MAT 121 Calculus of a Single Variable

Sample Exam 1 October 3, 2014

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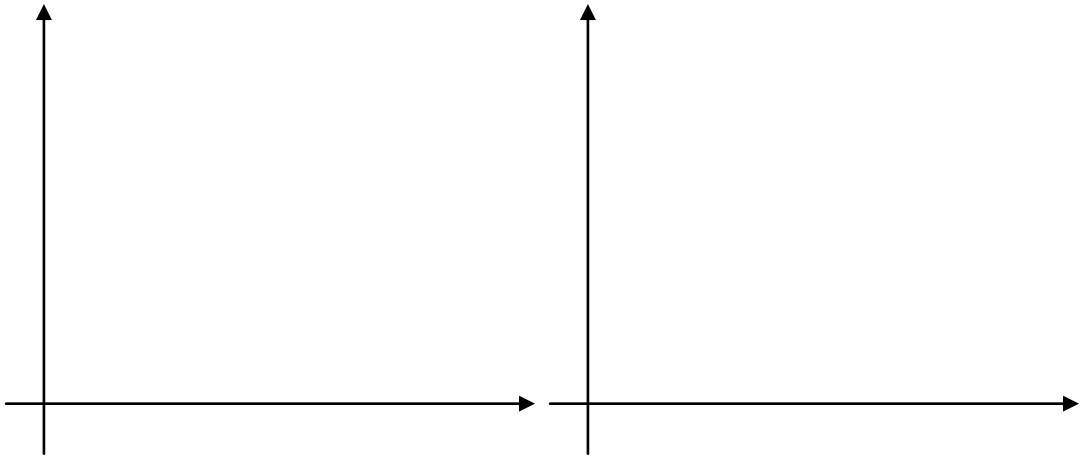
name

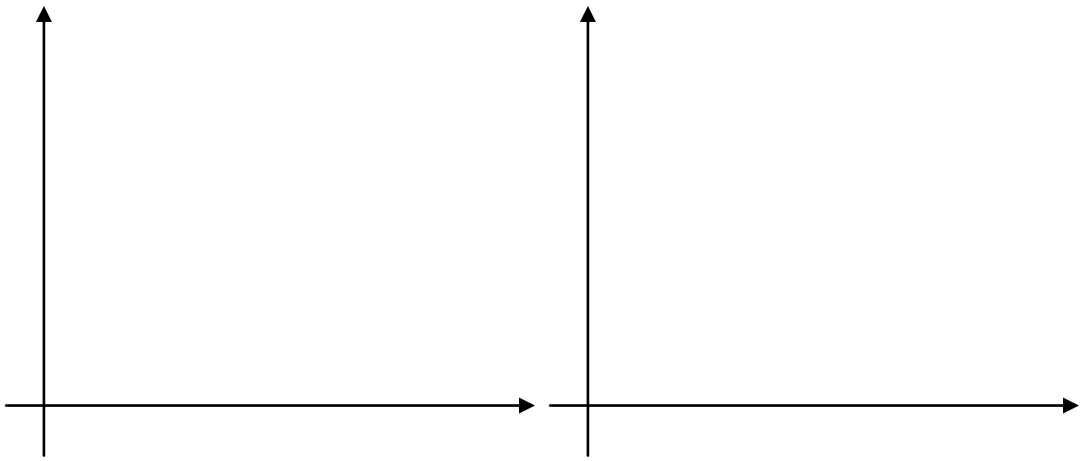
Read the problems carefully—on most problems you must either justify your answers and/or show your work. Don't approximate your answers unless directed to do so. Graphing calculators are allowed, but not calculators with symbolic capabilities, like the TI-89. 80 points possible.

1. (10 pts.) An airplane takes off from one airport, and 45 minutes later lands at another airport 300 miles away. Let t represent the time since the airplane took off, d(t) the horizontal distance traveled, and h(t) the altitude (height) of the plane.

a. What is the physical meaning of d'(t)? Sketch plausible graphs of d and d' (add axis labels and units.)



b. What is the physical meaning of h'(t)? Sketch plausible graphs of h and h' (add axis labels and units.)



