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C1B.1 Using the methods described in the Unit Conversions section and conversion factors from the inside front cover of the text, we have

$$\tfrac{1}{2}\,\text{ks} = \tfrac{1}{2}\,\text{ks}\left(\frac{1000\,\text{s}}{1\,\text{ks}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 8.3\,\text{min}$$

(Note how I am using different kinds of slashes to indicate what cancels with what.)

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C1B.5 Using the methods described in the Unit Conversions section and conversion factors from the inside front cover of the text, we have

$$1\ \cancel{\text{month}} = 1\ \cancel{\text{month}}\left(\frac{30\ \cancel{\text{d}}}{1\ \cancel{\text{month}}}\right)\left(\frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}}\right)\left(\frac{3600\ \text{s}}{1\ \cancel{\text{h}}}\right) = 2.59 \times 10^6\ \cancel{\text{s}}\left(\frac{1\ \text{Ms}}{10^6\ \cancel{\text{s}}}\right) = 2.59\ \text{Ms}$$

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C1B.6 An object's average density ρ is defined to be its mass m divided by its volume V ($\rho = m/V$), so the mass of an object with density ρ and volume V is $m = \rho V$.

(a) The volume of a cube of water that is 10 cm on a side is $(10\text{ cm})^3$. Using the methods described in the Unit Conversions section and conversion factors from the inside front cover of the text, we have

$$m = \rho V = \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)(10\ \text{cm})^3\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = \frac{1000\times(10)^3}{100^3}\,\text{kg} = 1.0\ \text{kg} \quad (1)$$

(Note how I am using different kinds of slashes to indicate what cancels with what.)

(b) Similarly, for a cube of water that is 1 cm on a side,

$$m = \rho V = \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)(1\ \text{cm})^3\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = \frac{1000\times 1^3}{100^3}\,\text{kg} = \frac{1}{1000}\,\text{kg}\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right) = 1.0\ \text{g} \quad (2)$$

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C1B.7 **(a)** In the equation $D = v\sin 2\theta / g$, the $\sin 2\theta$ is unitless, so v/g should have the same units as D. However,

$$\left\{\frac{v}{g}\right\} = \frac{\text{m/s}}{\text{m/s}^2} = \text{s} \neq \{D\} = \text{m} \tag{1}$$

so the expression cannot be correct.

(b) To get the correct units, we must get rid of the units of seconds. If v were squared, this would get rid of the seconds and correctly leave a remainder of one power of meters:

$$\left\{\frac{v^2}{g}\right\} = \frac{\text{m}^{2}/\cancel{\text{s}^2}}{\cancel{\text{m}}/\cancel{\text{s}^2}} = \text{m} = \{D\} \tag{1}$$

Therefore, a more plausible formula would be $D = v^2 \sin 2\theta / g$.

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C1M.5 Assume that the formula has the power law form $v_e = KG^iM^jR^k$, where v_e is the escape speed (in m/s), G is the universal gravitational constant (in $N \cdot m^2/kg^2$), M is the planet's mass (in kg), R is its radius (in m), and K is a unitless constant. To make the units work out, we must have

$$\{v_e\} = \frac{\text{m}}{s} = \{KG^iM^jR^k\} = \left(\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)^i (\text{kg})^j (\text{m})^k \left(\frac{1\ \text{kg}\cdot\text{m}/\text{s}^2}{1\ \text{N}}\right)^i = \frac{\text{kg}^{j-i}\text{m}^{3i+k}}{\text{s}^{2i}} \tag{1}$$

So, for the units to work out, we must have $j - i = 0$, $3i + k = 1$, and $2i = 1$. The last equation implies that $i = \frac{1}{2}$, so the first equation also implies that $j = \frac{1}{2}$, and the last that $\frac{3}{2} + k = 1 \Rightarrow k = -\frac{1}{2}$. Therefore the formula for the escape speed probably is

$$v_e = K\sqrt{\frac{GM}{R}} \tag{2}$$

As we will see later in this unit, this formula is correct (with $K = \sqrt{2}$).

