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C5B.3 Since I am on a flat, frictionless surface, I am functionally isolated. But if I am willing to sacrifice a shoe, I can consider myself and the shoe to be two interacting particles in a functionally isolated system. If I throw the shoe, the interaction between me and the shoe supplies an impulse to the shoe and an equal and opposite impulse to me. So I can get off the ice by throwing the shoe in a direction opposite to the direction that I want to go; the interaction will end up transferring momentum to me in the direction that I want to go, and thus changing my velocity from zero to something small in my preferred direction.

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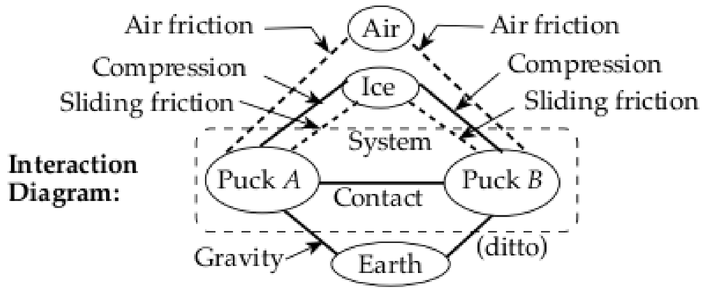
C5B.7 An interaction diagram for this situation might look like this:



(This is pretty much like the diagram that is the answer to exercise C5X.2.) If we ignore friction between the pucks and the ice, then this system is functionally isolated, but even if the sliding friction is not negligible, we can still treat it like a collision.

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C5B.8 As stated in the problem, let the carts' masses be m_1 and $m_2 = 2m_1$, the incoming cart's velocity be $\vec{v}_0$, and the lighter and more massive carts' final velocities be $\vec{v}_1$ and $\vec{v}_2$ respectively. Since both $\vec{v}_0$ and $\vec{v}_2$ are in the $+x$ direction, conservation of momentum for this functionally isolated system requires that

$$\begin{bmatrix} m_1|\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m_1 v_{1x} \\ m_1 v_{1y} \\ m_1 v_{1z} \end{bmatrix} + \begin{bmatrix} m_2|\vec{v}_2| \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

We see that $\vec{v}_1$ must also lie completely along the x axis (because $v_{1y} = v_{1z} = 0$). Solving the top component of equation 1 for v_{1x} and using the facts that $|\vec{v}_2| = \frac{5}{9}|\vec{v}_0|$ and $m_2 = 2m_1$ yields

$$m_1 v_{1x} = m_1|\vec{v}_0| - m_2|\vec{v}_2| \;\Rightarrow\; v_{1x} = |\vec{v}_0| - \frac{m_2}{m_1}|\vec{v}_2| = (1 - 2\tfrac{5}{9})|\vec{v}_0| = -\tfrac{1}{9}|\vec{v}_0| \tag{2}$$

Therefore, the lighter cart moves backwards (in the $-x$ direction) at a speed of $\frac{1}{9}|\vec{v}_0|$ after the collision.

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C5B.9 The carts are "identical" so their masses should be the same: call the common mass m. As stated in the problem, let the incoming cart's velocity be $\vec{v}_0$, and the rear and front carts' final velocities be $\vec{v}_1$ and $\vec{v}_2$ respectively (the "rear" cart is the originally moving cart, since it cannot pass through the cart it struck). Since both $\vec{v}_0$ and $\vec{v}_2$ are in the $+x$ direction, conservation of momentum for this functionally isolated system requires that

$$\begin{bmatrix} m|\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} mv_{1x} \\ m_1 v_{1y} \\ m_1 v_{1z} \end{bmatrix} + \begin{bmatrix} m|\vec{v}_2| \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

We see that $\vec{v}_1$ must also lie completely along the x axis (because $v_{1y} = v_{1z} = 0$). Solving the top component of equation 1 for v_{1x} and using the fact that $|\vec{v}_2| = \frac{5}{6}|\vec{v}_0|$ yields

$$mv_{1x} = m|\vec{v}_0| - m|\vec{v}_2| \;\Rightarrow\; v_{1x} = |\vec{v}_0| - |\vec{v}_2| = (1 - \tfrac{5}{6})|\vec{v}_0| = \tfrac{1}{6}|\vec{v}_0| \tag{2}$$

Therefore, the lighter cart moves forwards (in the $+x$ direction) at a speed of $\frac{1}{6}|\vec{v}_0|$ after the collision.

