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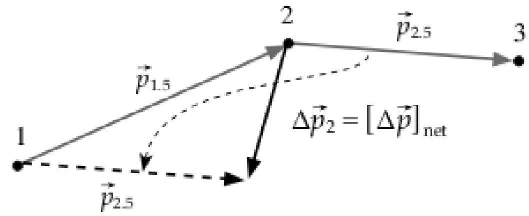
C2B.3 The net impulse $[\Delta\vec{p}]_{\text{net}}$ delivered during the interval from $t_{1.5}$ to $t_{2.5}$ will be the same as the object's change in momentum $\Delta\vec{p}_2$ during that interval (which is centered on time t_2). Because the change in momentum must be such that $\vec{p}_{1.5} + \Delta\vec{p}_2 = \vec{p}_{2.5}$ (as discussed in section C2.5), the change in momentum (and thus the net impulse) during that interval must have been as shown below:



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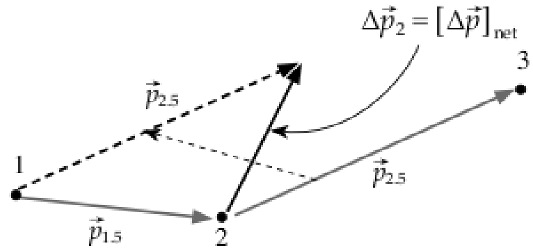
C2B.4 The net impulse $[\Delta\vec{p}]_{\text{net}}$ delivered during the interval from $t_{1.5}$ to $t_{2.5}$ will be the same as the object's change in momentum $\Delta\vec{p}_2$ during that interval (which is centered on time t_2). Because the change in momentum must be such that $\vec{p}_{1.5} + \Delta\vec{p}_2 = \vec{p}_{2.5}$ (as discussed in section C2.5), the change in momentum (and thus the net impulse) during that interval must have been as shown below:



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C2B.7 **(a)** In this process, the earth has delivered a downward impulse of (60 kg)(10 m/s) = 600 kg·m/s to the anvil. This must come from the earth's momentum, and subtracting 600 kg·m/s of downward momentum is the same as adding an upward momentum of magnitude $|[\Delta\vec{p}_e]_{net}|$ = 600 kg·m/s to the earth. If the earth has mass $M = 6.0 \times 10^{24}$ kg, was initially at rest (that is, the earth's initial velocity $\vec{v}_i = 0$), and this is the only interaction the earth participates in this time period (which is a ridiculous assumption, but let's go with it), then the earth's final velocity $\vec{v}_f$ must be such that

$$M\vec{v}_f - M\vec{v}_i = \Delta\vec{p}_e = [\Delta\vec{p}_e]_{\text{net}} \quad \Rightarrow \quad M\vec{v}_f + 0 = [\Delta\vec{p}_e]_{\text{net}} \tag{1}$$

so its final speed will be

$$|\vec{v}_f| = \left|\frac{[\Delta\vec{p}_e]_{net}}{M}\right| = \frac{600\ \cancel{\text{kg}}\cdot\text{m/s}}{6.0\times 10^{24}\ \cancel{\text{kg}}} = 1.0\times 10^{-22}\ \frac{\text{m}}{\text{s}} \tag{2}$$

(b) Assuming that the earth maintains this speed, then the distance D it travels in time T is $|\vec{v}_f| = D/T$. Therefore, if $D = 1.0\ \mu\text{m} = 1.0 \times 10^{-6}$ m, the time required will be

$$T = \frac{D}{|\vec{v}_f|} = \frac{1.0\times 10^{-6}\ \cancel{\text{m}}}{1.0\times 10^{-22}\ \cancel{\text{m}}/\text{s}} = 1.0\times 10^{16}\ \cancel{\text{s}}\left(\frac{1\ \text{y}}{3.16\times 10^{7}\ \cancel{\text{s}}}\right) = 3.16\times 10^{8}\ \text{y} = 316\ \text{million years} \tag{3}$$

Of course it is absurd to think that the earth would avoid receiving other impulses of comparable magnitude or greater during this time interval, so the entire question is unrealistic. But it does highlight just how negligible the effects of human-scale impulses on the earth's motion are.

