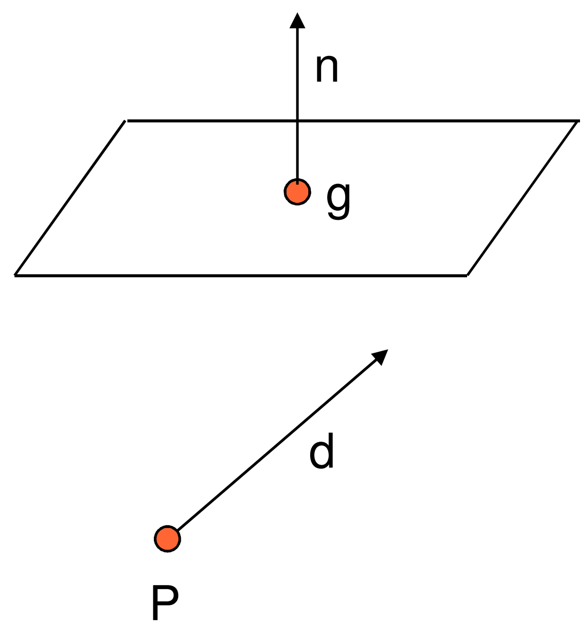
- Substituting ray into equation and solve

$$\mathbf{f}[\mathbf{P} + \mathbf{t}\,\mathbf{d}] = (\mathbf{P} + \mathbf{t}\,\mathbf{d} - \mathbf{g}) \cdot \mathbf{n} = 0$$

$$\mathbf{t} = \frac{(\mathbf{g} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$

- Substitute t back and find intersection

$$\mathbf{q} = \mathbf{P} + \mathbf{t}\,\mathbf{d} = \mathbf{P} + \frac{(\mathbf{g} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}\,\mathbf{d}$$



Example: Ray-Plane Intersects

Ray equation:
$q = P + t\,d$

- Plane definition

$$f[q] = (q - g) \cdot n = 0$$

- Substituting ray into equation and solve

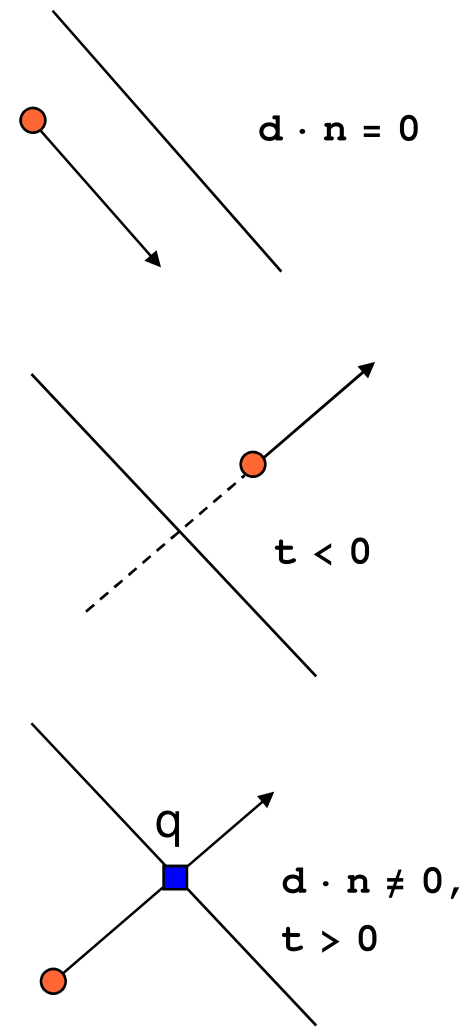
$$f[P + t\,d] = (P + t\,d - g) \cdot n = 0$$

$$t = \frac{(g - p) \cdot n}{d \cdot n}$$

- Substitute t back and find intersection

$$q = P + t\,d = P + \frac{(g - p) \cdot n}{d \cdot n}\,d$$

  – No intersection if
    - d.n=0 (Ray parallel to plane) or
    - t < 0 (behind the ray origin).



Example: Ray-Sphere Intersects

Ray equation:
$q = P + t\,d$

- Sphere definition

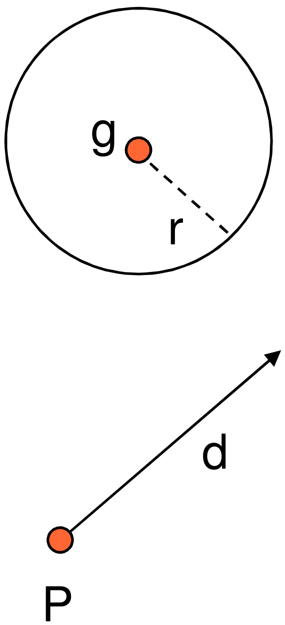
$f[q] = (q - g)^2 - r^2 = 0$

- Substituting ray into equation and solve

$f[P + t\,d] = (P + t\,d - g)^2 - r^2 = A\,t^2 + B\,t + C = 0$

where $A = d^2$, $B = 2\,d \cdot (P - g)$, $C = (P - g)^2 - r^2$

$$t_1 = \frac{-B - \sqrt{B^2 - 4\,A\,C}}{2\,A},\quad t_2 = \frac{-B + \sqrt{B^2 - 4\,A\,C}}{2\,A}\qquad (t_1 \le t_2)$$