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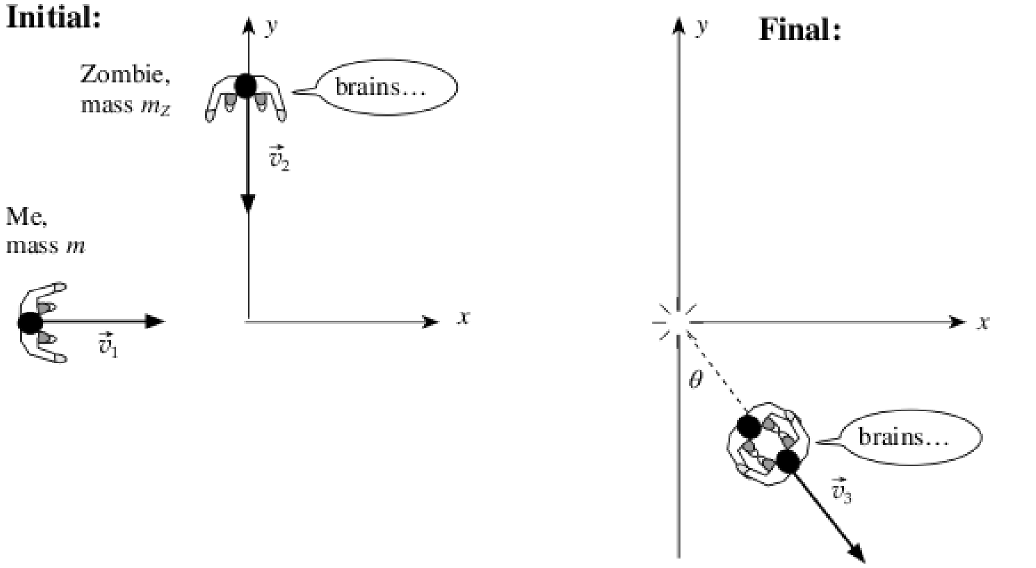
C5M.2 Let's set up a reference frame in standard orientation on the earth's surface with the origin at the point where you and the zombie collide. Initial and final diagrams of the situation look like this:



$m = 60$ kg
$m_Z = 50$ kg
$\vec{v}_1 = 2.0$ m/s east
$\vec{v}_2 = 3.2$ m/s south
$\vec{v}_3 = ?$
$\theta = ?$

The interacting system here is me and the zombie. We participate in a contact interaction with the ice and gravitationally with the earth (I am ignoring weak interactions with the air). If the ice is truly level and frictionless, then the gravitational interaction acting on each person (living and undead) will be canceled by that person's contact interaction with the ice, so the system is thus functionally isolated. (If there is significant friction, then we can treat this as a collision.)* In this case, where we have two objects initially but only one (composite) object after the collision, the system's final mass is $M = m + m_Z$ and conservation of momentum implies that

$$m\begin{bmatrix} |\vec{v}_1| \\ 0 \\ 0 \end{bmatrix} + m_Z\begin{bmatrix} 0 \\ -|\vec{v}_2| \\ 0 \end{bmatrix} = M\begin{bmatrix} v_{3x} \\ v_{3y} \\ v_{3z} \end{bmatrix} \quad (1)$$

where $M = m + m_Z$ (assuming that the zombie loses no parts in the collision). The three components of this equation provide enough equations to solve for the three unknown velocity components, and we can calculate the final velocity's direction from these components. The bottom component of this equation tells us that $v_{3z} = 0$, meaning that the zombie and I will stay on the ground. If we divide both sides of the other two component equations by $M = m + m_Z$, we get

$$v_{3x} = \frac{m|\vec{v}_1|}{M} = \frac{(60\ \cancel{\text{kg}})(2.0\ \text{m/s})}{60\ \cancel{\text{kg}} + 50\ \cancel{\text{kg}}} = 1.09\ \frac{\text{m}}{\text{s}}, \qquad v_{3y} = \frac{-m_Z|\vec{v}_2|}{M} = \frac{(50\ \cancel{\text{kg}})(3.2\ \text{m/s})}{60\ \cancel{\text{kg}} + 50\ \cancel{\text{kg}}} = -1.45\ \frac{\text{m}}{\text{s}} \quad (2)$$

By the definition of the tangent, the angle θ in the diagram has the value

$$\theta = \tan^{-1}\left|\frac{v_{3x}}{v_{3y}}\right| = \tan^{-1}\left|\frac{1.09}{-1.45}\right| = 37° \quad (3)$$

Since this angle is indeed the angle east of south, I might just have a chance! (Note that the units come out right, and signs of the final velocity components come out as expected.)
*Even if sliding friction is significant, as long as the quoted initial velocities are valid just before the collision, the direction of the final velocity will not change after the collision, because the sliding friction is opposite to that final velocity and so cannot change it. Therefore I will *still* slide toward the car, and my clever scheme might still work (as long as we actually slide all the way to the car!).

