Examination 3

CSC140 Foundations of Computer Science

26 February 2016

1. Look on the Web. Find a quotation attributed to either Donald E. Knuth or Frederick P. Brooks, Jr. Select something that appeals to you. Look for wisdom or inspiration.

   What did this very accomplished computer scientist say?

2. Our simulation of the Web allows a reader of Web pages to choose the next page to read in which two ways?

3. This is a discrete probability distribution function (PDF).

$$pdf = \{0.15, 0.25, 0.18, 0.12, 0.30\}$$

   This is the corresponding discrete cumulative distribution function (CDF).

$$cdf = \{0.15, 0.40, 0.58, 0.70, 1.00\}$$

   (a) Name a characteristic of all discrete probability distribution functions.

   (b) Name a characteristic of all discrete cumulative distribution functions.

4. Here is a discrete probability distribution function (PDF).

$$pdf = \{0.24, 0.12, 0.04, 0.40, 0.20\}$$

   What is the corresponding cumulative probability discribution function (PDF)?

5. Here is a matrix like the one that we constructed for our simulation of the Web.

$$\mathbf{M} = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.1 & 0.2 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$$

What is the sum of the values in each row?

6. Here is a matrix like the one that we constructed for our simulation of the Web.

$$\mathbf{M} = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.1 & 0.2 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$$

What is the meaning of the value in the row 1, column 0? (That value is 0.7.)

7. Here is a vector. Each element represents that a probability that a reader who starts on page 0 will visit pages 0, 1, and 2 next.

$$\vec{v} = \begin{bmatrix} 0.1 & 0.4 & 0.5 \end{bmatrix}$$

The vector is the first row of this matrix.

$$\mathbf{M} = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.1 & 0.2 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$$

Here is the vector-matrix product:

$$\begin{aligned} \vec{v}\mathbf{M} &= \begin{bmatrix} (0.1 \cdot 0.1 + 0.4 \cdot 0.7 + 0.5 \cdot 0.3) \\ (0.1 \cdot 0.4 + 0.4 \cdot 0.1 + 0.5 \cdot 0.6) \\ (0.1 \cdot 0.5 + 0.4 \cdot 0.2 + 0.5 \cdot 0.1) \end{bmatrix}^T \\ &= \begin{bmatrix} 0.44 \\ 0.38 \\ 0.18 \end{bmatrix}^T \\ &= \begin{bmatrix} 0.44 & 0.38 & 0.18 \end{bmatrix} \end{aligned}$$

What is the significance of this product?

8. What would you expect to see in the output of this program?

```
private static final Random RNG = new Random();

private static int simulate( int [] pdf ) {
    double r = RNG.nextDouble();

    int index = 0;
    while( r > pdf[index] ) {
        index = index + 1;
    } // while

    return index;
} // simulate()

public static void main( String [] args ) {
    int [] pdf = { 0.2, 0.5, 0.3 };

    for( int i = 0; i < 100; i++ ) {
        System.out.println( simulate(pdf) );
    } // for
} // main( String [] )
```

9. This method is recursive. How do you know?

```
double sqrt( double square, double guess ) {
  if( Math.abs( square - guess * guess) > 1E-8 ) {
      return sqrt( square, (guess + square/guess)/2 );
  } // if
  else {
      return guess;
  } // else
} // sqrt( double, double )
```

10. Write a method that computes this function:

$$
\begin{aligned}
f(0) &= 1 \\
f(1) &= 1 \\
f(n) &= n \cdot f(n-1) \quad \text{for all } n > 1
\end{aligned}
$$

11. Here is one way to compute 2^{16}.

$$2^{16} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Here is another way.

$$\begin{aligned}
2^{16} &= 2^8 \cdot 2^8 \\
2^8 &= 2^4 \cdot 2^4 \\
2^4 &= 2^2 \cdot 2^2 \\
2^2 &= 2^1 \cdot 2^1 \\
2^1 &= 2
\end{aligned}$$

This example suggests a reason for sometimes choosing to use recursion to solve a problem. What is one reason for sometimes choosing recursion?

12.

$$\begin{aligned}
fib(0) &= 1 \\
fib(1) &= 1 \\
fib(n) &= fib(n-1) + fib(n-2) \quad \text{for all } n > 1 \\
fib(5) &= fib(4) + fib(3) \\
&= (fib(3) + fib(2)) + (fib(2) + fib(1)) \\
&= ((fib(2) + fib(1)) + fib(2)) + ((fib(1) + fib(0)) + 1) \\
&= (((fib(1) + fib(0)) + 1) + (fib(1) + fib(0))) + (1 + 1 + 1) \\
&= (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1) \\
&= 8
\end{aligned}$$

This example suggest a reason for sometimes choosing not to use recursion. What is one possible adverse consequence that can follow from a naive use of recursion?

13. Indicate where the following comments belong in the program by annotating the program with the letters that label each comment.

A hold onto the first item in the unsorted part of the array until we are ready to insert it into the right place in the sorted part of the array

B make a hole in the sorted part of the array into which the next item can be inserted

C find the position at which to insert the the first item in the unsorted part of the array into the sorted part of the array

D insert the next item into the hole

E shift elements to one place to the right

F conduct a right-to-left search

G move one element at a time from the (shrinking) unsorted part of the array into the right place in the (growing) sorted part of the array

```java
public class Insertion {

    public static int positionGE( int i, int [] data ) {
        int j = i;
        while( (j > 0) && (data[j - 1] > data[i]) ) {
            j = j - 1;
        } // while
        return j;
    } // positionGE( int, int [] )

    public static void insert( int i, int j, int [] data ) {
        int temp = data[i];

        for( int k = i; k > j; k-- ) {
            data[k] = data[k - 1];
        } // for

        data[j] = temp;
    } // insert( int, int, int [] )

    public static void sort( int [] data ) {
        for( int i = 1; i < data.length; i++ ) {
            int j = positionGE( i, data );
            insert( i, j, data );
        } // for
    } // sort( int [] )

    public static void main( String [] args ) {
        int [] data = { 5, 2, 3, 1, 4 };
        sort( data );
        for( int i = 0; i < data.length; i++ ) {
            System.out.println( data[i] );
        } // for
    } // main( String [] )
} // Insertion
```