MAT 3-119 Calculus of a Single Variable I

Exam 1 November 4, 2015

solution

name

For credit on these problems, you must show your work. Don't approximate your answers unless directed to do so. Graphing calculators are allowed. 100 pts. possible

1. (8 pts.)Tell whether the following functions are even, odd, or neither.
   a. 2sin 4x

   odd

   b. f'(x) where f(x) = 3x + 4. $f'(x) = 3$ even

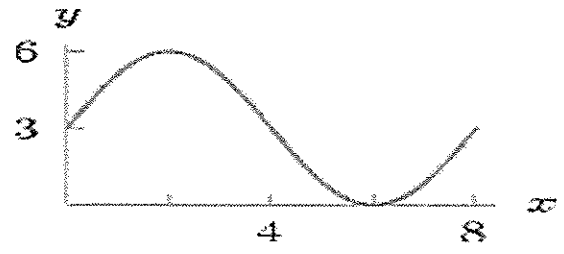
   c. $\frac{1}{x} + \tan x$ odd

   d. 2^x neither

2. (4 pts.) Let g be the function given by the rule: $g(x) = \sqrt{9 - x^2}$ Find the natural domain of g. Explain.

   $[-3, 3]$ can't take square root of neg. number.

3. (5 pts.) Give a formula for the following trigonometric function.

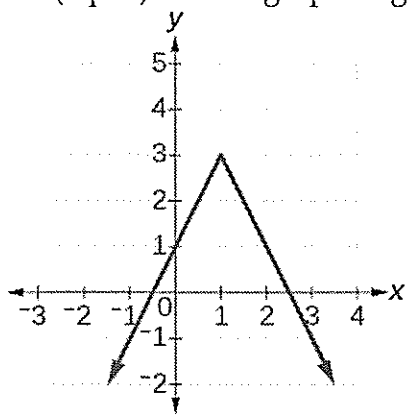


amp, period, vert sh.

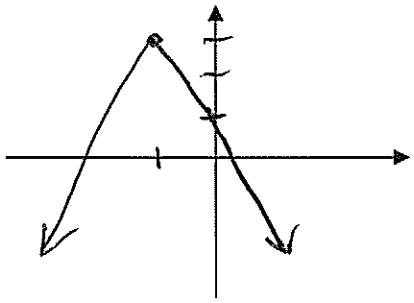
$3 \sin \frac{\pi}{4} x + 3$

$\frac{2\pi}{k} = 8 \quad k = \frac{\pi}{4}$

4. (7 pts.) Let the graph of g(x) pictured below:

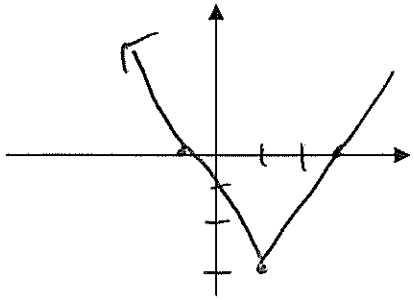


2 a. Sketch the graph of g(-x)



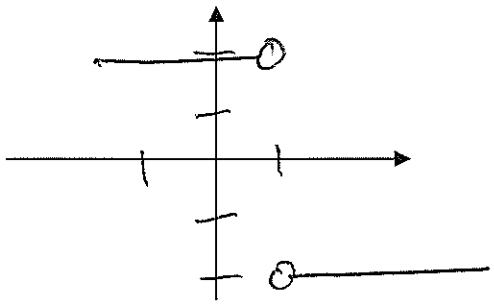
refl about y-axis

2 b. Sketch the graph of -g(x)



