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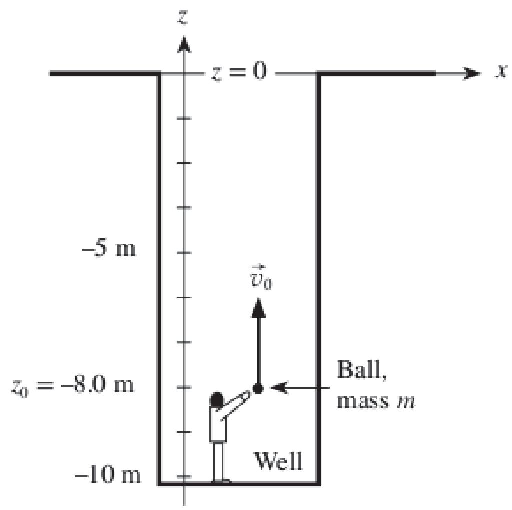
C8B.2 The diagram below shows the situation and defines some symbols:



The system here involves the ball interacting gravitationally with the earth (once it has left the person's hand). The ball is always close to the earth's surface here, we can use the potential energy formula $V_g(z) = m|\vec{g}|z$ to describe the energy of the gravitational interaction between the ball and earth, and $V_g(0)$ is already zero at the reference separation $z = 0$, so we don't need to add a constant to the formula. Ignoring internal energies and the earth's kinetic energy, then, the system's total energy initially is simply

$$E = \tfrac{1}{2}m|\vec{v}_0|^2 + V_g(z_0) = \tfrac{1}{2}(0.20\text{ kg})(6\text{ m/s})^2 + (0.20\text{ kg})(9.8\text{ m/s}^2)(-8.0\text{ m}) = -12\text{ kg}\cdot\text{m}^2/\text{s}^2 = -12\text{ J}$$

(Negative energy simply means that the system's energy is less than the energy it would have with the ball at rest at the reference position where $z = 0$. This means that the ball will never rise to that position.)

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C8B.3 If the car has mass m and initial speed $|\vec{v}_1| = 55$ mi/h and final speed $|\vec{v}_2| = 77$ mi/h, then the ratio of its kinetic energies K_1 and K_2 at those speeds is

$$\frac{K_2}{K_1} = \frac{\frac{1}{2}m|\vec{v}_2|^2}{\frac{1}{2}m|\vec{v}_1|^2} = \left(\frac{77\ \text{mi/h}}{55\ \text{mi/h}}\right)^2 = 1.96$$

This means that when a car's speed increases by 40%, its kinetic energy increases by 96%! (The extra increase is due to the fact that the car's kinetic energy depends on the square of its speed.) Since the severity of a crash depends on the kinetic energy involved, this means that even marginally higher speeds can lead to much more severe crashes.

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C8B.6 No matter how you throw it, the ball must climb a certain fixed vertical distance to get from you to the receiver. This means that the gravitational potential energy associated with the ball's interaction with the earth must increase by a fixed amount. Assuming that the ball's and earth's internal energies do not change in this process, that energy must come at the expense of the ball's kinetic energy (the change in the earth's kinetic energy is negligible). So the difference between the ball's initial and final kinetic energies must be the same no matter how you throw it. Since the final kinetic energy is fixed by the requirement that the ball have a final speed of 5 m/s, the initial kinetic energy must also be fixed. Since that kinetic energy depends only on the ball's mass (which is fixed) and its speed (which is independent of direction), the ball's initial speed must be the same no matter how you throw it. (Energy is an ordinary number that does not depend on the direction of motion.)

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C8B.9 The system here is the rock and the earth, which interact with each other gravitationally. This system is isolated because it floats in space. As the rock falls, conservation of energy implies that the gravitational interaction between the rock and the earth converts gravitational potential energy to other forms of energy. We don't expect the earth's kinetic energy K_e, the rock's internal energy U_r or the earth's internal energy U_e to change much while the rock is falling frictionlessly through the vacuum above the earth's atmosphere, so the converted gravitational potential energy will go virtually entirely to changing the rock's kinetic energy. Let the rock's mass be m and its unknown final speed as it enters the atmosphere be $|\vec{v}_f|$. We are told that the rock is falling from rest.

