Physics - 2

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Standing Waves

If two sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a standing wave.

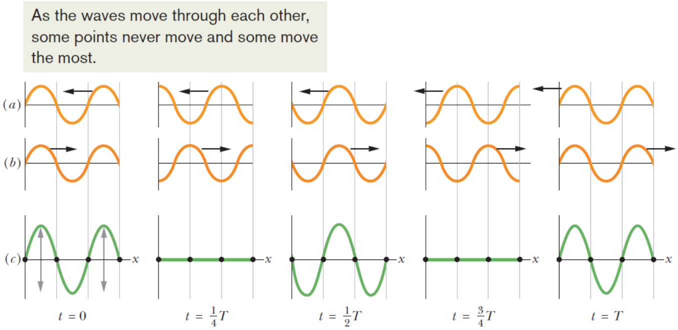


Figure 1: Figure adopted from HRW

Standing Waves

- The places along the string where the amplitude is always zero are called nodes.
- The places along the string where the amplitude is maximum are called antinodes.

Standing Waves

- To analyze a standing wave, let suppose that the two waves are

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

and

$$y_2(x,t) = y_m \sin(kx + \omega t)$$

- From superposition principle

$$y(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

$$\boxed{y'(x,t) = [2y_m \sin(kx)] \cos(\omega t)}$$

- This equation does not describe a traveling wave (because it does not have the term $(kx - \omega t)$). It describe a standing wave. The term $[2y_m sin(kx)]$ represent the amplitude.

Standing Waves

- The amplitude is zero when $sin(kx)$ is zero. So

$$sin(kx) = 0$$

$$kx = n\pi, \qquad for\ n = 0, 1, 2, ...$$

- Putting $k = 2\pi/\lambda$

$$x = n\frac{\lambda}{2} \quad ; \quad for\ n = 0, 1, 2, ...$$

- These are the position of zero amplitude. It shows that the adjacent nodes are separated by $\frac{\lambda}{2}$.

Standing Waves

- The maximum value of the amplitude is $2y_m$. This value occur when $sin(kx) = 1$. So

$$kx = \frac{1}{2}\pi,\ \frac{3}{2}\pi,\ \frac{5}{2}\pi, ...$$

or

$$kx = (n + \frac{1}{2})\pi, \quad for\ n = 0, 1, 2....$$

- Substituting $k = \frac{2\pi}{\lambda}$ and rearranging

$$x = (n + \frac{1}{2})\frac{\lambda}{2}, \qquad for\ n = 0, 1, 2, ...$$

- The antinodes are separated by $\frac{\lambda}{2}$

Standing Waves

Reflections at a Boundary: There are two kind of reflections at a boundary. **Hard Reflection** and **Soft Reflection**

- **Hard Reflection** is when a string wave the incident wave and reflected wave are out of phase. That is if the incident wave is a crest (peak) the reflected wave is a trough (valley). This happens because the one end of the string is fixed, it cannot move. A node is created at the fixed point.
- **Soft Reflection** is when the incident wave and reflected wave are in phase. That is if the incident wave is a crest (peak) the reflected wave is also a crest (peak). This happens because the reflection end is loose, it can move up and down. An antinode is created at the reflection point.

Standing Waves

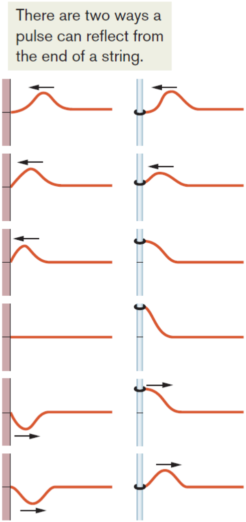
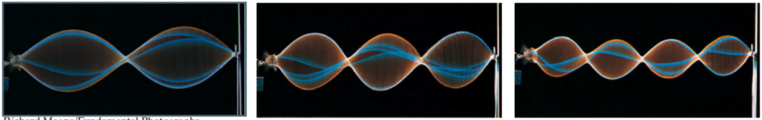


Figure 2: Figure adopted from HRW

Standing waves

- Let suppose we send a wave in a string which is fixed at the other end. When the wave reflect from the other end, then along the string the incident wave and the reflected can interfere with each other and will produce **Oscillation Modes**, as shown in the figure.



