MAT 2-121 Calculus of a Single Variable

Sample Exam 2 October 10, 2014

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name

For credit on these problems, you must show your work. Don't approximate your answers unless directed to do so. Graphing calculators are allowed, but not calculators with symbolic capabilities.

1. (15 pts.) Find the derivative of the following functions:

a. $f(x) = \dfrac{\ln x}{\sqrt{x}}$

b. $h(x) = 2^x \cos 3x$

c. $j(x) = \sin^3(2\pi x)$

d. $l(x) = \arcsin(x)$

2. (9 pts.) Calculate the anti-derivative of the following functions:

a. $f(x) = \dfrac{1}{1+x^2}$

b. $g(x) = \dfrac{1}{x^2} + \sqrt[3]{x}$

c. $h(x) = \dfrac{2x+3}{1+3x+x^2}$

3. (8 pts.) Find a polynomial function with exactly two stationary points—one at $x = -2$ and one at $x = 1$. Show your work.

4. (8 pts.) Find the equation of the tangent line to the curve $x^2 + xy - y^3 = xy^2$ at the point (1,1).

5. (8 pts.) Use implicit differentiation to derive the formula for the derivative of arcsin x.

6. (4 pts.) Simplify the following expression to algebraic form: cos(arcsin(3x)).

7. (9 pts.) Use the values:

X	-3	-2	-1	0	1	2	3
$f(x)$	2	-1	1	3	-2	4	5
$f'(x)$	1	0	2	-1	3	-3	-2

a. Let $g(x) = e^{f(x)}$ Evaluate $g'(-3)$. Show your work.

b. Let $h(x) = f(e^x)$. Find an equation of the line tangent to h at $x = \ln 2$. Show your work.

c. Let $j(x) = e^x f(x)$ and suppose that $f''(3) = 1$. Is the graph of j concave up at $x=3$? Justify your answer.

8. (6 pts.) Does the function $y(t) = 65 + 120e^{.2t}$ satisfy the initial value problem $y' = 0.2(65 - y)$ with $y(0) = 185$? Justify your answer.

9. (9 pts.) For this problem, give the Differential Equation or Initial Value Problem that models the given physical situation.

a. The change in the velocity of a falling object (ignoring friction) is equal to the constant -9.8. Give the differential equation for $v(t)$.

b. A flu epidemic spreads through a 30000-student college community at a rate proportional to the product of the number of members already infected (represented by $I(t)$) and the number of those not infected. The epidemic starts with just 2 cases.

c. The rate at which the temperature of an object changes is proportional to the difference between the temperature T of the object and the temperature S of the objects surroundings.

10. (10 pts.) Doubling time is defined to be the length of time necessary for a given quantity to double in size. Recall that money invested at a certain interest rate r which is compounded continuously is modeled by the differential equation $P' = rP$, with $P(0) = P_0$ as the initial amount invested. Consider a credit card with a balance of \$4,000 and an interest rate of 16.9% governed by this model. How long will it take until the balance doubles if no payments are made? (Assume no penalties; show your work including the statement and solution of the IVP.)

11. (9 pts.) Evaluate the following limits or explain why they don't exist. Show your work and justify your answer.

a. $\lim_{x \to \infty} xe^{-x}$

b. $\lim_{x \to 0} \frac{1-\cos x}{\sin(2x)}$

c. $\lim_{x \to 0^-} \frac{\cos x}{x}$

12. (7 pts.) Produce the slope field (at integer pairs for t from -1 to 1 and y from -1 to 1) for the differential equation y′ = t(1-y)