Unit: C Chapter: C10 Problem Number: C10B.2 Return to User Page

C10B.2 We are given that the two vectors $\vec{u}$ and $\vec{w}$ are perpendicular. This implies that their dot product must be zero, since $\vec{u}\cdot\vec{w} = |\vec{u}||\vec{w}|\cos\theta = 0$ no matter what the vector magnitudes are. Therefore, according to equation C10.5, we have

$$0 = \vec{u}\cdot\vec{w} = u_x w_x + u_y w_y + u_z w_z$$

$$\Rightarrow \quad w_z = \frac{-u_x w_x - u_y w_y}{u_z} = \frac{-(3\text{ m})(-4\text{ m}) - (-5\text{ m})(-2\text{ m})}{2\text{ m}} = +1\text{ m}.$$

Unit: C
Chapter: C10
Problem Number: C10B.6

Return to User Page

C10B.6 **(a)** Since the gravitational force $\vec{F}_g$ on the car is constant in magnitude (20,000 N) and direction (downward), the work that it does on the car during a finite displacement $\Delta\vec{r}$ is $W = \vec{F}_g \cdot \Delta\vec{r}$. During a one-second time interval, $|\Delta\vec{r}| = 25$ m, and the angle θ between $\vec{F}_g$ and $\Delta\vec{r}$ is $90° + 4° = 94°$. So the work that the gravitational force does on the car in a one-second time interval is

$$\vec{F}_g \cdot \Delta\vec{r} = |\vec{F}_g||\Delta\vec{r}|\cos\theta = (20{,}000\ \text{N})(25\ \text{m})\cos 94°\left(\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}}\right) = -34{,}900\ \text{J} \tag{1}$$

(b) Because the car's speed is not changing, its total kinetic energy is not changing, so $\vec{F}_{\text{ext}} \cdot \Delta\vec{r}_{\text{CM}} = 0$. This can either be because $\vec{F}_{\text{ext}} = 0$ or because $\vec{F}_{\text{ext}} \perp \Delta\vec{r}$, but the in the latter case, the car's direction of motion would be changing, which is not true here. So we must have $\vec{F}_{\text{ext}} = 0$, meaning that contact forces on the car's tires must be exerting an upward force that cancels the gravitational force.

(c) However, these forces are exerted on the points where the car's wheels touch the road, and these points do not move relative to the road as long as the contact lasts. Therefore, these external contact forces do no work on the car. Since the gravitational force does negative work on the car (meaning that energy is flowing *out* of the car), the internal energy of the car must be *decreasing*. (In this case, the energy comes from the chemical energy of the car's fuel.)

Unit: C
Chapter: C10
Problem Number: C10B.7
Return to User Page

C10B.7 **(a)** My butt does *positive* work on the ball, because I exert a downward force on the ball and the point on the ball where my butt pushes also moves downward, meaning that the dot product of the force and the displacement is positive.

(b) The floor does *no* work on the ball, because although the floor exerts an upward force on the ball, the point where the floor applies the force does not move.

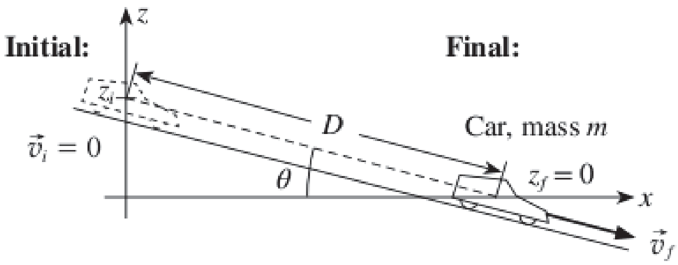
(c) The net change in the ball's kinetic energy is zero, because the ball is initially and finally at rest.

(d) The net change in the ball's internal energy is *positive*, because my butt does positive work on the ball and that energy has clearly not gone to the ball's kinetic energy, so it must have increased the ball's internal energy.