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- The oscillation frequencies with which these oscillation modes occur are called **resonant frequencies**.
- Not every frequency is resonant frequency.

Standing Waves

- Let's find an expression for the resonant frequencies.
- Let a string be stretched between two clamps(fixed) separated by a fixed distance L. Since the clamps are fixed, these positions must be nodes.
- Now, the simplest pattern that meet these requirements is shown in the figure below.

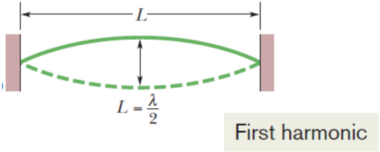

Figure 3: Figure adopted from HRW

Standing Waves

- From the above figure, we can see that

$$L = \frac{\lambda}{2}$$

or

$$\lambda = 2L$$

- The next pattern that we can get is shown below

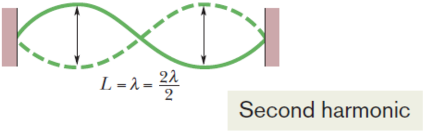

Figure 4: Figure adopted from HRW

- From the figure

$$\lambda = L = \frac{2L}{2}$$

Standing Waves

- The next pattern is

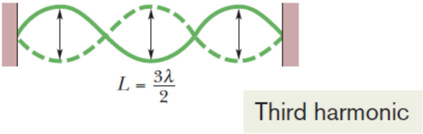

Figure 5: Figure adopted from HRW

- From the figure, we can see

$$\lambda = \frac{2L}{3}$$

- Looking at these equations, we can write a generalized expression

$$\boxed{\lambda = \frac{2L}{n}} \quad for \;\; n = 1, 2, 3....$$

Standing Wave

- Since

$$f = \frac{v}{\lambda}$$

- Then

$$\boxed{f = n\frac{v}{2L}} \qquad for \;\; n = 1, 2, 3.....$$

- These are the **Harmonic** or **Resonant Frequencies**.
- The n is called harmonic number.
- This means that the pattern of the standing waves will only appear if the harmonic frequency is an integral multiple of $\frac{v}{2L}$.

Example

Figure below shows resonant oscillation of a string of mass $m = 2.500g$ and length $L = 0.800m$ and that is under tension $T = 325.0N$. What is the wavelength λ of the transverse waves producing the standing wave pattern, and what is the harmonic number n? What is the frequency f of the transverse waves and of the oscillations of the moving string elements?

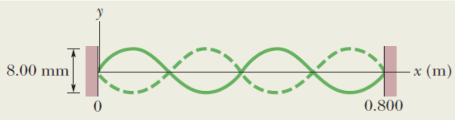

Figure 6: Figure Adopted from HRW

Example

Harmonic Number and Wavelength: By counting the number of loops in the figure, we can see that

$$\boxed{n = 4}$$

Since

$$\lambda = \frac{2L}{n} \tag{1}$$

$$\boxed{\lambda = \frac{2(0.8)}{4} = 0.400m}$$

Example

Frequency: We can find frequency from $v = f\lambda$, but first we need the speed of the wave.

- We can get the speed of the wave from the formula $v = \sqrt{\frac{T}{\mu}}$, where $\mu = \frac{m}{L}$ is the linear mass density of the sting, and T is the tension in the string.

$$v = \sqrt{\frac{TL}{m}} = \sqrt{\frac{(325)(0.8)}{2.5 \times 10^{-3}}} = 322.49\ m/s$$

- Now that we have the speed of the wave, we can find the frequency

$$f = \frac{v}{\lambda} = \frac{322.49}{0.400} = 806.2\ Hz$$

- Note that we can find the same answer from $f = n\frac{v}{2L}$

$$f = 4\frac{322.49}{2(0.8)} = 806\ Hz$$