Physics - 2

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Pendulums

- The motion of pendulums are another example of simple harmonic motion (SHM).
- In pendulums, the restoring force is gravitational force, which tries to bring the pendulum to it's equilibrium position.
- Next, we will go over two examples, a simple pendulum and a physical pendulum. We will show that the motion of these pendulum is indeed a simple harmonic motion (SHM).

The Simple Pendulum

Let's consider, a particle of mass m (called bob) which is suspended from a massless string of length L, that is fixed at the other end. As shown in the figure below.

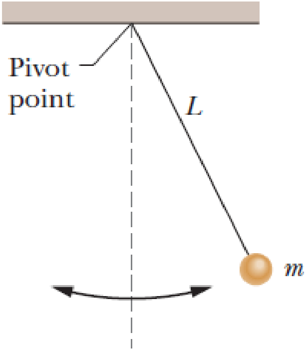

Figure 1: Figure Adopted from HRW

The Simple Pendulum

- We further assume that the string is unstretchable and the bob can swing freely in the plane of the page.
- There are two forces acting on the bob, the force of gravity and the tension in the string.
- Let's resolve the forces into it's components.

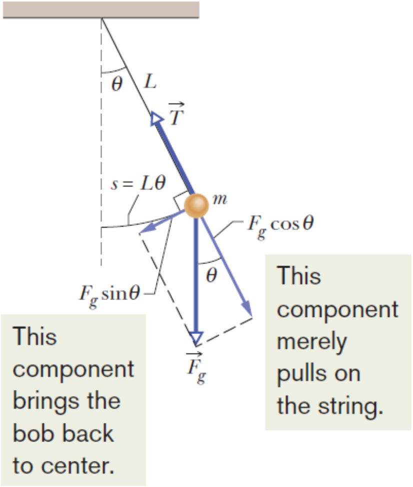

Figure 2: Figure Adopted from HRW

The Simple Pendulum

- The tangential component, $F_g sin\theta$ produces a restoring torque. Since

$$\tau = rFsin\theta$$

- In this case

$$\tau = -L(F_g sin\theta)\ sin90^0 = -LF_g sin\theta$$

- The minis sign means that torque acts to reduce θ

The Simple Pendulum

- Since torque is related to moment of inertia and angular acceleration by of the bob by

$$\tau = I\alpha$$

- Equating the two equation for torque

$$I\alpha = -L(F_g sin\theta) = -L(mgsin\theta)$$

- If we assume that the angle θ is very small, then

$$sin\theta \approx \theta \quad , when\ \theta\ is\ expressed\ in\ radians$$

The Simple Pendulum

- For small angles

$$I\alpha = -L(mg\ \theta)$$

- Rearranging

$$\boxed{\alpha = -\frac{mgL}{I}\theta}$$

- This is the definition of SHM. Acceleration is directly proportional and directed opposite to the displacement.

The Simple Pendulum

- We can find the angular frequency ω of the pendulum by comparing this equation with the general equation of the simple harmonic motion $(a = -\omega^2 x)$. By comparing, we find that

$$\omega = \sqrt{\frac{mgL}{I}}$$

- This is the angular frequency of the .simple pendulum. As $T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

- Since $I = ML^2$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

- This is the time period of a simple pendulum. Assuming the oscillation angle is very small.

The Physical Pendulum

- A physical pendulum can have a complicated mass distribution. As shown in the figure below

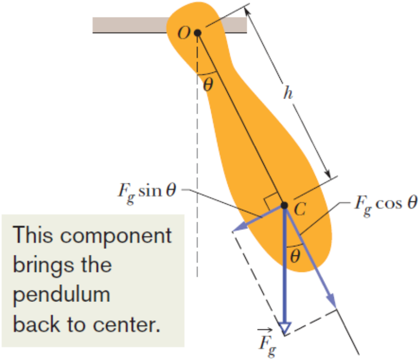

Figure 3: Figure Adopted from HRW

The Physical Pendulum

- Does a physical pendulum also undergo a SHM?
- If you pay attention to the two figures (Simple and physical pendulum), in both cases the restoring force $F_g sin\theta$ is bringing the pendulum to it's equilibrium position.
- The only difference is that in simple pendulum the component $F_g sin\theta$ act on the bob which is a distance L(length of the string) away from the pivot point. For physical pendulum, however this distance is h.
- h is the distance of from the pivot to the center of mass of the physical pendulum. Because this is where the component $F_g sin\theta$ will be acting.

The Physical Pendulum

- So, the only difference is we have to replace L by h for the physical pendulum.

$$\alpha = -\frac{mgh}{I}\theta$$

- The angular frequency will be

$$\omega = \sqrt{\frac{mgh}{I}}$$

- Time period

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgh}}}$$

- Note: We cannot simplify the expression for the time period more than this, because the moment of inertia will depend on the shape of the physical pendulum.

Example

In figure below, a meter stick swings about a pivot point at one end, at distance h from the stick's center of mass.

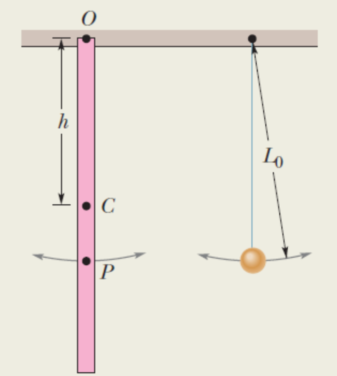

Figure 4: Figure Adopted from HRW

Example

(a) What is the period of oscillation T?

- Is this a simple pendulum or a physical pendulum ?
- For physical pendulum, the period is

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgh}}}$$

- Since for uniform rod $I = 1/3mL^2$ and $h = \frac{L}{2}$

Example

- Putting these values

$$T = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

- Since $L = 1m$ and $g = 9.8\frac{m}{s^2}$

$$\boxed{T = 1.64\ s}$$

Example

(b) What is the distance L_0 between the pivot point O of the stick and the center of oscillation of the stick?

- in other words, if we were to replace the physical pendulum with a simple pendulum wth the same time period, what would be the required length?

$$2\pi\sqrt{\frac{L_0}{g}} = 2\pi\sqrt{\frac{2L}{3g}}$$

$$\boxed{L_0 = \frac{2}{3}L = 66.7\ cm}$$