CSC151 Discrete Mathematics for Computer Science
Sample Exam 1

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name

For credit on these problems, you must show your work. On this exam, take the natural numbers to be $\boldsymbol{N} = \{0,1,2,3, \ldots\}$. This exam is closed book and closed notes. Calculators are allowed.

1. 1. (6 pts.) State and prove one of DeMorgan's Laws for propositional logic, using a truth table.

2. (8 pts.) Give a deduction for the following hypothesis and conclusion, justifying each step. Use these predicates: F(x) : x works at Facebook, S(x): x has a smart phone, T(x) : x has a tablet, domain: All people.

Hypothesis: Everyone working at Facebook either has a smart phone or a tablet. Sarah, who works at Facebook, doesn't have a smart phone. Everyone who has a tablet plays WordsWithFriends. Conclusion: Sarah plays WordsWithFriends.

3. (8 pts.) Let L(x,y) be the predicate: "x likes y", where the domains are given by: x is a CS student and y is a kind of music. Let D(x) be the predicate: x is a student in this discrete class, where the domain is all CS students.

Express the following statements using those predicates and any required quantifiers; you may not use the "there exists unique" quantifier.

a. Every CS student likes some kind of music.

b. Every student in this discrete class likes reggae.

c. There is no CS student who likes every kind of music.

d. Express the negation of the logical expression in part **b** with the negation moved as far inside as possible.

4. (2 pts. each) Let P(m,n) be the predicate "m + n is even" where the domain for each is the set **N.**

a. What is the truth value of $\forall m\ \exists n\ (P(m,n))$? Justify.

b. What is the truth value of $\exists m\ \forall n\ (P(m,n))$? Justify.

c. What is the truth value of $\forall m\ \forall n\ ((m = n) \rightarrow P(m,n))$? Justify.

5. (10 pts.) Carefully prove the following:

a. $3n^2 + 4n + 5$ is $O(n^2)$.

b. For any sets A and B, $A \cap B \subseteq A \cup B$.

6. (6 pts.) Let B be the set of all bit strings and f: B --> $\mathbb{Z}$ be defined by the number of 0's in the bit string minus the numbers of 1's.

a. Is f 1-1? Briefly justify your answer

b. Is f onto? Briefly justify your answer.

7. (6 pts.) Let $g: N \rightarrow O$, where O is the set of odd natural numbers, be given by the rule $g(n) = 2n+1$.
   a. Prove or disprove g is 1-1.

   b. Prove or disprove g is onto.

8. (3 pts. each) For which one of the growth functions, g(n), is f(n) $\Theta(g(n))$? **Briefly justify your answers.** For this problem choose among the following growth functions g(n):

   n, n^2, n^3, $\log n$, $n \log n$, 1, 2^n.

a. Let f(n) represent the worst case complexity of selection sort for a list of length n.

b. $f(n) = n \log n + 5n + 1$

c. Let f(n) represent the number of steps to determine set membership for a number when a set is implemented using a bit string.

d. The function $f: Z^+ \rightarrow N$ given by the rule $f(n) = \sum_{k=1}^{n} k$

9. (9 pts.) a. Consider the arithmetic sequence that begins with the five numbers 26, 37, 48, 59, 70. What is the sum of the first 100 terms of this sequence (give a number)? At the start, express the sum using sigma notation.

b. Give a recurrence relation that produces the geometric sequence 4, 20, 100, 500, ...

10. (3 pts. each) Give an example of the following. Carefully justify that your example satisfies the given conditions. If no example exists, state this and briefly explain why.

a. A set A such that $|\mathbb{P}(A)| = 5$.

b. Functions f and g such that g is $\mathbf{O}(f(x))$ but f is not $\mathbf{O}(g(x))$.

c. A pair of sets A and B with $A \subset B$ and a function $f: A \rightarrow B$ that is 1-1 and onto.

11. (4 pts.) a. In 1891 Georg Cantor published his famous proof that the real numbers are uncountably infinite. What is the first sentence of his proof?

b. Suppose you are going to prove the theorem: If x^3 is irrational, then x is irrational. If you choose to prove this using an indirect proof, what is the first sentence of your proof? (Do not complete this proof.)

12. (2 pts. each) True or false—circle one. No need to justify your answers.

True False a. For any real numbers x and y, $\lceil x \rceil + \lceil y \rceil = \lceil x + y \rceil$

True False b. Given any two sets A and B, $|A \times B| = |B \times A|$.

True False c. x is $\mathbf{O}(x \log x)$.

True False d. If f(x) is $\mathbf{\Theta}(g(x))$ then f(x) is $\mathbf{O}(g(x))$.

True False e. For any set A, $\emptyset \in A$.

True False f. $\{\wedge, \neg\}$ is a logically complete set of logical operators.

True False g. The set of all rational numbers is countably infinite.