MAT 4-120 Calculus of a Single Variable II

Exam 2 Part 2 December 11, 2015

solution

Name

This exam is closed book and closed notes. Don't approximate your answers unless directed to do so. Graphing calculators are allowed, but calculators with symbolic capabilities, like a TI-89 or TI-NSpire CAS may only be used for graphing or numerical calculations. 72 points possible on this part.

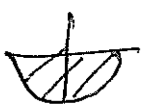
1. (3 pts.) Fill in the blanks. So far in our course we have considered three ways of looking at integrals. Integrals represent distance travelled, area, and anti-derivatives.

2. (15 pts) Find the following integrals exactly. **Show your work/justify your answers.**

a. $\int_0^1 e^x dx = e^x \Big|_0^1 = e^1 - e^0 = e - 1$

b. $\int_{-2}^{2} -\sqrt{4-x^2}\,dx$

negative area for semicircle of radius 2.



$-\frac{\pi \cdot 2^2}{2} = -2\pi$

c. $\int_0^{\pi/4} \cos 2x\,dx = \frac{\sin 2x}{2}\Big|_0^{\pi/4} = \frac{\sin \pi/2}{2} - \frac{\sin 0}{2} = \frac{1}{2}$

d. $\int_{-1}^{1} \tan x - 3dx = \int_{-1}^{1} -3dx = 2 \cdot -3 = -6$

odd fn so int. is 0

e. $\int \sqrt{x}\,(2-x)dx = \int 2x^{1/2} - x^{3/2} dx = \frac{2x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + C = \frac{4}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$

3. (5 pts.) The graphs of the functions $\frac{1}{x}$ and $\frac{1}{x^2}$ intersect at the point (1,1) and as x increases $\frac{1}{x}$ is slightly larger. Find the exact area under the graph of $\frac{1}{x}$ but above $\frac{1}{x^2}$ between x = 1 and x = 2.

$$\int_1^2 \frac{1}{x} - \frac{1}{x^2}\,dx = \ln|x| + \frac{1}{x}\Big|_1^2$$

$$= \ln 2 + \frac{1}{2} - (\ln 1 + 1)$$

$$= \ln 2 - \frac{1}{2}$$

4. (5 pts.) Suppose that f is a continuous function. If the average value of f over the interval [0,1] is 2 and the average value of f over the interval [1.3] is 4, what is the average value of f over the interval [0,3]?

$$\frac{\int_0^1 f}{1} = 2 \qquad \frac{\int_1^3 f}{2} = 4$$

$$\int_0^3 f = \int_0^1 f + \int_1^3 f = 2 + 8$$

$$\frac{\int_0^3 f}{3} = \frac{10}{3}$$

5. (6 pts.) The rate at which a coal plant releases CO2 into the atmosphere t days after 12:00 am on Jan 1, 2015 is given by the function E(t) measured in tons per day. Suppose $\int_0^{31} E(t)dt = 220$.

a. Give a practical interpretation of $\int_{31}^{59} E(t)dt$

This is the total release of CO2 by the plant in February.

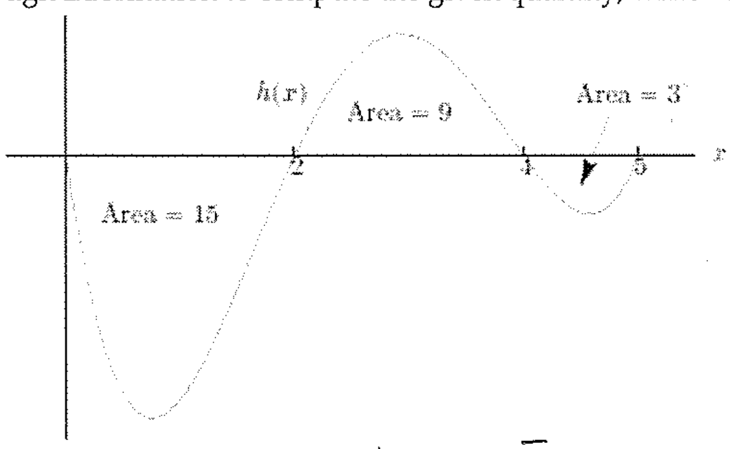
b. Give a practical interpretation of E(15) = 7.1.

The plant released 7.1 tons of CO2 on Jan 16.

c. The plant is upgrading to "clean coal" technology which will cause its July 2015 CO2 emissions to be one fourth of its January 2015 CO2 emissions. How much CO2 will the coal plant release into the atmosphere in July?

$\frac{1}{4} \cdot 220 = 55$ tons.

