MAT3-121 Calculus of a Single Variable

Quiz 5 November 17, 2016

solution

name

You must show your work for full credit on this quiz. Calculators are allowed.

1. (2 pts.) Give a parameterization of the line segment that starts at point (1,2) and ends at point (3,7)

$x(t) = 1+2t \quad 0 \le t \le 5$

$y(t) = 2+5t$

2. (2 pts.) Consider a particle moving along a curve whose motion is parameterized by

$x(t) = t^2 \qquad 0 \le t \le 5.$

$y(t) = t^3$

What is the speed of the particle at time t = 2?

$s(t) = \sqrt{[x'(t)]^2 + [y'(t)]^2} = \sqrt{(2t)^2 + (3t^2)^2}$

$s(2) = \sqrt{160}$

3. (5 pts) Find quadratic Taylor polynomial which approximates ln x at base point 1. Show all your work.

$f(x) = \ln x$

$f(1) = 0$

$f'(x) = \frac{1}{x}$

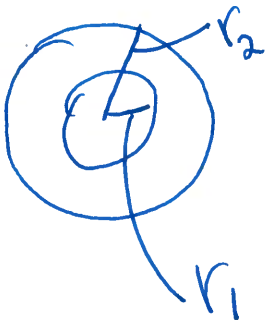
$f'(1) = \frac{1}{1}$

$f''(x) = -\frac{1}{x^2}$

$f''(1) = -1$

$q(x) = 0 + 1(x-1) - \frac{1}{2}(x-1)^2$

4. (5 pts.) Two concentric circles are expanding, the outer radius at a rate of 2 feet per second and the inner one at 5 feet per second. At a certain instant, the outer radius is 10 feet, and the inner radius is 3 feet. At this instant, is the area of the ring between the two circles increasing or decreasing? Justify your answer.



$A(t) = \pi r_2^2 - \pi r_1^2$ $\quad \frac{dr_2}{dt} = 2$

$\frac{dA}{dt} = 2\pi r_2 \frac{dr_2}{dt} - 2\pi r_1 \frac{dr_1}{dt}$ $\quad \frac{dr_1}{dt} = 5$

$\left.\frac{dA}{dt}\right|_{\text{certain instant}} = 2\pi \cdot 10 \cdot 2 - 2\pi 3 \cdot 5 = 10\pi > 0$

increasing

5. (4 pts.) Apply Newton's method to find the largest root of $f(x) = x^3 - 3x + 1$. Give an appropriate initial guess and one Newton iterate. Show your work.

$x_0 = 2$ $\quad f' = 3x^2 - 3$

$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{3}{9} = 1.\overline{6}$