2. (8 pts.) Use the definition of derivative as limit of a difference quotient to find f′(x) where f(x) = √x, for x > 0.

3. (5 pts.) The table below contains data about the average annual concentration in parts per million of carbon dioxide in the atmosphere. Us this data to estimate the rate at which the concentration of carbon dioxide in the atmosphere was increasing in 2000; include units in your answer.

Year	1998	1999	2000	2001
Conc.	356.9	358.8	360.9	362.7

4. (6 pts.) Let $f(x) = 5x^3 + \frac{3}{\sqrt{x}}$.

a. Find f′(x).

b. Find f′′(x).

5. (5 pts.) Let $g(x) = \sqrt{x} - 2/x$. Find the equation of the tangent line to the graph of g at x = 4.

6. (6 pts.) Let f be the function whose second derivative f'' is given by the rule: $f''(x) = e^{-x}$
   a. Where, if anywhere is f concave up?

   b. Does f have any inflection points? If so, where are they located?

7. (6 pts.) Suppose h is a function such that h(0) = 1, h(2) = 7, h(4) = 5, h'(0) = -2, h'(2) = 3 and h'(4) = -1.

a. What is the average rate of change of h over the interval [0,4]?

b. Evaluate $\lim_{t \to 2} \frac{h(t) - h(2)}{t - 2}$

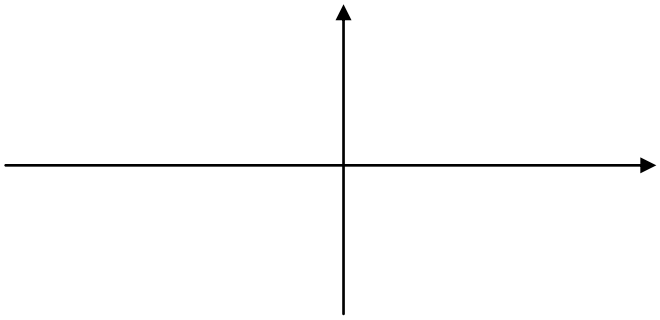
8. (6 pts.) True or false? If true, explain why. If false, give a counterexample.

a. If the limit of a function exists at point x=a, then the function is continuous at a.

b. If the limit exists at point x=a, then a must be in the domain of the function.

c. If a function is continuous at a point x=a, then the limit exists at x=a.

9. (12 pts.) The graph of a derivative of a function f is shown below. Use this graph to answer questions about f. Briefly justify ALL of your answers.



a. On which intervals, if any, is f increasing?

b. At which values of x, if any, does f have a stationary point?

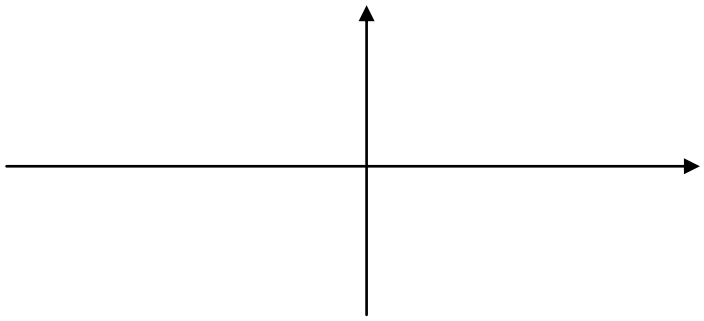
c. At which values of x, if any, does f have a local maximum point?

d. Where does f have points of inflection?

e. Is f concave up at x=2?

f. Suppose that f(-3) = 2 Find an equation of the line tangent to f at x = -3.

10. (10 pts.) Let f be the function given by the graph below; consider f only on the domain [-3,3]. Answer the following questions or find the following limits (or say they don't exist.)



a. $\lim_{x \to -1^-} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to -1} f(x)$

d. Where is f continuous? Briefly justify your answer.

e. Where is f differentiable? Briefly justify your answer.

11. (9 pts.) Let $f(w) = 7w^9 - 18w^7 + 63$. Use calculus techniques to answer questions about f; give exact values. Briefly justify ALL of your answers.

a. On which intervals, if any, is f increasing?

b. At which values of w, if any, does f have a stationary point?

c. At which values of w, if any, does f have a local maximum point?