MAT 7-221 Linear Algebra
Quiz 5 March 20, 2015

_____Solutions_________________

Name

This quiz is closed books and closed notes. No calculators or other devices allowed.

1. (4 pts.) Given the set is a basis for a subspace W, use the Gram-Schmidt process to produce an orthogonal basis of W: {(3,-4,5), (-3,14,-7)}

This is problem 4 section 6.4

4. Set $\mathbf{v}_1 = \mathbf{x}_1$ and compute that $\mathbf{v}_2 = \mathbf{x}_2 - \frac{\mathbf{x}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = \mathbf{x}_2 - (-2)\mathbf{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$. Thus an orthogonal basis for W is $\left\{ \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} \right\}$.

2. (4 pts.) Let W be the subspace spanned by the **u**'s, and write **y** as the sum of a vector in W and a vector orthogonal to W.

y = (-1,4,3), **u1** = (1,1,1), **u2** = (-1,3,-2)

This is problem 8, 6.3

8. Since $\mathbf{u}_1 \cdot \mathbf{u}_2 = -1 + 3 - 2 = 0$, $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set. By the Orthogonal Decomposition Theorem,

$$\hat{\mathbf{y}} = \frac{\mathbf{y} \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y} \cdot \mathbf{u}_2}{\mathbf{u}_2 \cdot \mathbf{u}_2}\mathbf{u}_2 = 2\mathbf{u}_1 + \frac{1}{2}\mathbf{u}_2 = \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix}, \mathbf{z} = \mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$$

and $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$, where $\hat{\mathbf{y}}$ is in W and $\mathbf{z}$ is in $W^{\perp}$.