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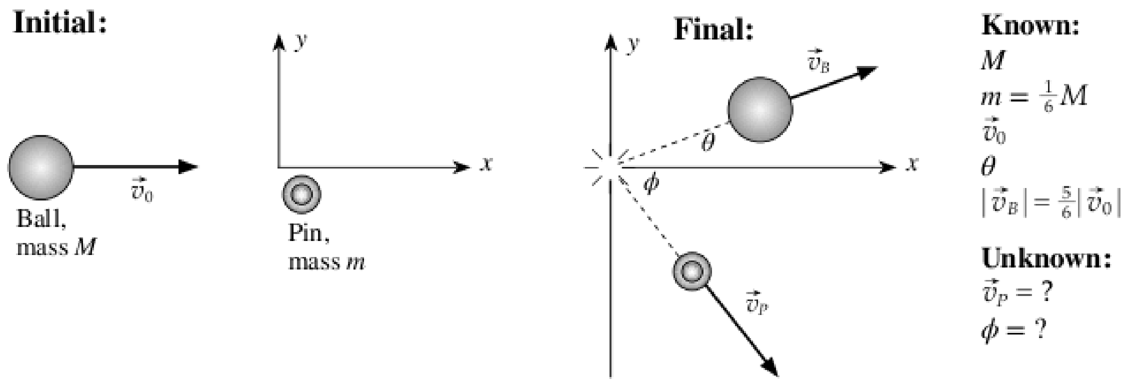
C5M.3 Let's set up a reference frame whose $+x$ direction coincides with the bowling ball's initial direction of motion, and the y direction perpendicular to that in the direction that the ball is deflected. Initial and final diagrams of the situation then look like this:



The interacting system here is the ball and pin. They both interact with the floor via a contact interaction and gravitationally with the earth (I am ignoring weak interactions with the air). Friction interactions with the floor could ultimately be significant, so we will treat this as a collision and assume that final velocities only apply right after the collision. Conservation of momentum in this situation implies that

$$M\begin{bmatrix} |\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + m\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = M\begin{bmatrix} |\vec{v}_B|\cos\theta \\ |\vec{v}_B|\sin\theta \\ 0 \end{bmatrix} + m\begin{bmatrix} v_{Px} \\ v_{Py} \\ v_{Pz} \end{bmatrix} \tag{1}$$

The three components of this equation provide enough equations to solve for the three unknown pin velocity components, and we can calculate that velocity's direction angle ϕ from these components. The bottom component of this equation tells us that $v_{3z} = 0$, meaning that ball and pin will stay on the ground.

(a) If we divide both sides of the other two component equations by M and use the fact that $m/M = \frac{1}{6}$, we find that

$$v_{Px} = \frac{M}{m}(|\vec{v}_0| - |\vec{v}_B|\cos\theta) = 6(|\vec{v}_0| - \tfrac{5}{6}|\vec{v}_0|\cos\theta) = (6 - 5\cos\theta)|\vec{v}_0| \tag{2a}$$

$$v_{Py} = \frac{M}{m}(-|\vec{v}_B|\sin\theta) = 6(-\tfrac{5}{6}|\vec{v}_0|\sin\theta) = (-5\sin\theta)|\vec{v}_0| \tag{2b}$$

(b) By the definition of the tangent, the angle ϕ in the diagram has the value

$$\phi = \tan^{-1}\left|\frac{v_{Py}}{v_{Px}}\right| = \tan^{-1}\left|\frac{-5\sin\theta}{6 - 5\cos\theta}\right| = \tan^{-1}\left|\frac{\sin\theta}{\frac{6}{5} - \cos\theta}\right| \tag{3}$$

Note that the argument of the inverse tangent is unitless (as it must be).

