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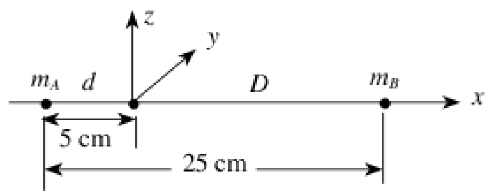
C4B.1 Let us choose our reference frame so that the line connecting the two particles coincides with the x axis, and let's put the system's center of mass at the origin, with particle A at the left (see below).



Note that the distances defined in the diagram are $d = 5$ cm and $D = 25$ cm – 5 cm = 20 cm. Finally, note that the system's total mass is $M = m_A + m_B$. The definition of the center of mass then tells us that

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{m_A + m_B}\left(m_A \begin{bmatrix} -d \\ 0 \\ 0 \end{bmatrix} + m_B \begin{bmatrix} +D \\ 0 \\ 0 \end{bmatrix} \right) \quad (1)$$

The top line of this equation tells us that

$$0 = \frac{m_B D - m_A d}{m_A + m_B} \quad \Rightarrow \quad m_B D - m_A d = 0 \quad \Rightarrow \quad m_B D = m_A d \quad \Rightarrow \quad \frac{m_B}{m_A} = \frac{d}{D} = \frac{5\ \cancel{\text{cm}}}{20\ \cancel{\text{cm}}} = \frac{1}{4} \quad (2)$$

(This makes sense, because the center of mass is closer to particle A, which we see by equation 2 is 4 times more massive than particle B.)

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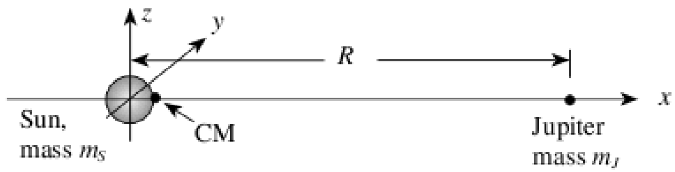
C4B.3 Let's choose our reference frame so that the centers of the sun and the Jupiter both lie on the x axis, with the sun's center at the origin, as shown in the diagram below.



Let the separation between the sun's center and Jupiter's center be $R = 779{,}000{,}000$ km, and let m_S be the sun's mass and m_J be Jupiter's mass, as shown in the diagram. Note that $m_S/m_J = 1047$ here, and that the system's total mass is $M = m_S + m_J$. In this reference frame, the definition of the center of mass implies that

$$\begin{bmatrix} x_{\text{CM}} \\ y_{\text{CM}} \\ z_{\text{CM}} \end{bmatrix} = \frac{1}{m_S + m_J}\left(m_S \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_J \begin{bmatrix} +R \\ 0 \\ 0 \end{bmatrix} \right) \tag{1}$$

The top line of this equation implies that $y_{\text{CM}} = z_{\text{CM}} = 0$, so the distance between the sun's center and the system's center of mass is

$$x_{\text{CM}} = \frac{0 + m_J R}{m_S + m_J} = \frac{R}{(m_S / m_J) + 1} = \frac{779{,}000{,}000 \text{ km}}{1047 + 1} = 743{,}000 \text{ km} \tag{2}$$

Since the sun's radius is about 696,000 km, the center of mass lies somewhat *outside* the sun's surface.

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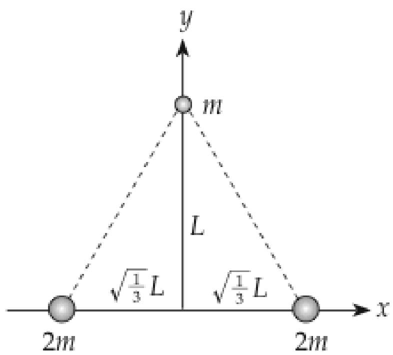
C4B.7 Taking the hint in the problem statement, let's define our reference frame so that the system looks as shown in the diagram below (note that $\tan 30° = \sin 30° / \cos 30° = (1/2)/(\sqrt{3}/2) = \sqrt{1/3}$.:



This choice of reference frame makes many position components conveniently zero. Note that the system's total mass is $M = 2m + 2m + m = 5m$ here. The definition of the center of mass implies that

$$\begin{bmatrix} x_{CM} \\ y_{CM} \\ z_{CM} \end{bmatrix} = \frac{1}{5m}\left(m\begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix} + 2m\begin{bmatrix} -\sqrt{\frac{1}{3}}L \\ 0 \\ 0 \end{bmatrix} + 2m\begin{bmatrix} +\sqrt{\frac{1}{3}}L \\ 0 \\ 0 \end{bmatrix}\right) = \frac{1}{5}\begin{bmatrix} 0 - 2\sqrt{\frac{1}{3}}L + 2\sqrt{\frac{1}{3}}L \\ L+0+0 \\ 0+0+0 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix}$$

So the system's center of mass lies on the y axis of our coordinate system, a distance $L/5$ above the line connecting the two massive particles. It makes sense that the center of mass should be much closer to the line connecting these massive particles than to light particle.

