MAT 4-120 Calculus of a Single Variable II

Sample Exam 1 December 2, 2015

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name

Read the problems carefully—on most problems you must either justify your answers and/or show your work. Don't approximate your answers unless directed to do so. Graphing calculators are allowed.

1. (10 pts.) Evaluate the following limits or state that they don't exist. (Answer only.)

a. $\lim\limits_{x \to \infty} \dfrac{7x^3 - x^2}{x^4 + 1}$

b. $\lim\limits_{x \to \infty} \dfrac{6x^3 - x^2}{x^2 + 1}$

c. $\lim\limits_{x \to 1} \dfrac{5}{x-1}$

d. $\lim\limits_{x \to 0^-} \dfrac{\cos x}{x}$

e. $\lim\limits_{x \to 0} \cos(\sin x)$

2. (9 pts.) Evaluate the following limits or explain why they don't exist. Carefully show your work/justify your answer.

a. $\lim\limits_{x \to \infty} \dfrac{\ln x}{x}$

b. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\sin 2x}$

c. $\lim\limits_{x \to \infty} x \cdot 2^{-x}$

3. (12 pts.) Match the slope fields shown below with the differential equations from this list. Briefly justify each answer.

a. $y' = t + 1$ _______________

b. $y' = 1 - y$_______________

c. $y' = \cos y$_______________

d. $y' = \cos t$ _______________

e. $y' = \cos(t + y)$ _______________

f. $y' = y^2 - 1$ _______________

4. (8 pts) A spherical weather balloon is inflated with helium at the rate of 100π cubic feet per minute. How fast is the balloon's radius increasing at the time when the balloon's radius is 5 ft?

5. (8 pts.) Boyle's Law states that for a fixed quantity of gas at a constant temperature, the pressure P and the volume V are inversely related—i.e. PV = k. Suppose a certain quantity of gas occupies 10 cm^3. The pressure is increasing at a rate of 0.05 atmospheres/minute at a time when the pressure is 2 atmospheres, while keeping the temperature constant. Is the volume increasing or decreasing and by what rate?

6. (6 pts.) Sketch the graph of curve given by the parametric equations

$x(t) = 1 + 2 \sin t$

$y(t) = 2 + 2 \cos t$ for $0 \leq t \leq \pi$

Clearly label the initial point and the ending point.

7. (6 pts.) The motion of a particle is given by the parametric equations

$x(t) = t^3 - 3t$

$y(t) = t^2 - 2t$ for $-5 \leq t \leq 5$

Compute the expression for the speed of the particle. Does the particle ever come to a stop? Justify your answer.

8. (8 pts.) Find the point on the right half of the parabola $1-x^2$ that is closest to the origin, using calculus techniques. Give your answer to 3 digits of accuracy. Show all your work.

9. (8 pts.) An ordinary soft drink can is cylindrical and has a volume of 355 cubic centimeters. Use calculus techniques to find the dimensions (radius r and height h) that minimizes the surface area of such a can. Give your answer to 3 digits of accuracy. Show all your work.

10. (6 pts.) The Extreme Value Theorem states that a continuous function on a closed interval achieves a minimum and a maximum there. Show that the conditions of the theorem are required by giving examples that satisfy the following conditions. Your function example may be given by some kind of function rule or a function given by a graph. You must justify that your example satisfies the given conditions.

a. A continuous function defined on an open interval that does not achieve a maximum there.

b. A function, not continuous, defined on a closed interval that does not achieve a maximum there.

11. (10 pts.) a. Apply Newton's method to find the largest root of $f(x) = x^3 - 3x + 1$. Give an appropriate initial guess and one Newton iterate. Show your work.

b. Use your calculator to solve the associated equation and give your answer here.

12. (6 pts.) Find the linear Taylor polynomial for $f(x) = \ln x$ based at 1. Show your work.

13. (9 pts.) Find the quadratic Taylor polynomial for f(x) = cos x based at 0. Show your work.

14. (6 pts.) Explain in words/equations and draw a sketch to illustrate what the Mean Value Theorem says for the specific function $f(x) = x^3$ on the interval [-1,3].