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C2B.13 The magnet participates in four macroscopic interactions: (1) a long-range gravitational interaction with the earth, (2) a long-range magnetic interaction with the refrigerator, (3) a compression contact interaction with the refrigerator, and (4) a friction contact interaction with the refrigerator. We can infer the existence of the compression interaction because the magnetic force on the magnet wants to pull it horizontally into the refrigerator, and something must be opposing that so that the magnet remains at rest horizontally. This interaction is a compression interaction because it prevents the magnet and refrigerator from merging. Finally, the gravitational interaction is constantly supplying downward momentum to the magnet, so something must be providing upward momentum at the same rate if the magnet is to remain at rest. Since this interaction is opposing the magnet sliding down the refrigerator (that is, the magnet moving relative to the refrigerator), we classify this as a friction interaction.

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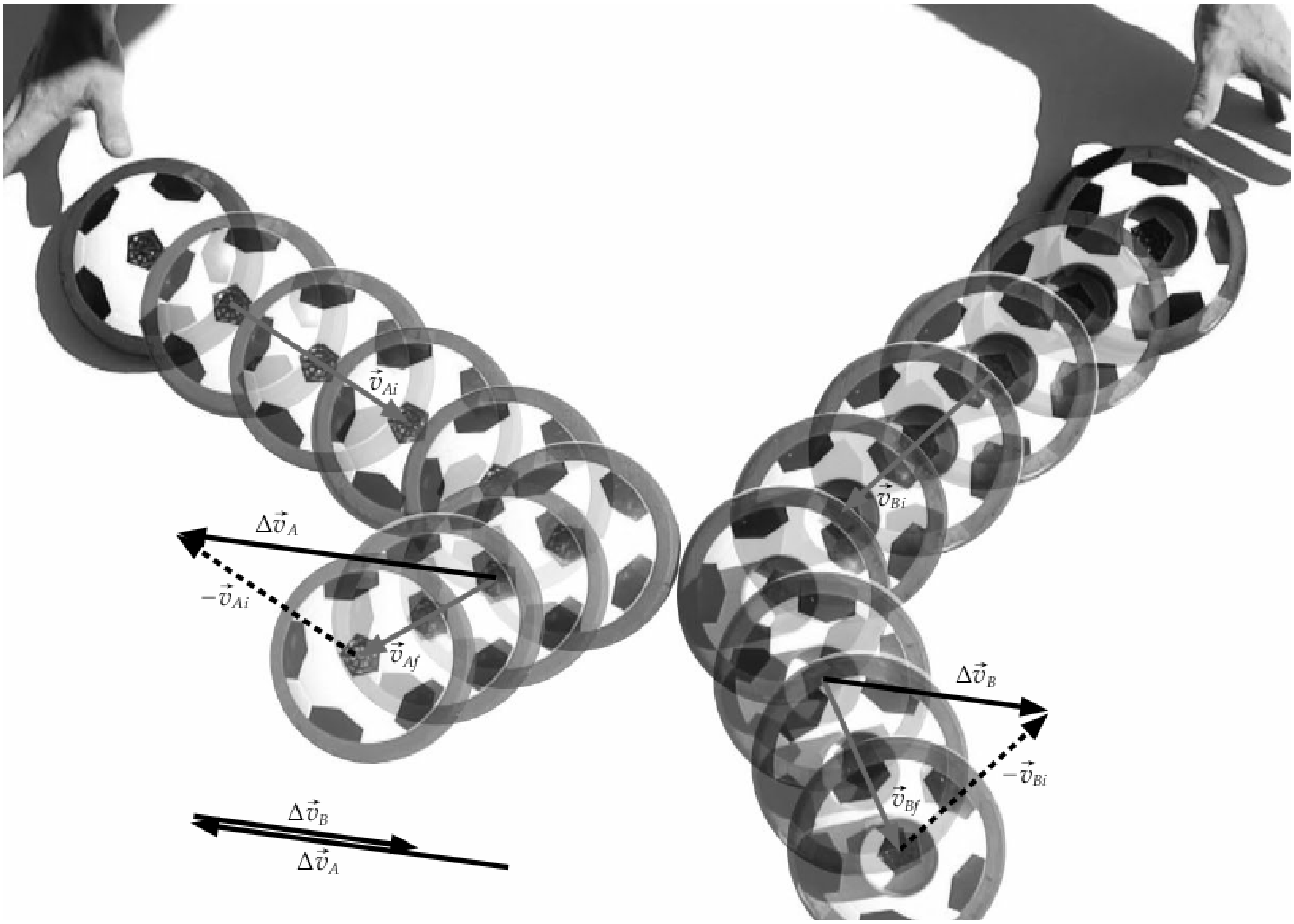
C2M.2 The diagram below illustrates the construction of the velocity vectors in this case:



