MAT 3-119 Calculus of a Single Variable I

Exam 2 November 11, 2015

Solution
name

Read the problems carefully—on most problems you must either justify your answers and/or show your work. Don't approximate your answers unless directed to do so. Graphing calculators are allowed. No other devices. YOU MAY NOT SHARE CALCULATORS.
100 points possible.

1. (8 pts.) Use the definition of derivative as limit of a difference quotient to find f′(x) where $f(x) = \frac{1}{x}$, for $x \neq 0$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x-(x+h)}{(x+h)x}}{h} = \lim_{h\to 0} \frac{\frac{-h}{(x+h)x}}{h}$$

$$= \lim_{h\to 0} \frac{-1}{x^2+xh} = \frac{-1}{x^2}$$

2. (9 pts.) Let $f(x) = 4x^3$, $g(x) = \frac{3}{\sqrt{x}}$, and $h(x) = \pi^2$.

a. Find f′(x). $12x^2$

b. Find g′(x). $g = 3 \cdot x^{-1/2}$ $g' = \frac{-3}{2} x^{-3/2} = \frac{-3}{2} \cdot \frac{1}{\sqrt{x^3}}$

c. Find h′(x). 0

3. (6 pts.) Let $g(x) = \sqrt{x}$. Find the equation of the tangent line to the graph of g at x = 4.

$g = x^{1/2}$ $g' = \frac{1}{2x^{1/2}}$

pt: $(4, 2)$

m: $\frac{1}{2 \cdot 2} = \frac{1}{4}$

$2 = \frac{1}{4} \cdot 4 + b$

$b = 1$

$y = \frac{1}{4}x + 1$

4. (6 pts.) Let f be the function whose second derivative f'' is given by the rule: $f''(x) = (x+2)^2(x-1)$

a. Where, if anywhere is f concave up? Briefly explain.

f conc↑ where $f'' > 0$. $x > 1$

b. Does f have any inflection points? If so, where are they located? Briefly explain.

f infl pt where f'' changes sign at 1

5. (9 pts.) Give an antiderivative of the following functions:

a. $f(x) = 10x^4 + 8x^3 + 7$

$$F = \frac{10x^5}{5} + \frac{8x^4}{4} + 7x = 2x^5 + 2x^4 + 7x$$

b. $g(x) = \sqrt[3]{x}$

$$g = x^{1/3} \quad G = \frac{x^{4/3}}{4/3} = \frac{3x^{4/3}}{4}$$

c. $h(x) = \dfrac{1}{x^3}$

$$h = x^{-3} \quad H = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$$

6. (6 pts.) True or false? No need to justify your answer.

____F____ a. If the limit of a function exists at point x=a, then a must be in the domain of the function.

____F____ b. If the limit of a function exists at point x=a, then the function is continuous at a.

____T____ c. If f and g are continuous functions for all real numbers, then so is the function f + g.

7. (9 pts.) Let $f(t) = 8t^9 - 9t^8 + 2$. Use calculus techniques to answer questions about f; give exact values. Briefly justify ALL of your answers.

a. On which intervals, if any, is f increasing?

$f' = 72t^8 - 72t^7 = 72t^7(t-1)$ $f'>0$ when $(-\infty,0)\cup(1,\infty)$

b. At which values of t, if any, does f have a stationary point?

$f'=0$ when $t=0$ or $t=1$

c. At which values of t, if any, does f have a local minimum point?

$t=1$ is a loc. min. $-0+$

8. (6 pts.) Made to order M-shaped function. Use calculus techniques to find a function rule for a function which has local maximums at x = -2 and x = 2 and a local minimum at x = 0. Show your work.

$y' = (x-2)x(x+2)$

$= x^3-4x$

$y = \frac{x^4}{4} - \frac{4x^2}{2}$

this is W so

$-\frac{x^4}{4} + \frac{4x^2}{2}$

9. (6 pts)Let $f(x) = \begin{cases} ax+1 & if\ x<2 \\ x^2 & if\ x \geq 2 \end{cases}$

a. Find a value of **a** which makes f a continuous function at x = 2.

4

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$ax+1=4$ $2a=3$

$a = 3/2$

b. If **a** has the value found in part a., does f′(2) exist? Justify your answer.

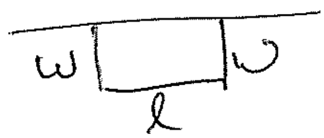
2

$f'(2)$ left $3/2$

no

$f'(2)$ right $2 \cdot 2 = 4$

10. (8 pts.) A farmer has 100 feet of fence and wishes to make a rectangular garden. She plans to put the garden next to a barn, so one side won't need a fence. What dimensions give the largest garden, in square feet. Show your work, using calculus techniques.



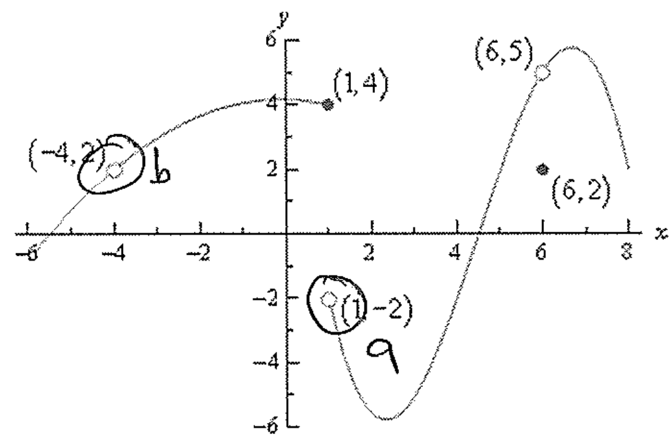
$100 = 2w + l$

$A(w) = l \cdot w = (100 - 2w) \cdot w$

$= 100w - 2w^2$

$A' = 0$ when $100 - 4w = 0 \quad w = \frac{50}{2} = 25 \quad l = 50$

11. (10 pts.) Let f be the function given by the graph to the right; consider f only on the domain [-6,8]. Answer the following questions or find the following limits (or say they don't exist.)



a. $\lim_{x \to 1^+} f(x)$ -2

b. $\lim_{x \to -4} f(x)$ 2

c. d. Is f continuous at x = 1? Briefly justify your answer.

No. $\lim_{x \to 1} f$ DNE

d. Is f continuous at x = 6? Briefly justify your answer.

No $\lim_{x \to 6} f(x) = 5 \neq f(6) = 2$

12. (6 pts.) Give the differential equation or initial value problem that models the following scenarios. Do not solve.

a. A flu epidemic spreads through a college community of size 1200 at a rate proportional to the product of the number of members already infected and the number of those not infected.

$$I' = kI(1200 - I)$$

b. A fungus grows at a rate proportional to its current size. Suppose that the fungus weighs 12 grams at time t = 0.

$F' = kF \qquad F(0) = 12$

13. (6 pts.) Solve the initial value problem y′ = 6x + 1, y(0) = 5.

$y = \frac{6x^2}{2} + \cancel{x} + x + C = 3x^2 + x + C$

$5 = y(0) = 3 \cdot 0 + 0 + C \qquad y = 3x^2 + x + 5$

14. (5 pts.) Is y(x) = $\sqrt{x}$ a solution of y′(x) = $\frac{1}{2y(x)}$? Justify your answer.

LHS: $y' = \frac{1}{2\sqrt{x}}$

RHS: $\frac{1}{2y} = \frac{1}{2\sqrt{x}}$

$) =$ so yes.