(a) If we use the near-earth gravitational potential energy formula $V_g = m|\vec{g}|z$ (where z is the rock's vertical position and $|\vec{g}|$ is the gravitational field strength) and the initial and final values of z specified in the problem, then conservation of energy tells us that

$$\tfrac{1}{2}m\cdot 0^2 + \cancel{U_r} + \overbrace{K_e}^{0} + \cancel{U_e} + m|\vec{g}|\tfrac{1}{4}R = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_r} + \overbrace{K_e}^{\approx 0} + \cancel{U_e} + m|\vec{g}|\tfrac{1}{100}R \;\Rightarrow\; |\vec{v}_f|^2 = 2|\vec{g}|(\tfrac{1}{4} - \tfrac{1}{100})R \qquad (1)$$

Remembering that $|\vec{g}| = GM/R^2$ (see equation C8.11), this becomes

$$|\vec{v}_f| = \sqrt{2\frac{GM}{R^2}\left(\frac{25}{100} - \frac{1}{100}\right)R} = \sqrt{2\frac{24}{100}\frac{GM}{R}} = \frac{\sqrt{12}}{5}\sqrt{\frac{GM}{R}} = 0.6928\sqrt{\frac{GM}{R}} \qquad (2)$$

(If you are curious about the numerical value, it is 5490 m/s.)

(b) If we instead use the universal gravitational potential energy equation $V_g = -GMm/r$, and note that r from the center of the earth is initially $r_i = R + \frac{1}{4}R = \frac{5}{4}R$ and finally $r_f = R + \frac{1}{100}R = \frac{101}{100}R$, conservation of energy now implies that

$$\tfrac{1}{2}m\cdot 0^2 + \cancel{U_r} + \overbrace{K_e}^{0} + \cancel{U_e} - \frac{GMm}{r_i} = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_r} + \overbrace{K_e}^{\approx 0} + \cancel{U_e} - \frac{GMm}{r_f} \;\Rightarrow\; |\vec{v}_f|^2 = \frac{2GM}{r_f} - \frac{2GM}{r_i}$$

$$\Rightarrow\; |\vec{v}_f|^2 = \frac{2GM}{\frac{101}{100}R} - \frac{2GM}{\frac{5}{4}R} = \frac{GM}{R}\left(\frac{200}{101} - \frac{8}{5}\right) = \frac{GM}{R}\left(\frac{1000-808}{505}\right) = \frac{192}{505}\frac{GM}{R}$$

$$\Rightarrow\; |\vec{v}_f| = \sqrt{\frac{192}{505}}\sqrt{\frac{GM}{R}} = 0.6166\sqrt{\frac{GM}{R}} \qquad (3)$$

This is the correct result, so the fractional error in the first result is

$$\frac{0.6928 - 0.6166}{0.6166} = 0.124 \qquad (4)$$

Using the near-earth formula therefore results in a bit more than a 12% error in predicting the rock's final speed.

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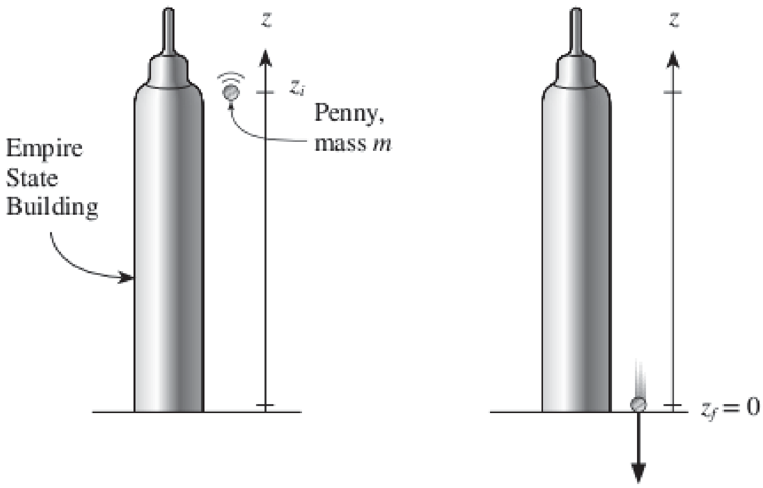
C8M.1 Initial and final drawings for this situation look like this:



