Physics - 2

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Waves

A disturbance that transfer energy from one point to another through a medium or vacuum without the transfer of the matter itself, is called a wave. There are three main types of waves

- **Mechanical Waves:** These waves require a material medium for it's propagation. For example water waves and sound waves.
- **Electromagnetic Waves:** These waves does not require any material medium for their propagation. For example the visible light, radio waves, x-rays, and microwaves.
- **Matter Waves:** These waves are associated with the matter itself. For example electron waves, proton waves and other fundamental particles.

Based on propagation, waves are of two types.

- **Transverse Waves:** In these waves the direction of propagation and the direction of the oscillation are perpendicular to each other. For example waves produced in a string, as shown in the figure below

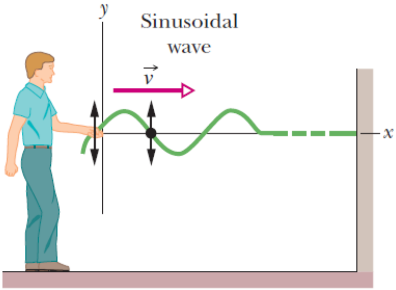

Figure 1: Figure adopted from HRW

- **Longitudinal Waves:** In these waves the direction of propegation and the direction of oscillation are parallel to each other. For example sound waves, as shown in the figure below

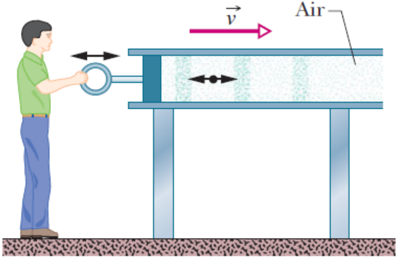

Figure 2: Figure adopted from HRW

Transverse Waves

From now on moving forward we will focus on the string waves.

- From the figure of waves in the sting, we can see that it's shape is sinusoidal. We can see that the displacement is a function of position (x) and time (t). So the general form of the waves on the string is

$$y(x,t) = y_m sin(kx - \omega t)$$

- This equation can tell us the shape of the wave at any given time.

Transverse Waves

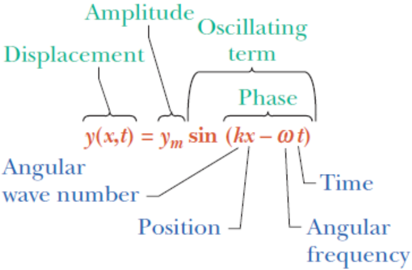

Figure 3: Figure adopted from HRW

Transverse Waves

Amplitude: The amplitude of the wave is the magnitude of the maximum displacement from the equilibrium position. In the equation it is represented by y_m.

Phase: The phase of the wave is $kx - \omega t$. As the wave sweeps through a string element at a particular position x, the phase changes linearly with time t.

Transverse Waves

Wavelength: The distance between two consecutive same point of a wave. it is denoted by λ and it's unit is m.

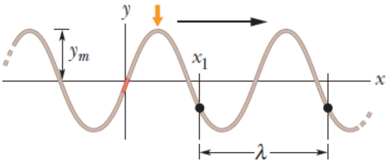

Figure 4: Figure adopted from HRW

Wave number: The number of waves per unit length. It is represented by k and it's unit is rad/m^{-1} (keep in mind this is not the spring constant)

Relation between wavelength and wave number:

- Let say the figure above represents a snapshot of the wave at $t = 0$.

$$y(x,0) = y_m sin(kx)$$

- The two points x_1 and $x_1 + \lambda$ represent the same displacement. So

$$y_m sinkx_1 = y_m sink(x_1 + \lambda) = y_m sin(kx_1 + k\lambda)$$

- Since, sine values repeat itself after every 2π *rad* then we can write the above equation as

$$y_m sinkx_1 = y_m sink(x_1 + \lambda) = y_m sin(kx_1 + 2\pi)$$

Transverse Waves

- From the above comparison

$$k\lambda = 2\pi$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

Period, Angular Frequency, and Frequency:

- If we were to observe one point on a string, then the wave equation for that point will become

$$y(0,t) = y_m sin(-\omega t) = -y_m sin\,\omega t$$

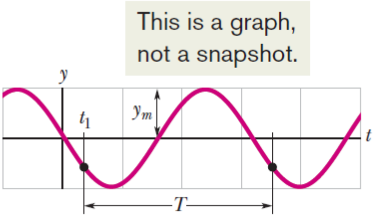

Figure 5: Figure adopted from HRW

Transverse Waves

- From the above figure

$$-y_m sin\omega t = -y_m sin\omega(t_1 + T) = -y_m sin(\omega t_1 + \omega T)$$

$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T}, \qquad AngularFrequency$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}, \qquad Frequency$$

Phase Constant: Phase constant give us information about the starting point of the wave. In the wave equation that we already have, the phase constant is zero. So, Generally

$$y = y_m sin(kx - \omega t + \phi)$$

Depending on the value of ϕ, the wave can start from anywhere along the

y-axis.

