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C3B.3 To compute the vector sum of the vectors $\vec{u}$ and $\vec{w}$ given in the problem, we can use equation C3.9:

$$\vec{u} + \vec{w} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} u_x + w_x \\ u_y + w_y \\ u_z + w_z \end{bmatrix} = \begin{bmatrix} 2\text{ m} + (-4\text{ m}) \\ -3\text{ m} + (-1\text{ m}) \\ 1\text{ m} + 3\text{ m} \end{bmatrix} = \begin{bmatrix} -2\text{ m} \\ -4\text{ m} \\ 4\text{ m} \end{bmatrix} \tag{1}$$

The magnitude of this sum (according to equation C3.13) is:

$$|\vec{u} + \vec{w}| = \sqrt{(u_x + w_x)^2 + (u_y + w_y)^2 + (u_z + w_z)^2} = \sqrt{(-2\text{ m})^2 + (-4\text{ m})^2 + (4\text{ m})^2} = 6\text{ m}. \tag{2}$$

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C3B.4 To compute the difference $\vec{w}-\vec{u}$ of the vectors $\vec{u}$ and $\vec{w}$ given in equation C3.20, we can use equation C3.10:

$$\vec{w}-\vec{u}=\vec{w}+(-\vec{u})=\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}+\begin{bmatrix} -u_x \\ -u_y \\ -u_z \end{bmatrix}=\begin{bmatrix} -4-2\text{ m} \\ -1\text{ m}-(-3\text{ m}) \\ 3\text{ m}-1\text{ m} \end{bmatrix}=\begin{bmatrix} -6\text{ m} \\ +2\text{ m} \\ +2\text{ m} \end{bmatrix} \quad (1)$$

The magnitude of this sum (according to equation C3.13) is:

$$|\vec{w}-\vec{u}|=\sqrt{(w_x-u_x)^2+(w_y-u_y)^2+(w_z-u_z)^2}=\sqrt{(-6\text{ m})^2+(2\text{ m})^2+(2\text{ m})^2}=6.6\text{ m} \quad (2)$$

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C3B.8 **(a)** The displacement vector $\Delta\vec{r}$ from the earth (at $\vec{r}_E$) to the alien ship (at $\vec{r}_A$) must be such that it carries us from $\vec{r}_E$ to $\vec{r}_A$: $\vec{r}_A = \vec{r}_E + \Delta\vec{r}$. Therefore

$$\Delta\vec{r} = \vec{r}_A - \vec{r}_E = \begin{bmatrix} -0.100\text{ Tm} \\ 0.288\text{ Tm} \\ 0.400\text{ Tm} \end{bmatrix} - \begin{bmatrix} 0.100\text{ Tm} \\ -0.112\text{ Tm} \\ 0\text{ Tm} \end{bmatrix} = \begin{bmatrix} (-0.100 - 0.100)\text{ Tm} \\ (0.288 + 0.112)\text{ Tm} \\ (0.400 - 0)\text{ Tm} \end{bmatrix} = \begin{bmatrix} -0.200\text{ Tm} \\ 0.400\text{ Tm} \\ 0.400\text{ Tm} \end{bmatrix} \tag{1}$$

(b) The magnitude of this vector is

$$|\Delta\vec{r}|^2 = \sqrt{(-0.20\text{ Tm})^2 + (0.40\text{ Tm})^2 + (0.40\text{ Tm})^2} = \sqrt{0.36\text{ Tm}^2} = 0.60\text{ Tm} \tag{2}$$

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C3M.1 The displacement vector $\Delta\vec{r}$ from the earth (at $\vec{r}_E$) to the alien ship (at $\vec{r}_A$) must be such that it carries us from $\vec{r}_E$ to $\vec{r}_A$: $\vec{r}_A = \vec{r}_E + \Delta\vec{r}$. Therefore

$$\Delta\vec{r} = \vec{r}_A - \vec{r}_E = \begin{bmatrix} -0.100\text{ Tm} \\ 0.288\text{ Tm} \\ 0.400\text{ Tm} \end{bmatrix} - \begin{bmatrix} 0.100\text{ Tm} \\ -0.112\text{ Tm} \\ 0\text{ Tm} \end{bmatrix} = \begin{bmatrix} (-0.100 - 0.100)\text{ Tm} \\ (0.288 + 0.112)\text{ Tm} \\ (0.400 - 0)\text{ Tm} \end{bmatrix} = \begin{bmatrix} -0.200\text{ Tm} \\ 0.400\text{ Tm} \\ 0.400\text{ Tm} \end{bmatrix} \quad (1)$$