$z_i = 380$ m
$z_f = 0$
initial speed $|\vec{v}_i| = 0$
$|\vec{g}| = 9.8$ m/s^2
$m = ?$
For part (a): $|\vec{v}_f| = ?$
For part (b):
$|\vec{v}_f| = 60$ m/s
Change in internal energy $U_f - U_i = ?$

Internal interactions:
- Gravitational $V_g = m|\vec{g}|z$

The system here is the penny and the earth, which interact with each other gravitationally. This system is isolated because it floats in space. We don't expect the earth's kinetic energy K_e to change much as the penny falls, because the earth's mass is enormously greater than that of the penny. Since the penny is always near the earth's surface, we can use the near-earth gravitational potential energy formula $V_g = m|\vec{g}|z$ handle the internal gravitational interaction's energy, as shown above.

(a) We will first assume that the penny's internal energy U_p and the earth's internal energy U_e do not change as the penny falls. Conservation of energy then tells us that

$$\tfrac{1}{2}m\overbrace{|\vec{v}_i|^2}^{0} + \cancel{U_p} + \overbrace{K_e}^{0} + \cancel{U_e} + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_p} + \overbrace{K_e}^{\approx 0} + \cancel{U_e} + m|\vec{g}|\overbrace{z_f}^{0} \quad\Rightarrow\quad \tfrac{1}{2}m|\vec{v}_f|^2 = m|\vec{g}|z_i$$

$$\Rightarrow\quad |\vec{v}_f| = \sqrt{2|\vec{g}|z_i} = \sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(380\text{ m})} = 86.3\,\frac{\text{m}}{\text{s}} \tag{1}$$

(b) Now let's assume that the internal thermal energy of the earth's atmosphere and/or the penny *does* change as a result of the penny's friction interaction with the air as it falls. This will probably affect the thermal energy of both and we don't know how much will go to each, so let's simply define U_i and U_f to be the *system's* initial and final internal energies, respectively. In this case, the conservation of energy master equation becomes

$$\tfrac{1}{2}m\overbrace{|\vec{v}_i|^2}^{0} + \overbrace{K_e}^{0} + U_i + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_f|^2 + \overbrace{K_e}^{\approx 0} + U_f + m|\vec{g}|\overbrace{z_f}^{0} \quad\Rightarrow\quad m|\vec{g}|z_i + U_i = \tfrac{1}{2}m|\vec{v}_f|^2 + U_f \tag{2}$$

The ratio of the change in the system's internal energy to the system's change in potential energy is

$$\frac{U_f - U_i}{m|\vec{g}|z_i - 0} = \frac{m|\vec{g}|z_i - \frac{1}{2}m|\vec{v}_f|^2}{m|\vec{g}|z_i} = 1 - \frac{|\vec{v}_f|^2}{2|\vec{g}|z_i} = 1 - \frac{(60\ \cancel{\text{m/s}})^2}{(86.3\ \cancel{\text{m/s}})^2} = 0.52 \tag{3}$$

where I used the result for $\sqrt{2|\vec{g}|z_i}$ from part (a). This means that a bit more than half of the system's change in potential energy was converted into internal energy in this process.