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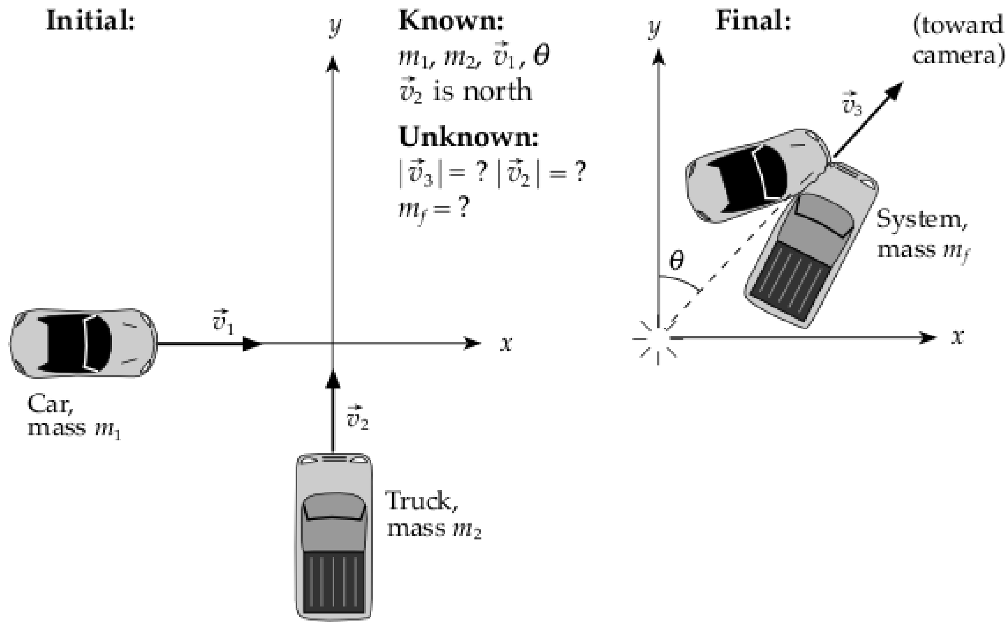
C5M.6 Let's set up our reference frame in standard orientation on the earth's surface, with its origin located at the position of impact. Initial and final diagrams for this situation therefore look like this:



The system here is the car and truck. Both interact gravitationally with the earth and in compression and friction contact interactions with the ground. The momentum flows from the compression and gravitational interactions will mostly cancel, but the friction interaction is likely to be significant, especially after the crash. Fortunately, initial friction interactions will be taken care of by the drivers, who will use the vehicles' engines to maintain a constant speed. Afterward, the friction interactions will drain momentum from the system, but since friction will exert a force directly opposite to the interlocked vehicles' velocity, it will only change their speed, not their direction of motion, which is what we are concerned with here. So we can model this situation as a collision, applying the angle θ that we want just after the collision, and be confident that the interlocked vehicles will continue to move in that direction subsequently. Assuming that no significant chunks of the car and/or truck fly off, carrying significant amounts of momentum away from the collision, then conservation of momentum requires that

$$m_1 \begin{bmatrix} |\vec{v}_1| \\ 0 \\ 0 \end{bmatrix} + m_2 \begin{bmatrix} 0 \\ |\vec{v}_2| \\ 0 \end{bmatrix} = m_f \begin{bmatrix} |\vec{v}_3|\sin\theta \\ |\vec{v}_3|\cos\theta \\ 0 \end{bmatrix} \tag{1}$$

The top two components of these equations provide two equations in our three unknowns m_f, $|\vec{v}_3|$, and $|\vec{v}_2|$. If we assume that no significant pieces even drop off the car and/or truck, the third equation $m_f = m_1 + m_2$ would provide enough additional information to solve for all three unknowns. But we are really only interested in $|\vec{v}_2|$. We can get rid of the unknowns m_f and $|\vec{v}_3|$ if we simply divide the y component equation by the x component equation

$$\frac{m_2|\vec{v}_2|}{m_1|\vec{v}_1|} = \frac{\cancel{m_f|\vec{v}_3|}\cos\theta}{\cancel{m_f|\vec{v}_3|}\sin\theta} = \frac{1}{\tan\theta} \quad \Rightarrow \quad |\vec{v}_2| = \frac{m_1|\vec{v}_1|}{m_2\tan\theta} \tag{2}$$

We know m_1, m_2, $|\vec{v}_1|$, and θ, so substituting those numbers into the equation above will yield the value of $|\vec{v}_2|$ that we need. (Note that since the units of m_1 and m_2 will cancel and $\tan\theta$ is unitless, $|\vec{v}_2|$ will have the same units as $|\vec{v}_1|$, as it must.)