(Note that to get maximum accuracy, I skipped one intervening image when constructing both the initial and final velocity vectors.) Note that $\Delta\vec{v}_A$ and $\Delta\vec{v}_B$ appear to be almost exactly equal opposite in direction, but $|\Delta\vec{v}_A| > |\Delta\vec{v}_B|$. The principle of momentum transfer, however, requires that the impulses delivered must have equal magnitudes, so the changes in the disks' momenta must be equal in magnitude. This means that we should have $m_A|\Delta\vec{v}_A| = m_B|\Delta\vec{v}_B|$. I measure the lengths of the $\Delta\vec{v}_A$ and $\Delta\vec{v}_B$ arrows on the diagram above to be 3.05 cm and 4.30 cm, respectively (to the nearest half-millimeter). Since the magnitude of the velocities will be proportional to these arrow lengths, we must have

$$\frac{m_B}{m_A} = \frac{|\Delta\vec{v}_A|}{|\Delta\vec{v}_B|} = \frac{4.30 \text{ cm}}{3.05 \text{ cm}} = 1.41$$

When we took this photograph, we carefully measured the disk masses and found the ratio to be 1.42, consistent with the result above. (It is not surprising that the results don't agree in the last decimal place. Both the construction and measurement processes, even if they are done as carefully as possible, are likely involve errors of a few percent.)

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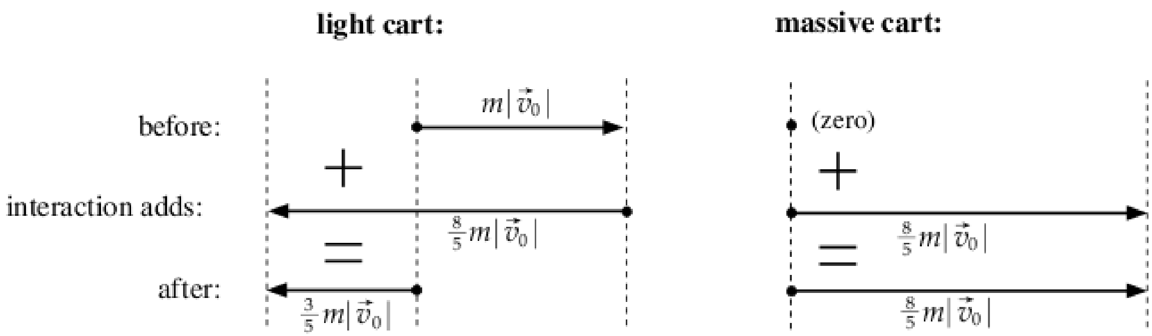
C2M.3 The diagram below illustrates the situation. Let m be the mass of the incoming cart, $|\vec{v}_0|$ be its initial speed, and M be the unknown mass of the cart at rest. We are given that the first cart rebounds leftward with a speed of $|\vec{v}_1| = \frac{3}{5}|\vec{v}_0|$ and the second cart moves rightward with a speed of $|\vec{v}_2| = \frac{2}{5}|\vec{v}_0|$ after the collision. We can see from the left side of the diagram below that the interaction between the carts must have added $\frac{8}{5}m|\vec{v}_0|$ of leftward momentum to the first cart, because $m|\vec{v}_0|$ rightward plus the momentum delivered must end up being $\frac{3}{5}m|\vec{v}_0|$ leftward. Assuming that the collision interaction simply transfers momentum from one cart to the other (and no external interactions affect either carts' momenta during the process) the right side of the diagram below then implies that the final momentum of the second cart has a magnitude of $\frac{8}{5}m|\vec{v}_0|$. Since the magnitude of the second cart's momentum must be equal to $M|\vec{v}_2|$ and we know what $|\vec{v}_2|$ is, we can solve for M in terms of the "known" quantities m and $|\vec{v}_0|$:

$$\frac{8}{5}m|\vec{v}_0| = M|\vec{v}_2| = M\frac{2}{5}|\vec{v}_0| \quad \Rightarrow \quad M = \frac{\frac{8}{5}m|\vec{v}_0|}{\frac{2}{5}|\vec{v}_0|} = 4m.$$



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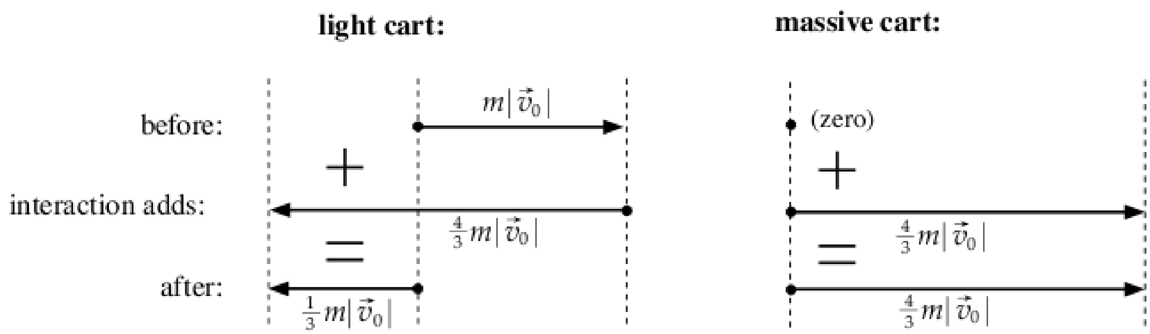
C2M.4 The diagram below illustrates the situation. Let m be the mass of the incoming cart, $|\vec{v}_0|$ be its initial speed, and M be the mass of the cart at rest and $\vec{v}_2$ is its unknown final velocity. We are given that $M = 2m$ and that the first cart rebounds leftward from the collision with a speed of $|\vec{v}_1| = \frac{1}{3}|\vec{v}_0|$. We can see from the left side of the diagram below that the interaction between the carts must have added $\frac{4}{3}m|\vec{v}_0|$ of leftward momentum to the first cart, because $m|\vec{v}_0|$ rightward plus the momentum delivered must end up being $\frac{1}{3}m|\vec{v}_0|$ leftward. Assuming that the collision interaction simply transfers momentum from one cart to the other (and no external interactions affect either carts' momenta during the process) the right side of the diagram below then implies that the final momentum of the second cart has a magnitude of $\frac{4}{3}m|\vec{v}_0|$. Since the magnitude of the second cart's momentum must be equal to $M|\vec{v}_2|$ and we know what M is in terms of m, we can solve for $|\vec{v}_2|$ in terms of the "known" quantities m and $|\vec{v}_0|$:

$$\frac{4}{3}m|\vec{v}_0| = M|\vec{v}_2| = 2m|\vec{v}_2| \quad \Rightarrow \quad |\vec{v}_2| = \frac{\frac{4}{3}\cancel{m}|\vec{v}_0|}{2\cancel{m}} = \frac{2}{3}|\vec{v}_0|.$$

Therefore, the massive cart's final velocity is $\vec{v}_2 = \frac{2}{3}|\vec{v}_0|$ rightward.