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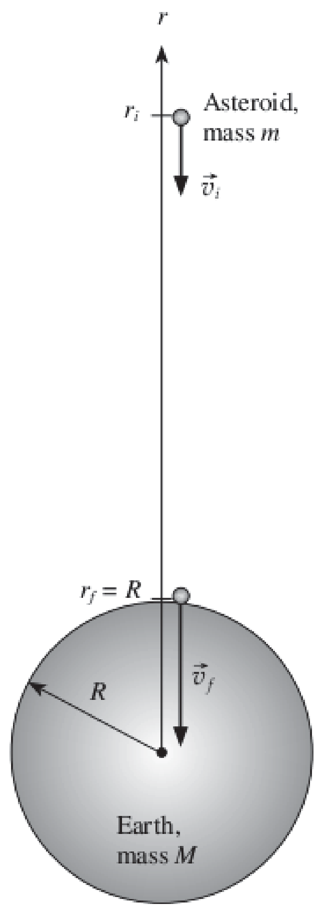
C8M.3 Initial and final drawings of the situation might look as shown below. Note that I have drawn the asteroid coming directly toward the earth, but it could have easily followed a more complicated trajectory: only the initial and final *speeds* and initial and final radii are relevant.



$r_i = 384{,}000$ km
$|\vec{v}_i| = 15$ km/s
$r_f = R = 6380$ km
$M = 6.0 \times 10^{24}$ kg
$G = 6.67 \times 10^{-11}$ N·m²/kg
$|\vec{v}_f| = ?$
$m = ?$

<u>Internal interactions:</u>

- Gravitational $V_g = -GMm/r$

The system here is the asteroid and the earth, which interact with each other gravitationally. This system is isolated because it floats in space. We don't expect the earth's kinetic energy K_e to change much as it interacts with the asteroid, because the earth's mass is much greater. Since the distance between the asteroid and the earth is initially much greater than the earth's radius R, we should use the universal gravitational potential energy formula $V_g = -GMm/r$ handle the internal gravitational interaction's energy in this case, as noted above. Neither the asteroid's internal energy U_a and nor the earth's internal energy U_e should change much until the asteroid hits. The conservation of energy master equation therefore becomes

$$\tfrac{1}{2}m|\vec{v}_i|^2 + \cancel{U_a} + \overbrace{K_e}^{0} + \cancel{U_e} - \frac{GMm}{r_i} = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_a} + \overbrace{K_e}^{\approx 0} + \cancel{U_e} - \frac{GMm}{r_f} \quad \Rightarrow \quad \tfrac{1}{2}m|\vec{v}_i|^2 - \frac{GMm}{r_i} = \tfrac{1}{2}m|\vec{v}_f|^2 - \frac{GMm}{r_f}$$

$$\Rightarrow \quad |\vec{v}_f|^2 = |\vec{v}_i|^2 + \frac{2GM}{r_f} - \frac{2GM}{r_i} \quad \Rightarrow \quad |\vec{v}_f| = \sqrt{|\vec{v}_i|^2 + 2GM\left(\frac{1}{R} - \frac{1}{r_i}\right)} \tag{1}$$

$$\Rightarrow \quad |\vec{v}_f| = \sqrt{\left(15000\,\frac{\text{m}}{\text{s}}\right)^2 + \left(6.67\times10^{-11}\,\frac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(6.0\times10^{24}\,\cancel{\text{kg}})\left(\frac{1\,\cancel{\text{kg}}\cdot\cancel{\text{m}}/\text{s}^2}{1\,\cancel{\text{N}}}\right)\left(\frac{1}{6{,}380{,}000\,\cancel{\text{m}}} - \frac{1}{384{,}000{,}000\,\cancel{\text{m}}}\right)}$$

$$= 16900\,\frac{\cancel{\text{m}}}{\text{s}}\left(\frac{1\text{ km}}{1000\,\cancel{\text{m}}}\right) = 16.9\,\frac{\text{km}}{\text{s}} \tag{2}$$

So the earth's gravitational field does increase the asteroid's speed somewhat.