Now, if the ship is heading directly *toward* the earth, its velocity $\vec{v}$ must be directly opposite to $\Delta\vec{r}$. Moreover, since it must cover the displacement in time $\Delta t = 5$ h, the velocity vector must be

$$\vec{v} = \frac{-\Delta\vec{r}}{\Delta t} = -\frac{1}{5.0\text{ h}} \begin{bmatrix} -0.200\text{ Tm} \\ 0.400\text{ Tm} \\ 0.400\text{ Tm} \end{bmatrix} = \begin{bmatrix} +0.040\ \cancel{\text{Tm}}/\cancel{\text{h}} \\ -0.080\ \cancel{\text{Tm}}/\cancel{\text{h}} \\ -0.080\ \cancel{\text{Tm}}/\cancel{\text{h}} \end{bmatrix} \left(\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\right)\left(\frac{10^{12}\text{ m}}{1\ \cancel{\text{Tm}}}\right) = \begin{bmatrix} +1.11\times 10^{7}\text{ m/s} \\ -2.22\times 10^{7}\text{ m/s} \\ -2.22\times 10^{7}\text{ m/s} \end{bmatrix} \quad (2)$$

The ship's speed (3.33×10^7 m/s) is about 1/9 times the speed of light.

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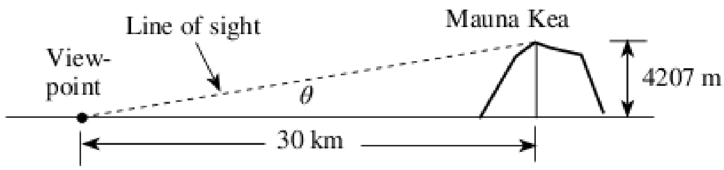
C3M.3 The distance between the viewpoint and the peak is

$$d = \sqrt{(24\text{ km})^2 + (18\text{ km})^2} = 30\text{ km} \tag{1}$$

As the drawing below shows, this distance will be the base of a right triangle and the peak represents the vertical leg of that triangle, so the angle above the horizon θ must be such that

$$\theta = \tan^{-1}\left[\frac{4207\text{ m}}{30\text{ km}}\left(\frac{1\text{ km}}{1000\text{ m}}\right)\right] = \tan^{-1}(0.140) = 8.0° \tag{2}$$



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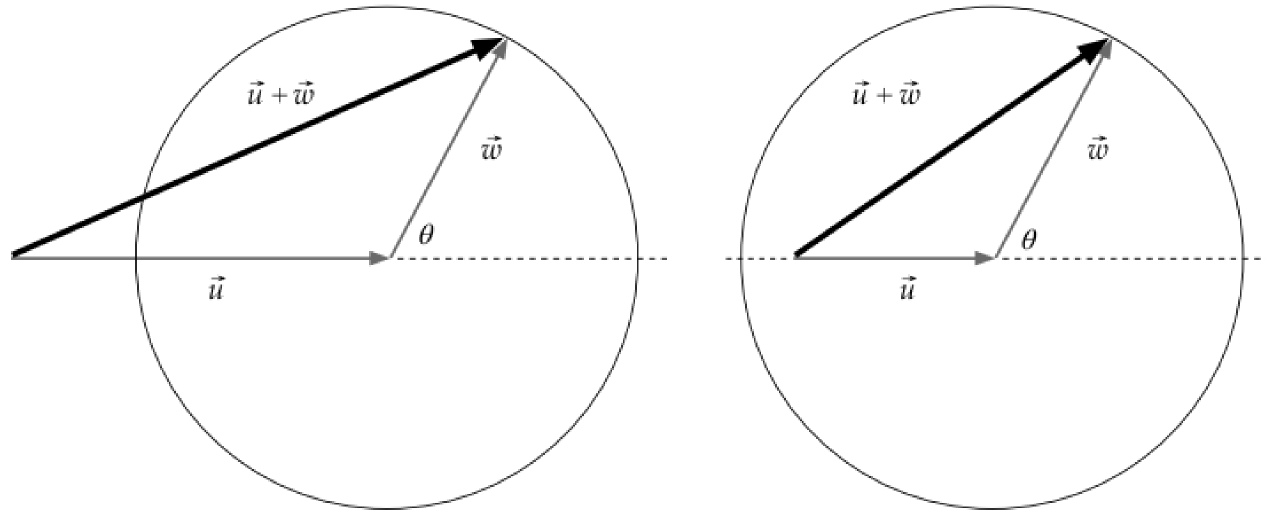
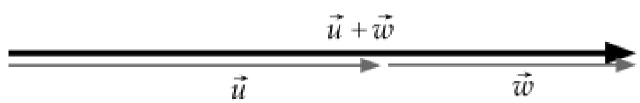
C3D.1 Consider the drawings below.