- The following figure shows a snapshot of a wave at two different times separated by Δt

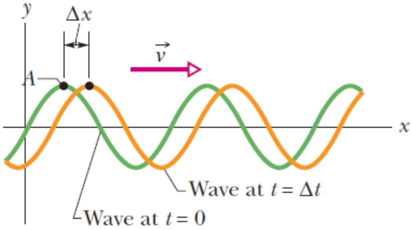

Figure 6: Adopted figure from HRW

- The entire wave moves a distance Δx in time duration Δt. This means that the ratio $\frac{\Delta x}{\Delta t}$ is the speed of the wave.

The Speed of a Traveling Wave

- Let suppose a point (say point A) on the wave form. As the wave move, the displacement of the point A remains constant, it does not change. The from the wave equation

$$y(x,t) = y_m sin(kx - \omega t)$$

- We can conclude that the phase is constant. Hence

$$kx - \omega t = constant$$

The Speed of a Traveling Wave

- Taking time derivative on both sides.

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}$$

$$\boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f}$$

- We can see that the wave speed is one wavelength per period. This means the wave travel a distance of one wavelength in time duration of one time period.

The Speed of a Traveling Wave

- If the wave is traveling in the negative x-direction, then

$$kx + \omega t = constant$$

- From this equation we can see that x decreases as time passes.
- This means that a wave traveling in the negative x-direction is defined by a wave equation

$$y(x,t) = y_m sin(kx + \omega t)$$

- The wave speed will be

$$\frac{dx}{dt} = -\frac{\omega}{k}$$

Example

A transverse wave traveling along an x axis has the form given by

$$y = y_m sin(kx \pm \omega t + \phi)$$

Figure 1 (on left side) gives the displacements of string elements as a function of x, all at time $t = 0$. Figure 2 (on right side) gives the displacements of the element at $x = 0$ as a function of t.

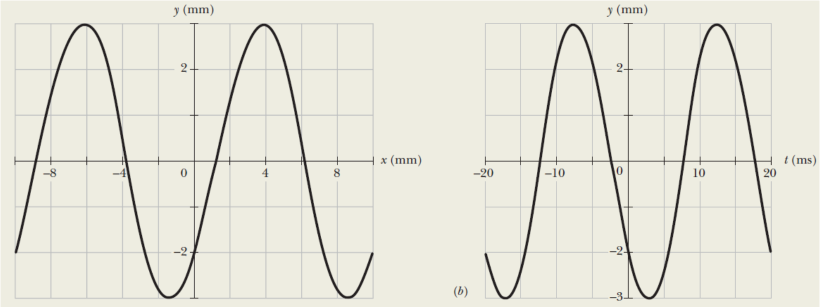

Figure 7: Figure adopted from HRW

Example

Find the values of the quantities shown in the equation above, including the correct choice of sign.

Amplitude: We can see from the figure, the amplitude is

$$y_m = 3.0\ mm$$

Wavelength: We can find the wavelength from the $t = 0$ graph. Wavelength is the distance between two same consecutive points.Pay attention to the cycle that is starting from 0 and going to 10 mm on the x-axis. So

$$\lambda = 10\ mm$$

Example

Period: We can find the period from the graph on the right side when $x = 0$. As, period the time required for one cycle, lets pay attention to the cycle starting at 0 and going upto 20 ms. Hence

$$T = 20\ ms$$

Direction of Travel: Since the left side graph is at $t = 0$ and the right side is over some time duration (x-axis is time). If the wave is traveling to the right, then the depth of the curve should increase as the wave proceed in time. If we look at the figure on the right side, this is what we see. After $t = 0$, as we proceed on the x-axis the depth increases. Hence the wave is moving in the positive x-direction.

Example

<u>**Phase constant:**</u> From both figures we can see that at $t = 0$ and $x = 0$, the wave equation become

$$-2\ mm = (3\ mm)\sin(0 - 0 + \phi)$$

$$\phi = sin^{-1}(-2/3) = -0.73 rad$$

<u>**Equation:**</u>

$$y = (3.0\ mm) sin(200\pi x - 100\pi t - 0.73\ rad)$$

Interference of Waves

The Principle of Superposition for Waves: Overlapping waves algebraically add to produce a resultant wave (or net wave).

- Suppose two waves are traveling simultaneously along the same stretched string. As shown in the figure below.

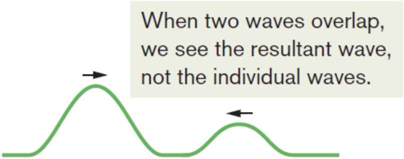

Figure 8: Figure adopted from HRW

- Let suppose that the displacement of one is $y_1(x, t)$ and the displacement of the other wave is $y_2(x, t)$.

- Then the superposition principle is

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

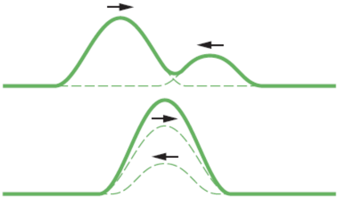

Figure 9: Figure adopted from HRW

Interference of Waves

- Overlapping waves do not in any way alter the travel of each other.

