MAT 5-119 Calculus of a Single Variable I

Exam 2 November 9, 2015

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name

Read the problems carefully—on most problems you must either justify your answers and/or show your work. Don't approximate your answers unless directed to do so. Graphing calculators are allowed. 75 points possible.

1. (8 pts.) Use the definition of derivative as limit of a difference quotient to find f′(x) where $f(x) = \frac{3}{x}$, for $x \neq 0$.

2. (5 pts.) Let $f(x) = 5x^3$, $g(x) = \frac{3}{\sqrt{x}}$, and $h(x) = 7\pi$.

a. Find f′(x).

b. Find g′(x).

c. Find h′(x).

3. (5 pts.) Let $g(x) = 3\sqrt{x}$. Find the equation of the tangent line to the graph of g at x = 9.

4. (6 pts.) Let f be the function whose second derivative f'' is given by the rule: $f''(x) = (x+1)^2(x-2)$

a. Where, if anywhere is f concave up?

b. Does f have any inflection points? If so, where are they located?

5. (6 pts.) Give an antiderivative of the following functions:

a. $f(x) = 10x^4 + 8x^3 + 6x^2 + 4x + 7$

b. $g(x) = \sqrt[3]{x}$

c. $h(x) = \dfrac{1}{x^3}$

6. (6 pts.) True or false? If true, explain why. If false, give a counterexample.

a. If the limit exists at point x=a, then a must be in the domain of the function.

b. If the limit of a function exists at point x=a, then the function is continuous at a.

c. The limit of a sum equals the sum of the limits.

7. (9 pts.) Let $f(t) = -8t^9 + 9t^8 - 7$. Use calculus techniques to answer questions about f; give exact values. Briefly justify ALL of your answers.

a. On which intervals, if any, is f increasing?

b. At which values of t, if any, does f have a stationary point?

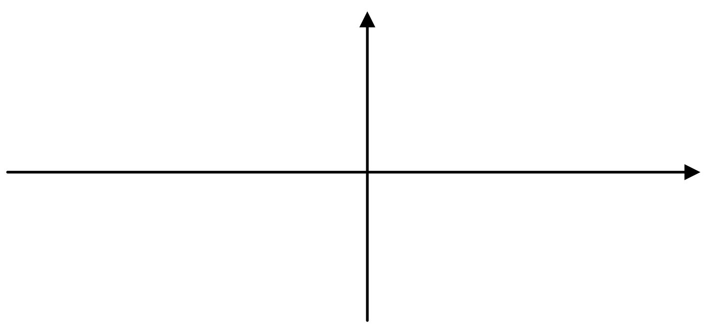
c. At which values of t, if any, does f have a local maximum point?

8. (6 pts.) Made to order M-shaped function. Use calculus techniques to find a function rule for a function which has local maximums at $x = -1$ and $x = 1$ and a local minimum at $x = 0$. Show your work.

9. (6 pts.) Solve the initial value problem $y' = 4x + 3$, $y(0) = 2$.

10. (8 pts.) An open box with capacity 36000 cubic inches is to be twice as long as it is wide. The material for the box costs $0.10 per square foot. What are the dimensions of the least expensive box?

11. (10 pts.) Let f be the function given by the graph below; consider f only on the domain [-3,3]. Answer the following questions or find the following limits (or say they don't exist.)



a. $\lim_{x \to -1^-} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to -1} f(x)$

d. Where is f continuous? Briefly justify your answer.

e. Where is f differentiable? Briefly justify your answer.