Bézier Curves

CSC321 Computer Graphics

08 December 2016

1. Here is a product of a row vector, a matrix, and a column vector that yields the x coordinate of a point on a cubic Bézier curve.

$$x(u) = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} rx_0 \\ 3(x_1 - x_0) \\ 3(x_0 - 2x_1 + x_2) \\ 3(x_1 - x_2) + (x_3 - x_0) \end{bmatrix}$$

$$= x_0 + 3(x_1 - x_0)u + 3(x_0 - 2x_1 + x_2)u^2 + (3(x_1 - x_2) + (x_3 - x_0))u^3$$

$$x(0) = x_0$$

$$\begin{aligned} x(1) &= x_0 + 3(x_1 - x_0) + 3(x_0 - 2x_1 + x_2) + 3(x_1 - x_2) + (x_3 - x_0) \\ &= (x_0 - 3x_0 + 3x_0 - x_0) + (3x_1 - 6x_1 + 3x_1) + (3x_2 - 3X_2) + x_3 \\ &= x_3 \end{aligned}$$

$$x'(u) = 3(x_1 - x_0) + 6(x_0 - 2x_1 + x_2)u + 3(3(x_1 - x_2) + (x_3 - x_0))u^2$$

$$\begin{aligned} x'(0) &= 3(x_1 - x_0) \\ x'(1) &= 3(x_3 - x_2) \end{aligned}$$