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C1M.7 Assume that the formula has the power law form $T = Km^i L^j g^k$, where T is the pendulum's period (in s), L is the pendulum's length (in m), m is the pendulum's mass (in kg), g is the gravitational field strength (in m/s^2), and K is a unitless constant. We must include the gravitational field strength here because that is what causes the pendulum to swing. To make the units work out, we must have

$$\{T\} = \text{s} = \{Km^i L^j g^k\} = (\text{kg})^i (\text{m})^j \left(\frac{\text{m}}{\text{s}^2}\right)^k = \frac{\text{kg}^i \text{m}^{j+k}}{\text{s}^{2k}} \tag{1}$$

So, for the units to work out, we must have $i = 0$, $j + k = 0$, and $2k = -1$. The last equation implies that $k = -\frac{1}{2}$, so the second equation implies that $j = \frac{1}{2}$. The formula for the rocking period should therefore be

$$T = KL^{1/2} g^{-1/2} = \sqrt{\frac{L}{g}} \tag{2}$$

You can check experimentally that this works out. (Calculating the exact unitless constant here depends on the exact pendulum shape, but for a uniform rod, $K = \sqrt{8\pi^2/3} \approx 5.13$.)

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C1M.10 **(a)** The missing quantity must be the universal gravitational constant G, because gravitation is what might prevent the universe's infinite expansion.

(b) Assume, therefore, that the formula has the power law form $\rho_c = KG^iH^jc^k$, where ρ_c is the critical density (in kg/m^3), G is the universal gravitational constant (with units of N·m^2/kg^2), c is the speed of light (in m/s), and K is a unitless constant. To make the units work out, we must have

$$\{\rho_c\} = \frac{\text{kg}}{\text{m}^3} = \{KG^iH^jc^k\} = \left(\frac{\cancel{\text{N}}\cdot\text{m}^2}{\text{kg}^2}\right)^i\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\,\cancel{\text{N}}}\right)^i\left(\frac{1}{\text{s}}\right)^j\left(\frac{\text{m}}{\text{s}}\right)^k = \frac{\text{kg}^{-i}\text{m}^{3i+k}}{\text{s}^{2i+j+k}} \tag{1}$$

So, for the units to work out, we must have $-i = 1$, $3i + k = -3$, and $2i + j + k = 0$. The first equation tells us that $i = -1$, the second then tells us that $k = 0$, so the last equation tells us that $-2 + j + 0 = 0$, so $j = 2$. The formula for the critical density should therefore be

$$\rho_c = KG^{-1}H^2 = \frac{KH^2}{G} \tag{2}$$

Note that the speed of light cannot actually factor in at all! According to the theory of general relativity applied to the universe, this formula is correct, with $K = 3/8\pi \approx 0.1194$.

(c) If (in our ignorance) we set $K = 1$, we would find the critical density to be

$$\rho_c = \frac{H^2}{G} = \left(\frac{2.28\times10^{-18}}{\cancel{\text{s}}}\right)^2\left(\frac{\text{kg}^2}{6.67\times10^{-11}\,\cancel{\text{N}}\cdot\text{m}^2}\right)\left(\frac{1\,\cancel{\text{N}}}{1\text{ kg}\cdot\text{m}/\cancel{\text{s}}^2}\right) = 7.8\times10^{-26}\,\frac{\text{kg}}{\text{m}^3} \tag{3}$$

This is about 47 hydrogen atoms per cubic meter. (Including the correct factor of K yields a critical density more like 5.5 hydrogen atoms per cubic meter. Physicists currently believe that the universe actually contains an average of less than 0.25 hydrogen atoms of normal mass per cubic meter, but that mysterious substances known as "dark matter" and "dark energy" combine enough so that the density of our universe is experimentally indistinguishable from the critical density.)