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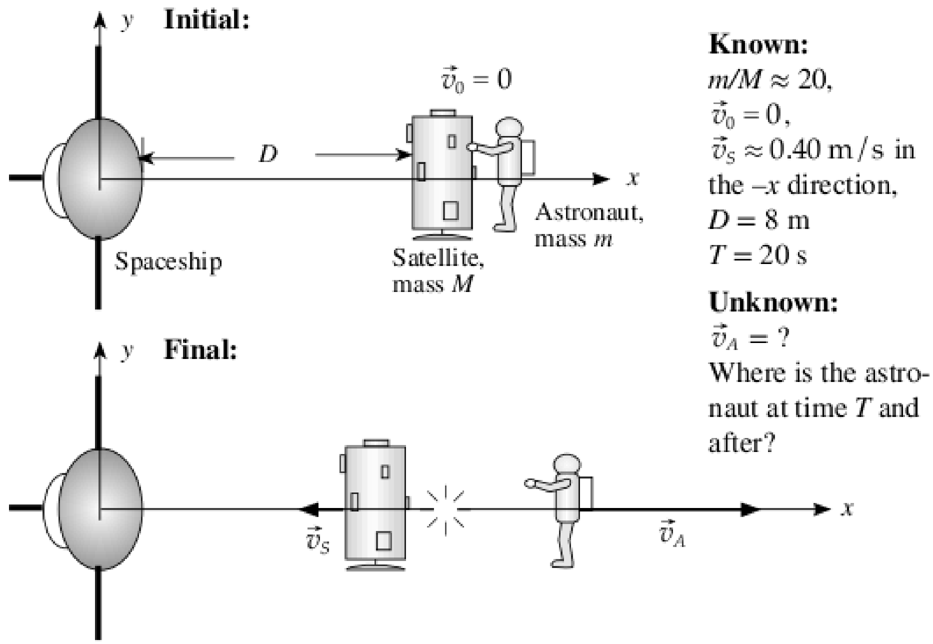
C5M.7 Let's set up a reference frame attached to the astronauts' spaceship with the origin at that spaceship, and the x axis going through the satellite's original position. After the spring is released, the satellite covers a distance of $D = 8$ m in $T = 20$ s, so it must be moving with a speed $|\vec{v}_S| = D/T = 8\text{ m}/20\text{ s} = 0.40\text{ m/s}$ in the $-x$ direction (toward the spaceship) after the interaction. A diagram of this situation then looks like:



The key is that the astronaut-satellite system is isolated because it floats in space. Therefore this system's momentum must be conserved, which implies that

$$m\begin{bmatrix}0\\0\\0\end{bmatrix} + M\begin{bmatrix}0\\0\\0\end{bmatrix} = m\begin{bmatrix}v_{Ax}\\v_{Ay}\\v_{Az}\end{bmatrix} + M\begin{bmatrix}-|\vec{v}_S|\\0\\0\end{bmatrix} \Rightarrow m\begin{bmatrix}v_{Ax}\\v_{Ay}\\v_{Az}\end{bmatrix} = M\begin{bmatrix}|\vec{v}_S|\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}v_{Ax}\\v_{Ay}\\v_{Az}\end{bmatrix} = \frac{M}{m}\begin{bmatrix}|\vec{v}_S|\\0\\0\end{bmatrix} \tag{1}$$

The y and z components of this equation tell us that the astronaut is moving along the x axis. The x component of this equation tells us that the astronaut's speed is

$$|\vec{v}_A| = v_{Ax} = \frac{M}{m}|\vec{v}_S| = 20\left(0.40\ \frac{\text{m}}{\text{s}}\right) = 8.0\ \frac{\text{m}}{\text{s}} \tag{2}$$

So after 20 seconds, the partner should look for the astronaut at a distance of $|\vec{v}_A|T + D = 168$ m in the $+x$ direction away from the spaceship. Of course, in the 60 seconds it took the partner (a very bright physics graduate) to solve the problem, the astronaut has moved another 480 m away!

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C5D.1 **(a)** Momentum has SI units of kg·m/s, while weight (because it is a force) has SI units of newtons, so momentum transferred per unit propellant weight has SI units of

$$\frac{\text{kg}\cdot\text{m/s}}{\text{N}}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \frac{\text{s}^2}{\text{s}} = \text{s} \tag{1}$$

(b) Let the propellant exhaust speed be $|\vec{v}_e|$. If a rocket engine ejects propellant having mass m during a certain time period, the momentum transferred to the propellant will have a magnitude of $m|\vec{v}_e|$, and the specific impulse will be $m|\vec{v}_e|/m|\vec{g}| = |\vec{v}_e|/|\vec{g}|$, where $|\vec{g}|$ is the gravitational field strength of what ever gravitational field is used to determine the propellant's weight. Therefore, a fuel's specific impulse is indeed proportional to $|\vec{v}_e|$, and the constant of proportionality is $1/|\vec{g}|$.

