Sample questions from Old Discrete Exams
December 2013

1. (16 pts.) For each of the following relations, tell whether or not they are reflexive, symmetric, or transitive, by putting a T or F in the appropriate slot. No need to justify your answers.

a. The proper subset relation $\subset$ on $P(\{1,2,3\})$.

___________ Reflexive ___________ Symmetric ___________ Transitive ___________ Anti-symmetric

b. Let R be the relation on the set of all sets where A R B if and only if there exists a bijection from A to B.

___________ Reflexive ___________ Symmetric ___________ Transitive ___________ Anti-symmetric

c. Let R be the relation on the set of all logical propositions, where p R q if and only if p --> q.

___________ Reflexive ___________ Symmetric ___________ Transitive ___________ Anti-symmetric

d. The relation on {a,b,c} represented by the following graph:

a ▪ ▪ b

▪ c

___________ Reflexive ___________ Symmetric ___________ Transitive ___________ Anti-symmetric

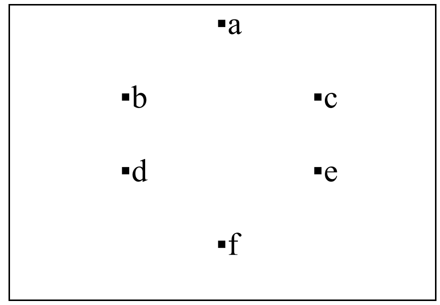
2. (9 pts.) Let R be the relation on {1,2,3} given by the following matrix: $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

a. Give the matrix that represents the reflexive closure of R.

b. Give the matrix that represents the symmetric closure of R.

c. Give the matrix that represents the transitive closure of R.

3. (20 points) Consider the following weighted graph:



a. Illustrate Dijkstra's shortest path algorithm to find the minimal cost path from a to f on this graph. Briefly explain how the algorithm works.

b. Choose either Prim's algorithm or Kruskall's algorithm and produce the least cost spanning tree for this graph. Briefly explain how your chosen algorithm works.

c. What is the computational complexity of Dijkstra's algorithm on a graph with n nodes (disregard edges)?

4. (8 pts.) a. Suppose G = {V,E} is a simple, (undirected), **connected** graph, and suppose |V| = 50. **Justify your answers to the following questions.**

a. What is the smallest possible number of edges in G?

b. What is the largest possible number of edges in G?

5. (10 pts.) Prove the following using the Principle of Mathematical Induction:

$1 + 3 + 5 + \ldots + (2n-1) = n^2$ for $n \geq 1$.

6. (8 pts) For this problem choose among the following growth functions g(n): n, n^2, n^3, log n, n log n, 1, 2^n, n!. For which one of the above growth functions, g(n), is $f(n) = \Theta(g(n))$?

a. Let f(n) represent the complexity of the standard algorithm to check if a relation, represented by an n by n matrix, is symmetric. Briefly justify your answer.

b. Let f(n) represent how many times the statement ExecuteThis is encountered in the following code, from the algorithm for exponentiation by repeated squaring:

```
result = 1
x = a
while (n > 0) {
        if (n mod 2 == 1)
                result = result * x
        x = x * x
        ExecuteThis;
        n = ⌊n/2⌋
        }
```

7. (10 pts.) Give a recursive definition of each of the following.

   a. Give a recursive specification of "c choose k"-- the binomial coefficient-- which we write as C(n,k) or ${}_nC_k$

b. The set/language of all bit strings that start with one or more 0's then finishes with one or more 1's. Example: 0000000000000011111 is in the language, 010101 is not.

8. (9 pts.) Consider the complete graph with 100 vertices, $K_{100.}$

   a. How many edges does this graph have? Briefly justify your answer.

   b. Does the graph have an Euler cycle? Briefly justify your answer.

   c. Does the graph have a Hamilton cycle? Briefly justify your answer.

On problems 9 through 11, you may leave your answers in combination, permutation, or power format.

9. (5 pts.) Currently on the Internet, some addressing of computers is handled by IPv4 addresses which are strings of binary digits (0 or 1) of length 32. One group of these addresses—the class B addresses--has the following format. Each class B address begins with 10. The next 14 bits are the net ID. The final 16 bits are the Host ID. Host IDs of all 0s or all 1s are not available for use. How many total Class B addresses are available for computers, ipods, toasters, etc? Show your work.

10. (3 pts each) Let A = (1,2,3,4,5,6,7,8} and B = {X,Y,Z} Show your work.

a. How many functions are there from A to B?

b. How many functions are there from B to A?

c. How many 1-1 functions are there from A to B?

a. How many 1-1 functions are there from A to A?

b. How many subsets of A contain the number 3?

c. How many subsets of A contain exactly 5 elements?

11. (3 pts. each) a. A committee of 8 students is to be selected from a class consisting of 10 sophomores and 18 juniors. In how many ways can 3 sophomores and 5 juniors be selected?

b. How many positive integers between 100 and 999 inclusive are divisible by 7?