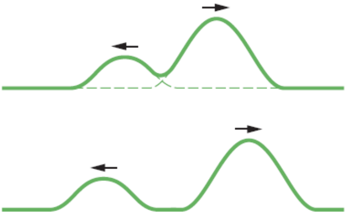

Figure 10: Figure adopted from HRW

- It means that after they pass each other, they keep their direction of travel, their speed and displacement. They don't change each other.

Interference of Waves: A phenomenon or pattern that results when two (or more) waves superpose or combine with each other.

- Suppose we send two sinusoidal waves of the same wavelength and amplitude in the same direction along a stretched string.
- If the waves are exactly in phase (so that the peaks and valleys of one are exactly aligned with those of the other), they combine to double the displacement of either wave acting alone.
- If they are exactly out of phase (the peaks of one are exactly aligned with the valleys of the other), they combine to cancel everywhere, and the string remains straight.

Interference of Waves

- Let suppose the one traveling wave is

$$y_1(x, t) = y_m sin(kx - \omega t)$$

- The other wave is

$$y_2(x, t) = y_m sin(kx - \omega t + \phi)$$

- These waves are travelling in the same direction, they have the same frequency, wavelength and amplitude. They only differ a phase constant ϕ.
- Equivalently we can say that the two waves have a phase difference of ϕ.

- From the superposition principle

$$y^{'}(x,t) = y_1(x,t) + y_2(x,t)$$

$$y^{'}(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

- As

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$

- Then

$$\boxed{y'(x,t) = \left[2y_m \cos\tfrac{1}{2}\phi\right]\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)}$$

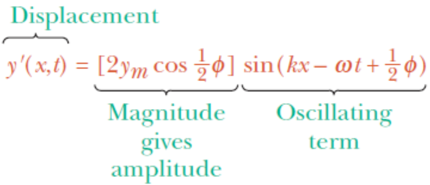

Figure 11: Figure adopted from HRW

The resultant wave differs from the interfering waves in two ways.

1. It's phase constant is $\frac{1}{2}\phi$
2. It's amplitude is $2y_m cos\frac{1}{2}\phi$.

- If $\phi = 0\ rad$, this means the two waves are in phase or we can say there is no phase difference. Then the resultant wave will be

$$y'(x,t) = \left[2y_m \cos\left(\tfrac{1}{2}0\right)\right] \sin\left(kx - \omega t + \tfrac{1}{2}(0)\right)$$

$$\boxed{y'(x,t) = 2y_m \sin(kx - \omega t)}$$

- This means the amplitude became double. This kind of interference is called **Constructive Interference**.

Interference of Waves

- If $\phi = \pi$ *rad*, then we say that the two waves are out of phase. The resultant wave will become.

$$y'(x,t) = \left[2y_m \cos\left(\tfrac{1}{2}\pi\right)\right] \sin\left(kx - \omega t + \tfrac{1}{2}(\pi)\right)$$

- Since $cos(\pi/2) = 0$

$$y^{'}(x,t) = 0$$

- This means that the amplitude become zero. This kind of interference is called **Destructive Interference**.

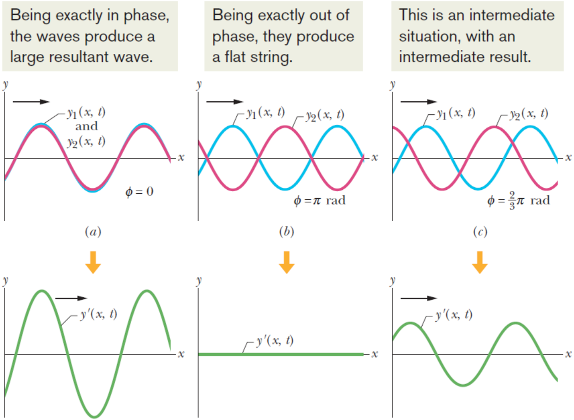

Figure 12: Figure adopted from HRW

Example

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is $9.8mm$, and the phase difference f between them is 100^o.

Example

(a) What is the amplitude $y_m^{'}$ of the resultant wave due to the interference, and what is the type of this interference?

- Identical waves means they have the same amplitude. So the resultant will become

$$y_m^{'} = 2y_m(cos\frac{1}{2}\phi) = (2)(9.8\ mm)cos(100^0/2)$$

$$\boxed{y_m^{'} = 13\ mm}$$

- From the amplitude we can conclude that the interference is intermediate.

Example

(b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of $4.9mm$?

- Now the resultant amplitude is given and we have to find the ϕ. Since

$$y_m' = 2y_m(cos\frac{1}{2}\phi)$$

$$4.9\ mm = (2)(9.8)(cos\frac{1}{2}\phi)$$

$$\phi = 2cos^{-1}(\frac{4.9}{(20)(9.8)})$$

$$\boxed{\phi = \pm 2.6\ rad}$$