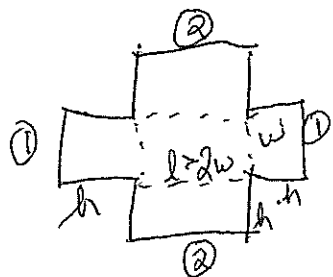Sample exam solution to problem 10

10. (8 pts.) An open box with capacity 36000 cubic inches is to be twice as long as it is wide. The material for the box costs $0.10 per square foot. What are the dimensions of the least expensive box?

$$l \cdot w \cdot h = 36\,000$$

$$h = \frac{36000}{lw} = \frac{36000}{2w^2} = \frac{18000}{w^2}$$

Surface area : $2 \cdot wh + 2 \cdot l \cdot h + l \cdot w$

$$S(w) = 2 \cdot w \cdot \frac{18000}{w^2} + 2 \cdot 2w \cdot \frac{18000}{w^2} + 2w \cdot w$$

$$= \frac{36000}{w} + \frac{72000}{w} + 2w^2 = \frac{108000}{w} + 2w^2$$

$$S'(w) = \frac{-108,000}{w^2} + 4w$$

$$S'(w) = 0 \text{ when } 4w = \frac{108000}{w^2}$$

$$w^3 = 27,000$$

$$\boxed{w = 30}$$

$$l = 60$$

$$h = 20$$