Example: Ray-Sphere Intersects

Ray equation:
$q = P + t\,d$

- Sphere definition

  $f[q] = (q - g)^2 - r^2 = 0$

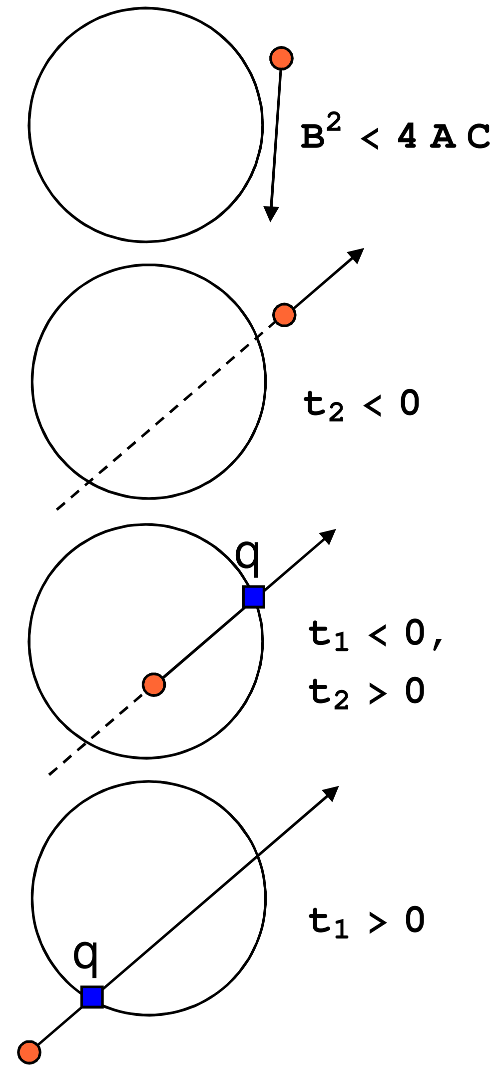
- Substituting ray into equation and solve

  $f[P + t\,d] = (P + t\,d - g)^2 - r^2 = A\,t^2 + B\,t + C = 0$

  where $A = d^2$, $B = 2\,d \cdot (P - g)$, $C = (P - g)^2 - r^2$

  $$t_1 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, \quad t_2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad (t_1 \le t_2)$$

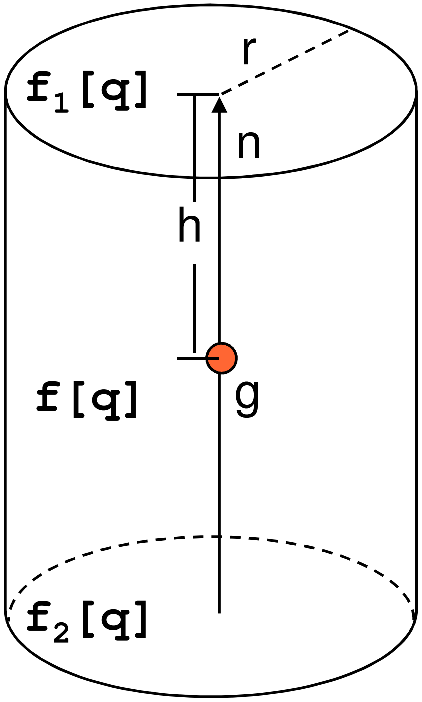
- Find Intersection
  - No intersection if $B^2 < 4AC$ or $t_2 < 0$
  - If $t_1 < 0$ and $t_2 > 0$: $q = P + t_2\,d$
  - If $t_1 > 0$: $q = P + t_1\,d$



Constructive Solid Geometry (CSG)

- Boolean results of basic shapes
  - Example: capped cylinder = cylinder ∩ 2 half-spaces
  - Defined by multiple implicit functions
    - Side: $\mathbf{f[q]} = ((\mathbf{q} - \mathbf{g}) - ((\mathbf{q} - \mathbf{g}) \cdot \mathbf{n})\,\mathbf{n})^2 - \mathbf{r}^2$
    - Top: $\mathbf{f_1[q]} = (\mathbf{q} - \mathbf{g} - \mathbf{h}\,\mathbf{n}) \cdot \mathbf{n}$
    - Bottom: $\mathbf{f_2[q]} = (\mathbf{q} - \mathbf{g} + \mathbf{h}\,\mathbf{n}) \cdot \mathbf{n}$
  - Point q lies inside or on the capped cylinder if

$$\mathbf{f[q]} \le 0 \ \&\& \ \mathbf{f_1[q]} \le 0 \ \&\& \ \mathbf{f_2[q]} \ge 0$$