6. (9 pts.) Using the graph of h(x) shown below, compute each of the following quantities. If there is not enough information to compute the given quantity, write "not enough information".



a. $\int_0^2 h(t) - 3dt$ $= -15 + 2 \cdot 3 = -21$

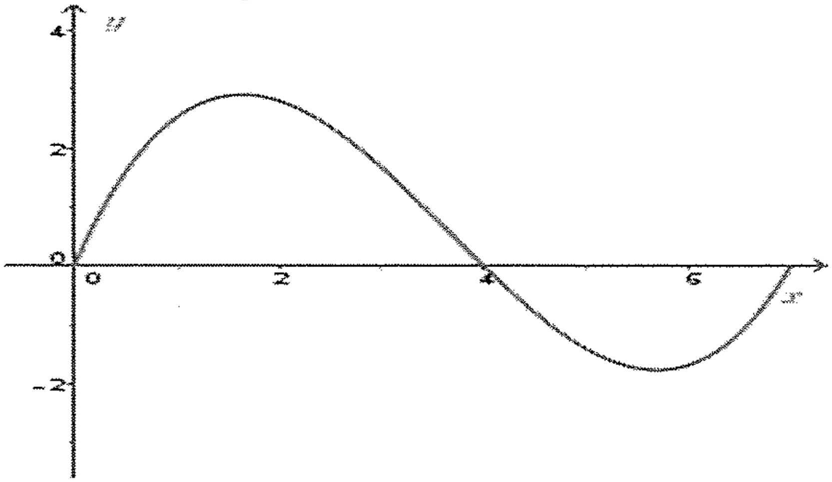
b. $\int_0^5 2h(x)dx$ $= 2\int_0^5 h(x)dx = 2(-15+9-3) = -18$

c. H(4) – H(0) where H is an anti-derivative of h.

FToC says $H(4) - H(0) = \int_0^4 h(x)dx = -15 + 9 = -6$

7. (10 pts.) Let f be given by the graph below defined on [0,7] and consider the area function

$$A_f(x) = \int_2^x f(t)dt$$



a. Which is larger $A_f(4)$ or $A_f(5)$? Justify your answer.

$A_f(4)$. $A_f(5)$ includes negative area.

b. Is $A_f(1)$ positive, negative, or zero? Justify your answer.

Negative. Integrate pos. fn. in neg. direction.

c. When is the derivative of $A_f(x) = 0$? Justify your answer.

$\frac{d}{dx}(A_f(x)) = f(x)$ (FToC) so 0, 4, 7.

d. Does $A_f(x)$ have any local mins or maxs in the interval (0,7)? If so, where? Justify your answer.

local max at 4. Adding pos. area changes to adding neg area. so $A_f(x)$ peaks.

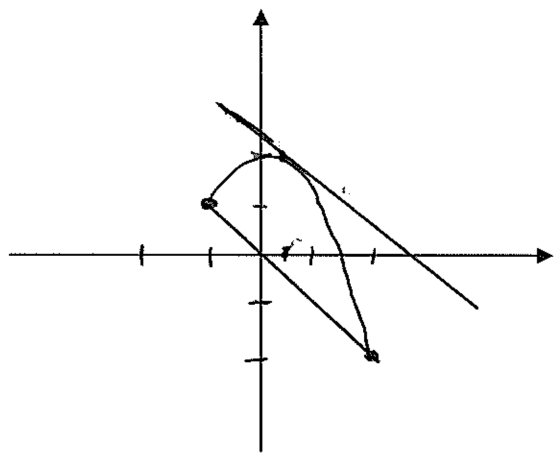
8. (3 pts.) The Extreme Value Theorem states that a continuous function on a closed interval achieves a minimum and a maximum there. Show that the conditions of the theorem are required by giving examples that satisfy the following conditions. Your function example may be given by some kind of function rule or a function given by a graph. You must justify that your example satisfies the given conditions.

A continuous function defined on an open interval that does not achieve a maximum there.

$f(x) = x$ on $(0,1)$.

It gets close to 1 near 1 but never achieves a max value.

9. (5 pts.) Explain in words/equations and draw a sketch to illustrate what the Mean Value Theorem says for the specific function $f(x) = 2 - x^2$ on the interval [-1,2].



There exists c s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(-1)}{2 - -1} = \frac{-2 - 1}{-3} = -1$$

$f'(x) = -2x$

$f'(c) = -1$ is $-2c = -1$
so $c = 1/2$.

10. (2 pt. each) True or false? No need to justify your answers.

T a. The integral of a sum is the sum of the integrals.

F b. The integral of a product is the product of the integrals.

T c. If a = b, then $\int_a^b f(x)dx = 0$.

F d. The units for a definite integral of a function f(x) are the same as the units for f(x).

T e. If $f(x) \leq g(x)$ on the interval [a,b], then the average value of f is less than or equal to the average value of g on the interval [a,b].

T f. If f is a differentiable on [a,b], then f is continuous on [a,b]