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C2R.3 I estimate that the hose equipped with a high-speed nozzle could fill up a 2-liter jar in roughly 10 seconds. This estimate is probably good within a factor of two or so: I am fairly confident that it would not take as long as 20 s and I am somewhat less confident that it would take more than 5 s, so I am making, I think, a fairly conservative estimate here. This would imply a flow rate of 200 cm^3/s = 0.2 kg/s. I also estimate that the flow velocity is about 10 m/s, because I think that if I interrupt the spray, it takes roughly 1 s to travel the width of my yard, which is about 30 ft ≈ 10 m. (Again, this estimate is probably good to within a factor of two or three.)

Since the water only flows fairly slowly through the hose itself (if you take the nozzle off, the water does not travel nearly as fast), I am going to assume that the water is essentially at rest when it enters the nozzle.* To the extent that this model is correct, the water's contact interaction with the nozzle therefore takes approximately 0.2 kg of water from essentially rest to 10 m/s every second, thus increasing this chunk of water's momentum by $|[\Delta\vec{p}]_N| = m|\Delta\vec{v}| \approx (0.2\text{ kg})\cdot(10\text{ m/s}) = 2\text{ kg}\cdot\text{m/s}$ during this time. The *rate* at which the interaction adds momentum to the water (i.e. the force $\vec{F}_N$ the interaction exerts on the water) therefore has a magnitude of (2 kg·m/s)/s = 2 N, which is roughly equal to 0.5 lb, which is about the weight of a large hamburger.

The forward momentum the nozzle gives to the water must come at the expense of the nozzle's momentum, meaning that the nozzle receives backward momentum at a rate of 2 N. To keep the nozzle at rest, I must therefore push forward on the nozzle with my hand so that my hand supplies forward momentum at exactly the same rate: if I do not, the nozzle will accumulate either forward or backward momentum, and therefore will not remain at rest.**

When I have used such a nozzle, I recall having to push forward on the nozzle to keep it at rest: about 0.5 lb seems to be a plausible force magnitude: this would be noticeable, but not difficult to apply.

*The nozzle accelerates the water because it constricts the opening that the water must flow through. The hose itself has a diameter of maybe 2 cm, while the nozzle opening may be something like 0.4 cm in diameter. Since the volume of water that flows out the nozzle every second must be the same as the volume that enters via the hose, the water must increase in speed in inverse proportion to the ratio of the nozzle and hose cross sectional areas so that the same amount of water flows through each in a given second (think about it). Therefore the final speed of water coming out the nozzle is about 25 times that of the water flowing in from the hose, so we make only a 4% error by assuming that the latter speed is zero. The crudeness of this model does not deserve a more precise estimate. Note that this means that if the hose opening is no smaller than the diameter of the hose itself, then there is *no* acceleration of the water at the nozzle, so the water will exert no net force on the hose opening (aside from possible transient effects when the water is first turned on): the water will simply flow through it at a constant rate.

**The force on the nozzle exerted by the water is NOT equal in magnitude to the force on the nozzle due to the hand by Newton's third law, which some of you might remember from high school and be tempted to use. We haven't mentioned this law yet in this course, but we will see later that this law only connects forces that a single interaction exerts on the two separate objects connected by that single interaction. Therefore, this law connects the forces that the contact interaction between the water and nozzle exerts on each, or the forces that the contact interaction between the hand and nozzle exerts on each, but it does NOT connect the force exerted by the hand on the nozzle with the force the water exerts on the nozzle: these are forces exerted on the *same* object by different interactions. If these forces are equal in magnitude (as they are likely to be in this case), it is because we observe that object to remain at rest: we can then conclude that the forces must be balanced so that the momentum flow rates are equal. (If one insists on appealing to one of Newton's laws to arrive at this conclusion, it is Newton's *second* law, not the third, that tells one that the net force on an object that does not accumulate momentum must be zero.)