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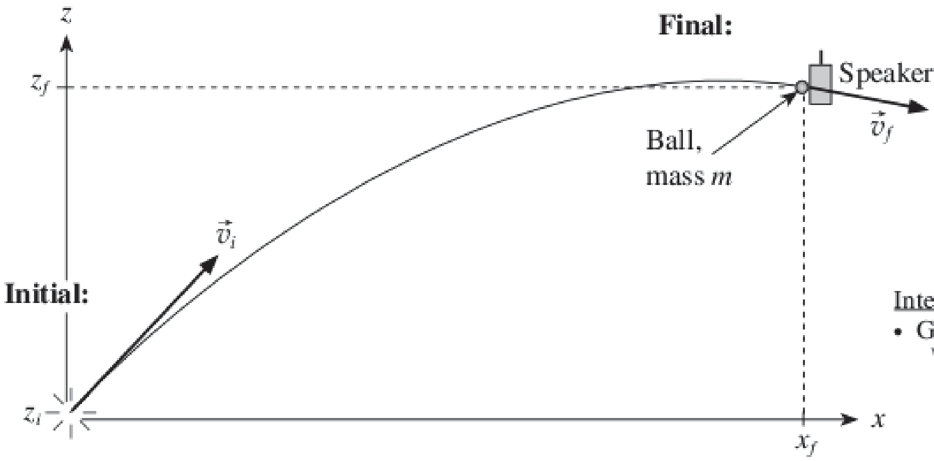
C8M.5 Initial and final drawings for this situation appear below:



$z_f = 36$ m
$z_i = 1$ m
$x_f = 100$ m
$|\vec{v}_i| = 47$ m/s
$|\vec{g}| = 9.8$ m/s^2
$m = ?$
$|\vec{v}_f| = ?$

Internal interactions:

- Gravitational $V_g = m|\vec{g}|z$

The system here is the ball and the earth, which interact with each other gravitationally. This system is isolated because it floats in space. The earth's kinetic energy K_e will be negligible in this process because it is so massive compared to the ball. If we ignore friction, we are essentially saying that neither the ball's internal energy U_b nor the earth's internal energy U_e changes much as the ball coasts along its trajectory. Since the ball is always near the earth's surface, we can use the near-earth gravitational potential energy formula $V_g = m|\vec{g}|z$. Note that have defined $z = 0$ to be the ground level, but I am guessing that the ball was hit about $z_i = 1$ m off the ground. The conservation of energy master equation for this case is thus

$$\tfrac{1}{2}m|\vec{v}_i|^2 + \cancel{U_b} + \overbrace{K_e}^{0} + \cancel{U_e} + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_b} + \overbrace{K_e}^{\approx 0} + \cancel{U_e} + m|\vec{g}|z_f \quad \Rightarrow \quad |\vec{v}_i|^2 + 2|\vec{g}|z_i = |\vec{v}_f|^2 + 2|\vec{g}|z_f \qquad (1)$$

$$\Rightarrow \quad |\vec{v}_f| = \sqrt{|\vec{v}_i|^2 - 2|\vec{g}|(z_f - z_i)} = \sqrt{\left(47\,\frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(35\text{ m})} = 39\,\frac{\text{m}}{\text{s}} \qquad (2)$$

Note that the horizontal distance is irrelevant to this calculation.