Constructive Solid Geometry (CSG)

- Finding intersections with CSG
  - Intersect with each composing implicit function
  - Make sure the intersection lies ON the shape

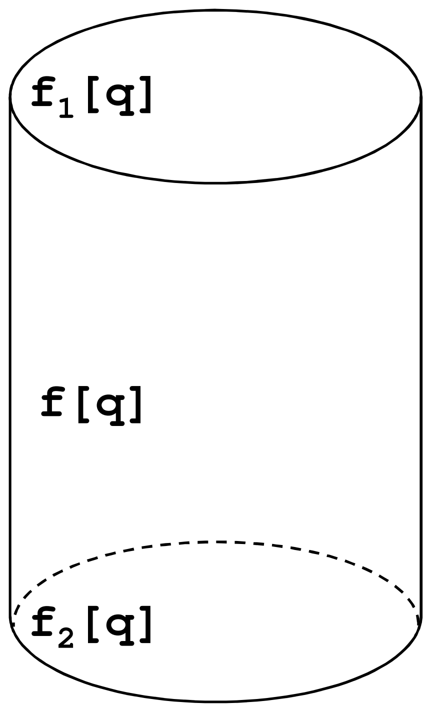
Constructive Solid Geometry (CSG)

- Example: a capped cylinder
  - A point **q** lies on the top plane if

    $f_1[q] = 0,\ f[q] \le 0$
  - A point **q** lies on the bottom plane if

    $f_2[q] = 0,\ f[q] \le 0$
  - A point **q** lies on the side if

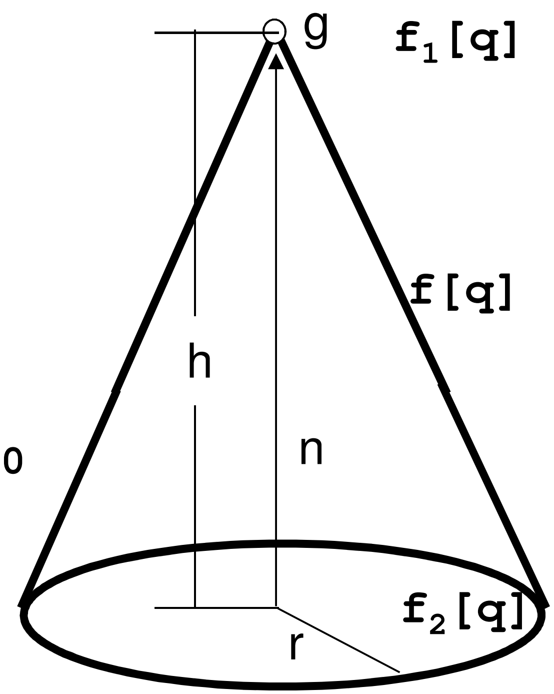
    $f[q] = 0,\ f_1[q] \le 0\ \&\&\ f_2[q] \ge 0$



Constructive Solid Geometry (CSG)

- Example: A capped cone
  - How many implicit functions define this shape?
    - Infinite cone: $\mathbf{f[q] = (v - (v \cdot n)\, n)^2 - r^2\, (v \cdot n)^2}$, **where** $\mathbf{v = q - g}$
    - Top plane: $\mathbf{f_1[q] = (q - g) \cdot n}$
    - Bottom plane: $\mathbf{f_2[q] = (q - g - h\, n) \cdot n}$
  - Point q lies inside or on the capped cone if

    $\mathbf{f[p] \le 0 \;\&\&\; f_1[p] \le 0 \;\&\&\; f_2[p] \ge 0}$
  - Point q lies
    - On the side if: $\mathbf{f[q] = 0,\; f_1[q] \le 0 \;\&\&\; f_2[q] \ge 0}$
    - On the base if: $\mathbf{f_2[q] = 0,\; f[q] \le 0}$



Triangles

- First test if ray intersects the plane
  - Triangle vertices $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$
  - Plane defined by point $\mathbf{g}_1$ and normal $\mathbf{n} = (\mathbf{g}_2 - \mathbf{g}_1) \times (\mathbf{g}_3 - \mathbf{g}_2)$
- Next test if the intersection lies in the triangle
  - Let intersection be q
  - If q lies inside, the triangles $\mathbf{q}\ \mathbf{g}_i\ \mathbf{g}_{i+1}$ have same orientation as the triangle $\mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3$

$$\mathbf{n} \cdot ((\mathbf{g}_{i+1} - \mathbf{g}_i) \times (\mathbf{q} - \mathbf{g}_{i+1})) > 0$$

$$i = 1, 2, 3$$