The drawings show the two possible cases $|\vec{u}|>|\vec{w}|$ and $|\vec{u}|<|\vec{w}|$. The circles in each case show all of the possible positions of the tip of $\vec{w}$ (and thus the tip of $\vec{u}+\vec{w}$) depending on the angle θ that $\vec{w}$ makes with the direction of $\vec{u}$.

(a) From the diagrams, we see that $|\vec{u}+\vec{w}|=|\vec{u}|+|\vec{w}|$ if and only if $\theta=0$ (that is, the two vectors point in the same direction), because then we have

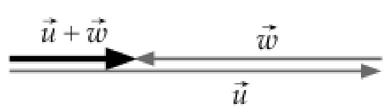


and the length of the vector sum is clearly the same as the arithmetic sum of the lengths.

(b) We also see that $|\vec{u}+\vec{w}|=0$ if and only if θ is 180° (the vectors are opposite) and $|\vec{u}|=|\vec{w}|$. Only then will the circle for the tip of $\vec{w}$ coincide with the tail of $\vec{u}$.

(c) We can never have $|\vec{u}+\vec{w}|>|\vec{u}|+|\vec{w}|$, because the length of $\vec{u}+\vec{w}$ is maximum when $\theta=0$, but that corresponds to the case when $|\vec{u}+\vec{w}|=|\vec{u}|+|\vec{w}|$ as discussed in part (a).

(d) Yes, in fact $|\vec{u}+\vec{w}|$ is *always* greater than or equal to $|\vec{u}|-|\vec{w}|$. We can see from the diagrams that $|\vec{u}+\vec{w}|$ has its minimum value when $\theta=180°$. In the case where $|\vec{u}|>|\vec{w}|$, the diagram below shows that when $\theta=180°$, $|\vec{u}+\vec{w}|=|\vec{u}|-|\vec{w}|$.



For all other θ, $|\vec{u}+\vec{w}|$ is larger. If $|\vec{u}|<|\vec{w}|$, then $|\vec{u}|-|\vec{w}|$ is negative, so $|\vec{u}+\vec{w}|>|\vec{u}|-|\vec{w}|$ simply because $|\vec{u}+\vec{w}|$ is positive. So $|\vec{u}+\vec{w}|\geq|\vec{u}|-|\vec{w}|$ always.

(e) Note that $\vec{u}-\vec{w}=\vec{u}+(-\vec{w})$, so all of the statements above apply to $|\vec{u}-\vec{w}|$ as long as we reverse the direction of $\vec{w}$ (that is, replace θ by $\theta+180°$). In particular, part (d) with this adjustment implies that $|\vec{u}-\vec{w}|\geq|\vec{u}|-|-\vec{w}|=|\vec{u}|-|\vec{w}|$. In this case, the equality will happen when $\theta=0°$ instead of $\theta=180°$.

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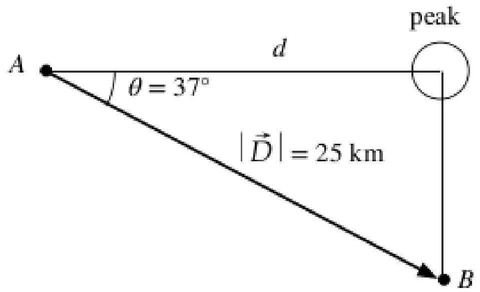
C3R.3 A top view of the situation is shown below. Point A is the ship's initial position (when it is due west of the peak), and point B is the second position (when the ship is due south of the peak). The side view shows the peak and the ship at its initial position, as viewed from the south.



To solve the problem, we have to neglect the curvature of the earth, which over distances of tens of kilometers is not going to be all that significant. Note that we have two right angles in these drawing. According to the triangle in the right-hand drawing, $h = d\tan\phi$, so if we knew d, we could find h. But according to the left-hand picture, $d = |\vec{D}|\cos\theta$. We can use these two equations to eliminate d and solve for h in terms of the known quantities $|\vec{D}|$ and θ. Using the second equation to eliminate d in the first yields

$$h = d\tan\phi = |\vec{D}|\cos\theta\tan\phi = (25\text{ km})\cos 37°\tan 12° = 4.2\text{ km}$$

Note that the units are correct, and 4.2 km (just under 14,000 ft) is a plausible height for a mountain.