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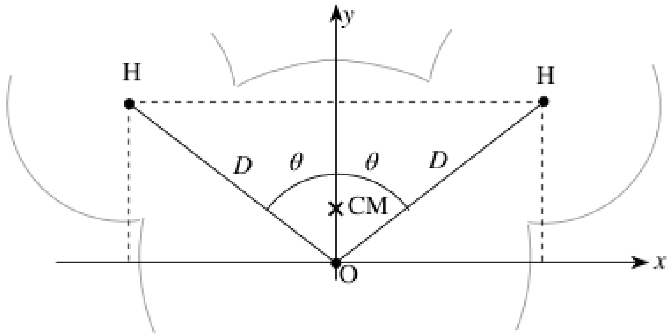
C4M.4 Taking the hint in the problem, a revised diagram of the water molecule looks like this:



Note again that the hydrogen nuclei have mass m while the oxygen nucleus has mass $16m$. Therefore, the definition of the center of mass implies that in this reference frame

$$\begin{bmatrix} x_{CM} \\ y_{CM} \\ z_{CM} \end{bmatrix} = \frac{1}{18m}\left(m\begin{bmatrix} D\sin\theta \\ D\cos\theta \\ 0 \end{bmatrix} + m\begin{bmatrix} -D\sin\theta \\ D\cos\theta \\ 0 \end{bmatrix} + 16m\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = \frac{1}{18}\begin{bmatrix} D\sin\theta - D\sin\theta + 0 \\ D\cos\theta + D\cos\theta + 0 \\ 0+0+0 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 0 \\ 2D\cos\theta \\ 0 \end{bmatrix} \quad (1)$$

$$\Rightarrow \quad y_{CM} = \tfrac{1}{9}D\cos\theta = \tfrac{1}{9}(96\text{ pm})\cos(52.5°) = 6.50\text{ pm} \quad (2)$$

and $x_{CM} = z_{CM} = 0$. Therefore, the center of mass lies 6.5 pm from the oxygen nucleus on a line perpendicular to the line connecting the hydrogen nuclei and halfway between those nuclei. This is a bit less than the 7.5 pm calculated in example C4.2, but this makes sense, because the vertical spacing between the hydrogen and oxygen atoms was reduced when we changed the angle between atoms from 90° to 105°.

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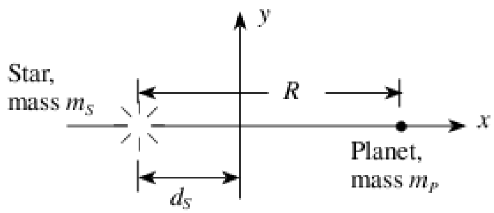
C4M.7 Assuming that the star and planet are far away from other stars, the effects of external interactions will be negligible. Thus the system's center of mass should move in a straight line, as shown in figure C4.6 in the text. Let us set up our reference frame so that its origin is at the center and so that the star and planet both lie along the x axis at some given instant of time. We know that the star's mass is $m_S = (0.3)(2.0 \times 10^{30}\text{ kg}) = 6.0 \times 10^{29}$ kg, that the planet's mass is $m_P = (1.5)(1.9 \times 10^{27}\text{ kg}) = 2.9 \times 10^{27}$ kg, and that the distance between the star and planet is $R = 1.5 \times 10^{12}$ m. Let the unknown distance between the star and the center of mass be d_S. The diagram below illustrates the situation.



(a) The definition of the center of mass implies that

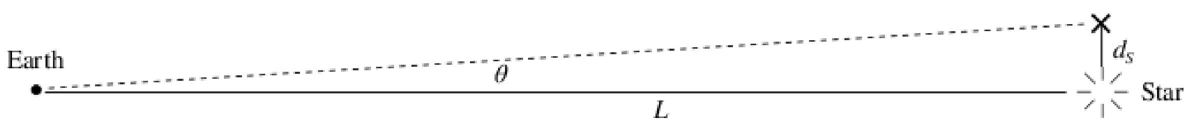
$$\frac{1}{M}\left(m_S\begin{bmatrix}-d_S\\0\\0\end{bmatrix}+m_P\begin{bmatrix}R-d_S\\0\\0\end{bmatrix}\right)=\begin{bmatrix}x_{CM}\\y_{CM}\\z_{CM}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{1}$$