refl about x-axis

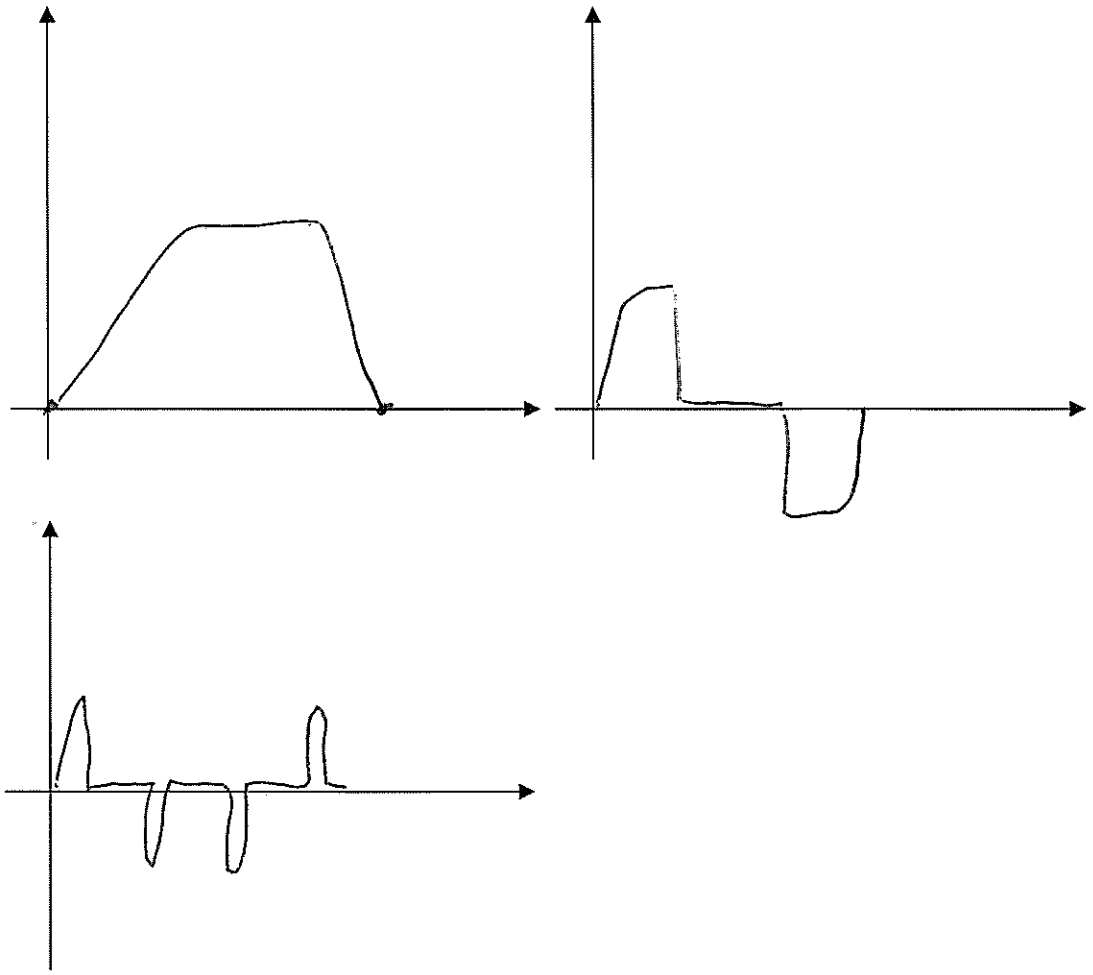
3 c. Sketch the graph of g'(x)



$\begin{cases} 2 & x<1 \\ -2 & x>1 \end{cases}$

5. (4 pts.) Fill in the blanks. We have learned that, given a function f(x), its derivative f'(x) may be viewed as the rate of change of the function f(x) or as the slope of the graph of f(x).

6. (6 pts.) Suppose that an object can move along a straight line to the right (positive) or to the left (negative). On separate graphs below, sketch plausible graphs of position vs. time, velocity vs. time, and acceleration vs. time if the object start at the origin, first moves steadily to the right, stops for a short time, then moves steadily to the left until it gets back to the origin.



7. (4 pts.) The table below contains data about the average annual concentration in parts per million of carbon dioxide in the atmosphere as observed at the Mauna Loa Observatory. Use this data to estimate the rate at which the concentration of carbon dioxide in the atmosphere was increasing (in ppm/yr) in 2013. Show your work.

Year	2011	2012	2013	2014
Concentration	391.63	393.82	396.48	398.55

$2.07 = \frac{398.55 - 396.48}{1}$ or $2.365 = \frac{398.55 - 393.82}{2}$

or $2.66 = \frac{396.48 - 393.82}{1}$

8. (8 pts.) The equations of motion of a ball shot straight up are given by $h(t) = 6 + 64t - 16t^2$ and $h'(t) = v(t) = 64 - 32t$, where height is in feet and time is in seconds.
   a. What is the initial height of the ball (include units)?

   6 ft.

   b. What is the initial velocity of the ball (include units)?

   64 ft/sec

   c. At what time does the ball reach its maximum height? What is the height of the ball at that time? Justify your answer with calculus techniques.

when $v(t)=0$. $64-32t=0$ $t=2$.

$h(2)=6+64\cdot 2-16\cdot 4 = 70$ ft.

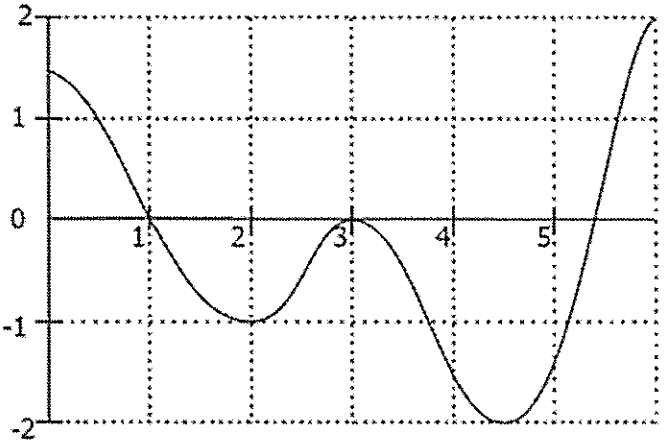
9. (4 pts.) Let f be a function such that $f(0) = 1$ and $f'(x) \leq 4$ for all x. What can be said about the value of f(3)? Justify your answer.