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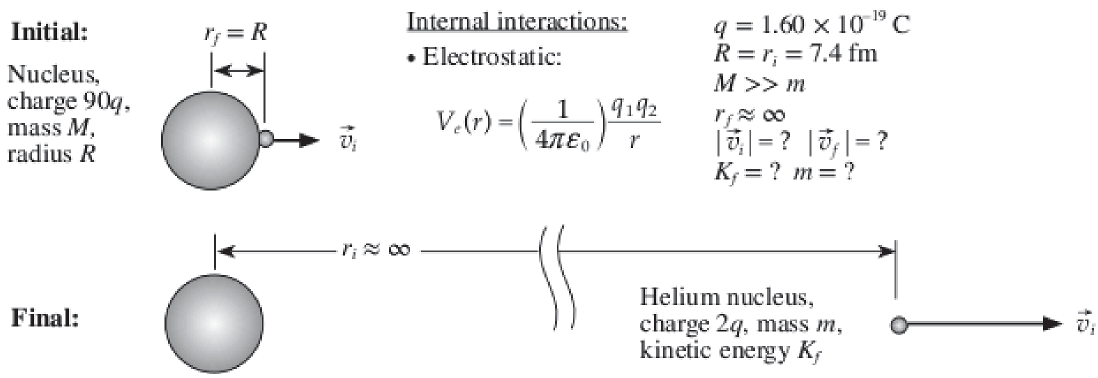
C8M.11 Since the helium and the uranium nucleus both have positive charge, they electrostatically repel each other. Once the helium nucleus emerges from the uranium nucleus, it is therefore strongly repelled by that nucleus and will therefore accelerate away from it. Initial and final drawings for this situation appear below. (Note that $q \equiv$ charge of a proton here.)



The system is the helium and the nucleus that remains (a thorium nucleus, actually) after the uranium nucleus spits out the helium nucleus. Even though we are observing this process in a laboratory immersed in a gravitational field, the whole process will happen so quickly that we can consider the process a "collision" (rather a collision in reverse, really!). The two nuclei will interact gravitationally as well as electrostatically, but the gravitational interaction is negligible (see problem C8D.3). Note that by the time the helium nucleus is only a few millimeters from the other nucleus, r_i will be enormously larger than $R = 7.4$ fm, so we can consider $r_f \approx \infty$. We will assume that the helium and thorium's internal energies U_{He} and U_n are not significantly affected by the electrostatic interaction after the helium nucleus emerges. We could calculate the other nucleus's mass M, but the only thing we really need to know about it is that $M \approx (90 + 142)m >> m$, meaning that the nucleus's kinetic energy K_n remains essentially zero throughout the process. Let $K_i = \frac{1}{2}m|\vec{v}_i|^2$ be the helium nucleus's initial kinetic energy as it emerges and $K_f = \frac{1}{2}m|\vec{v}_f|^2$ be the same when it is far away. The conservation of energy master equation for this process is therefore

$$K_i + \cancel{U_{He}} + \overset{0}{K_n} + \cancel{U_n} + \frac{2q(90q)}{4\pi\varepsilon_0 r_i} = K_f + \cancel{U_{He}} + \overset{\approx 0}{K_n} + \cancel{U_n} + \overbrace{\frac{2q(90q)}{4\pi\varepsilon_0 r_f}}^{\approx 0} \Rightarrow K_f = K_i + \frac{180q^2}{4\pi\varepsilon_0 R} \quad (1)$$

Now, note that the smallest value that K_i can possibly have is zero. Therefore

$$K_f \geq \frac{180q^2}{4\pi\varepsilon_0 R} = \left(8.99\times 10^9 \frac{\cancel{N}\cdot\cancel{m}^2}{\cancel{C}^2}\right)\frac{180(1.60\times 10^{-19}\ \cancel{C})^2}{7.4\ \cancel{fm}}\left(\frac{1\ \cancel{fm}}{10^{-15}\ \cancel{m}}\right)\left(\frac{1\ J}{1\ \cancel{N}\cdot\cancel{m}}\right) = 5.6\times 10^{-12}\ J \quad (1)$$

Note that the units all work out correctly, and the result is plausible (a helium nucleus with an energy as large as a joule or even a microjoule would be suspicious.) This result is actually quite consistent with the energies observed in such cases, so it seems like the model we have developed here makes good sense. Since we did not actually use the formula $K_f = \frac{1}{2}m|\vec{v}_f|^2$ but only conservation of energy, this result would apply even if the helium nucleus were relativistic (though it is not in this case).