(c) Assuming that we use the value of $|\vec{g}|$ at the earth's surface to determine the propellant's weight, the specific impulse of a chemical rocket that ejects its fuel at $|\vec{v}_e| = 4.5$ km/s will be

$$\frac{|\vec{v}_e|}{|\vec{g}|} = \frac{4500\ \text{m/s}}{9.8\ \text{N/kg}}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 460\ \text{s} \tag{2}$$

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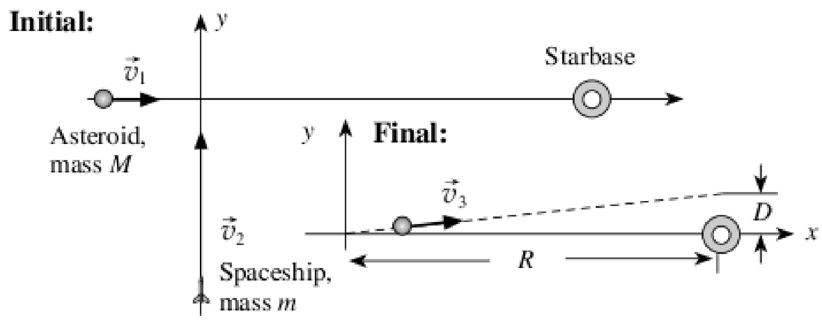
C5R.1 Let's set up a reference frame so that the asteroid's original direction of motion and the starbase both lie on the x axis. The problem does not explicitly state the the spaceship's trajectory, but we will plausibly get the largest deflection of the asteroid's trajectory if the ship is traveling perpendicular to the x axis when the ship hits: assume this. Pictures of our system before and after the collision then look like:



Known:
$m = M/10^6$, $|\vec{v}_1|$
$\vec{v}_1$ is in the $+x$ direction,
$\vec{v}_2$ is in the $+y$ direction,
$|\vec{v}_2| = 5|\vec{v}_1|$, D

Unknown:
$R = ?$
Minimum time T between initial and final situations $= ?$

The system here is the asteroid and rocket, which is isolated because it floats in space. If the ship buries itself in the asteroid without shedding fragments, then the system after the impact will have a total mass of $M + M/10^6 \approx M$ (to six decimal places). Therefore conservation of momentum requires that

$$M\begin{bmatrix}|\vec{v}_1|\\0\\0\end{bmatrix} + m\begin{bmatrix}0\\|\vec{v}_2|\\0\end{bmatrix} = M\begin{bmatrix}v_{3x}\\v_{3y}\\v_{3z}\end{bmatrix} \quad\Rightarrow\quad \begin{bmatrix}|\vec{v}_1|\\0\\0\end{bmatrix} + \frac{m}{M}\begin{bmatrix}0\\5|\vec{v}_1|\\0\end{bmatrix} = \begin{bmatrix}v_{3x}\\v_{3y}\\v_{3z}\end{bmatrix} \tag{1}$$

We also know that since the asteroid's velocity will be constant after the impact, $v_{3x} = R/T$ and $v_{3y} = D/T$. These two equations, in combination with the three component equations in equation 1, yield five equations in our five unknowns v_{3x}, v_{3y}, v_{3z}, R, and T (note that while we don't know m and M separately, we do know the ratio m/M. So we can in principle solve for all unknowns.

(a) However, the y component equation and $v_{3y} = D/T$ are all we really need:

$$\frac{m}{M}5|\vec{v}_1| = v_{3y} = \frac{D}{T} \quad\Rightarrow\quad T = \left(\frac{M}{m}\right)\frac{D}{5|\vec{v}_1|} \tag{2}$$

(b) With the specified values of D and $|\vec{v}_1|$, we get

$$T = \frac{(10^6)D}{5|\vec{v}_1|} = \frac{(10^6)(1800\,\text{m})}{5(18{,}000\,\text{m/s})} = 20{,}000\,\text{s}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 5.6\,\text{h} \tag{6}$$

The units of the last equation are correct for a time, and the result is plausible. Note that this is the *minimum* time (any impact time earlier than this will be fine!).