The z component of this equation, after we multiply both sides by the system's total mass M, says that

$$-m_S d_S + m_P(R-d_S) = 0 \quad\Rightarrow\quad m_S d_S = m_P(R-d_S) = m_P R - m_P d_S$$

$$\Rightarrow\quad (m_S+m_P)d_S = m_P R \quad\Rightarrow\quad d_S = \frac{m_P R}{m_S+m_P} = \frac{(2.9\times10^{27}\,\cancel{\text{kg}})(1.5\times10^{12}\text{ m})}{(6.0+0.029)\times10^{29}\,\cancel{\text{kg}}} = 7.2\times10^{9}\text{ m} \tag{2}$$

(b) The drawing below shows that this distance spans an angle θ such that $\tan\theta = d_s/L$:



Therefore:

$$\theta = \tan^{-1}\left[\frac{d_S}{L}\right] = \tan^{-1}\left[\frac{7.2\times10^{9}\,\cancel{\text{m}}}{8.1\,\cancel{\text{ly}}}\left(\frac{1\,\cancel{\text{ly}}}{0.946\times10^{16}\,\cancel{\text{m}}}\right)\right] = 5.4\times10^{-6}\text{ deg}\left(\frac{3.6\times10^{6}\text{ mas}}{1\text{ deg}}\right) = 19\text{ mas} \tag{3}$$

(c) This angle is a tiny fraction of the size of the star's image after it has been smeared out by atmospheric disturbances, so it is going to be very difficult to observe. This is part of the reason that G. Gatewood's tentative discovery of a planet orbiting Lalande 21185 remains tentative.

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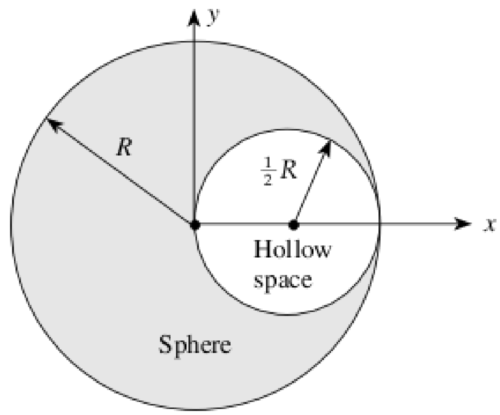
C4R.1 A cross-sectional diagram of the described object looks like this:



Note that I have defined my reference frame so that the main sphere is centered on the origin and the center of the spherical hollow space lies on the x axis. Now, note that if the sphere were whole, then its center of mass would be at the origin. So lets imagine filling the hollow space with the same material as the rest of the sphere, and let's call this object A, while the original sphere with the hollow space is object B. If the mass of the complete sphere (objects A + B) is M, then the mass of the material filling the hollow space will be $m_B = \frac{1}{8}M$ (because its volume is 1/8 of the volume of the whole sphere) and the mass of the rest of the sphere is $m_B = \frac{7}{8}M$. Let x_{CMB}, y_{CMB}, z_{CMB} be the unknown components of the latter object's center of mass. Since the center of mass of the material filling the hollow space will be at $x = \frac{1}{2}R$, and since when calculating the center of mass of a system of objects, we can treat each object as if it were a particle concentrated at its center of mass, the center of mass of the complete sphere ($A + B$) is

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\cancel{M}}\left(\frac{7}{8}\cancel{M}\begin{bmatrix} x_{CMB} \\ y_{CMB} \\ z_{CMB} \end{bmatrix} + \frac{1}{8}\cancel{M}\begin{bmatrix} +\frac{1}{2}R \\ 0 \\ 0 \end{bmatrix} \right) \Rightarrow \frac{7}{8}\begin{bmatrix} x_{CMB} \\ y_{CMB} \\ z_{CMB} \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -\frac{1}{2}R \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_{CMB} \\ y_{CMB} \\ z_{CMB} \end{bmatrix} = \begin{bmatrix} -\frac{1}{14}R \\ 0 \\ 0 \end{bmatrix}$$

We see, therefore, that the center of mass of object B (the sphere with the hollow space) is located at a point on the x axis that is $\frac{1}{14}R$ to the left of the origin. This makes sense, because when we remove the material in the hollow space, the center of mass should shift a bit away from the hollow space: more material will be to the left of the origin than to the right.

(An alternative method for solving this problem would be to add a small sphere with negative mass $m_A = -\frac{1}{8}M$ to a complete sphere. Adding a small sphere with negative mass to a complete large sphere is equivalent to removing the material in the hollow space.)