$f(3) \leq 1 + 3\cdot 4 = 1 + 12 = 13$

10. (3 pts.) Suppose that C(T) is the cost to heat a house, in dollars per day, when the average outside temperature is T degrees. Interpret the statement $C'(23) = -0.17$ in this context.

When the outside temperature is 23 deg, for every 1 deg increase in temp the cost of heating goes down by 17 cents.

11. (18 pts.) The graph of the DERIVATIVE of a function f is shown below for the interval (0,6). Use this graph to answer questions about f. Briefly justify ALL of your answers. Approximate answers are appropriate.



a. On which intervals, if any, is f increasing?

$(0,1) \cup (5.3,6)$ or 5.4 $\quad f' > 0$

b. At which values of x, if any, does f have a stationary point?

1, 3, 5.3 or 5.4 $\quad f' = 0$

c. At which values of x, if any, does f have a local maximum point?

1 $\quad + 0 -$: 1st der test

d. Where does f have points of inflection?

2, 3, 4.5 $\quad$ change f' fr. ↑ to ↓ or ↓ to ↑

e. Is f concave up at x=1?

no, conc ↓ $\quad f'$ ↓

f. Suppose that f(2) = 3. Find an equation of the line tangent to f at x = 2.

pt: (2,3) $\quad y = mx + b$

m: −1 $\quad 3 = -1 \cdot 2 + b$

$b = 5$

$y = -x + 5$

12. (8 pts.) The derivative of a function f is given by the rule $f'(x) = x^2 - 1$. Use this definition to answer questions about f. Briefly justify ALL of your answers.

a. On which intervals, if any, is f decreasing?

$(-1, 1)$ $\quad f' < 0$

b. At which values of x, if any, does f have a stationary point?

$-1, +1 \quad f' = 0$

c. At which values of x, if any, does f have a local minimum point?

1st der test : $- \; 0 \; +$ (1)

d. At which values of x, if any, does f have a terrace point?

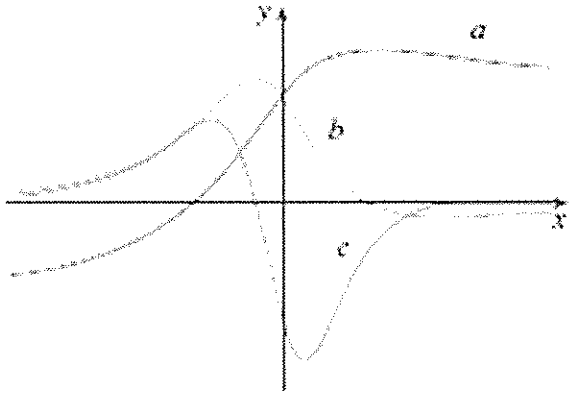
st. pt $+ \; 0 \; +$ $\quad - \; 0 \; -$ none

13. (5 pts.) At the right is the graph of three functions. One is f(x), one is f'(x), and one is f''(x). Fill in the blanks with which is which. Briefly justify your answers.

f(x) is the graph labeled a

f'(x) is the graph labeled b

f''(x) is the graph labeled c



c = 0 at b loc max

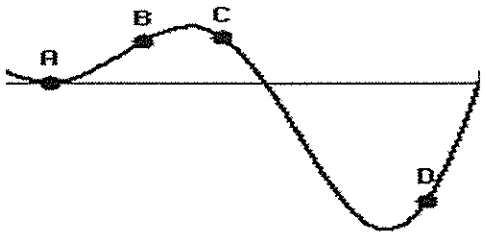
$+ \; 0 \; -$

b = 0 at a loc max

14. (3 pts.) Suppose f(x) is a function such that f''(1) = -2. What, if anything, does this imply about f?

f is conc ↓ at 1.

15. (8 pts) Given the graph of a function at labeled points answer the following questions. Briefly justify your answers.



a. At which point(s) is f' positive?

B, D f↑

b. At which point(s) is f'' positive?

A, D conc ↑

c. One of these points is a stationary point of f. Which is it?

A der = 0 loc min

d. One of these points is an inflection point of f. Which is it?

B changes concd to conc↑

16. (5 pts.) Suppose that $h'(w) = \sqrt{w} - 3$

a. The function h has a stationary point at 9. Is it a local maximum, a local minimum, or neither? Justify.

− 0 + loc min by f.d.t.

b. Suppose h(4) = 2. Find the equation of the tanget line to h at 4.

pt: (4, 2)

m: $\sqrt{4} - 3 = 2 - 3 = -1$

$y = mx + b$

$2 = -1 \cdot 4 + b$

$b = 6$

$y = -x + 6$