

MAT4-120 Calculus of a Single Variable

Quiz 2 December 1, 2015

solution

name

You must show your work for full credit on this quiz.

1. (6 pts.) Find the following limits. Show your work.

a.  $\lim_{x \rightarrow 0} \frac{5x - \sin x}{x}$   $\frac{L'H}{\left(\frac{0}{0}\right)}$   $\lim_{x \rightarrow 0} \frac{5 - \cos x}{1} \stackrel{\text{cont.}}{=} 5 - \cos 0 = 4$

b.  $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 + 3}$   $\frac{L'H}{\left(\frac{\infty}{\infty}\right)}$   $\lim_{x \rightarrow \infty} \frac{2x}{4x} = \lim_{x \rightarrow \infty} \frac{2}{4} = \frac{1}{2}$

c.  $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$   
 not indeterminate  $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$   $\lim_{x \rightarrow \infty} -\frac{1}{x} = 0 = \lim_{x \rightarrow \infty} \frac{1}{x}$   
 Sandwich/squeeze play.

2. (6 pts.) What is the minimum value of the expression  $x^3 + y$  if  $x + y = 4$  and  $x \geq 0$ ?

constraint:  $x + y = 4$  &  $x \geq 0$   $y = 4 - x$

objective  $\theta(x) = x^3 + y = x^3 + (4 - x)$

$\theta'(x) = 3x^2 - 1$  st. pt:  $3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$  since  $x \geq 0$ .

$x = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

value:  $\left(\frac{\sqrt{3}}{3}\right)^3 + \left(4 - \frac{\sqrt{3}}{3}\right)$