Unit: C
Chapter: C10
Problem Number: C10M.3

Return to User Page

C10M.3 Initial and final diagrams for this situation appear below:



Known:
D, $|\vec{g}|$, θ
$\vec{v}_i = 0$
Unknown:
m, $|\vec{v}_f|$

Internal Interactions:
- Gravitational: $V_g = m|\vec{g}|z$
- Contact with road: ⊥

We could take the system to be either just the car (and keep track of work done on the car by the external forces acting on) or we can take the system to be the car an the earth, similar to what we have done in previous chapters. Let's do the latter. This system is isolated because it floats in space. The system participates in two significant internal interactions, a gravitational interaction between the car and the earth (which we will handle using the near-earth gravitational potential energy formula $V_g = m|\vec{g}|z$ and a contact interaction between the car and the road. We will assume that the latter interaction is frictionless, meaning (1) that we can handle it by ignoring it (because it acts perpendicular to the car's motion) and (2) that the system's total internal energy U will not change in this process. The earth's kinetic energy K_e will remain negligible because it is much more massive than the car. Our conservation of energy master equation therefore specifies that

$$\tfrac{1}{2}m\overbrace{|\vec{v}_i|^2}^{0} + \overbrace{K_e}^{0} + \cancel{U} + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_f|^2 + \overbrace{K_e}^{\approx 0} + \cancel{U} + m|\vec{g}|z_f \quad \Rightarrow \quad \tfrac{1}{2}m|\vec{v}_f|^2 = m|\vec{g}|(z_i - z_f) \tag{1}$$

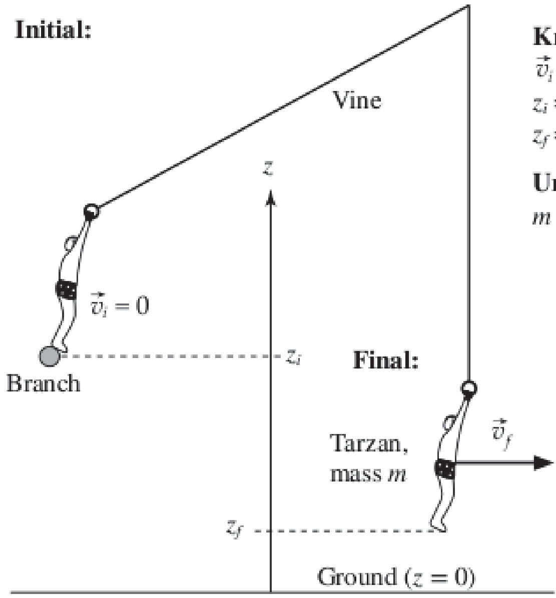
Note from the diagram that $z_i - z_f = D\sin\theta$, so substituting this into equation 1 and multiplying both sides by $2/m$ yields

$$|\vec{v}_f|^2 = 2|\vec{g}|D\sin\theta \quad \Rightarrow \quad |\vec{v}_f| = \sqrt{2|\vec{g}|D\sin\theta} \tag{2}$$

(Note that the units of $|\vec{g}|D = \text{m}^2/\text{s}^2$ so the units make sense. Also note that the final speed goes to zero if θ goes to zero or D goes to zero, which also makes sense.)

Unit: C ▼ Chapter: C10 ▼ Problem Number: C10M.4 ▼ Return to User Page

C10M.4 Initial and final diagrams for this situation appear below. In this case, defining $z = 0$ to be the ground level and Tarzan's initial and final vertical positions z_i and z_f to be the positions of his *feet* (instead of his center of mass) is convenient. Ultimately, we will see that the answer depends only on the change in his center of mass's vertical position, which will be the same as the change of position of his feet (assuming that he remains essentially vertical).



Known:
$\vec{v}_i = 0$
$z_i = 3.0$ m
$z_f = 1.0$ m

Unknown:
$m = ?$ $|\vec{v}_f| = ?$

Internal Interactions:
- Gravitational: $V_g = m|\vec{g}|z$
- Tension force from rope: $\perp$

Take the system to be Tarzan and the earth, which is isolated because it floats in space. If we ignore air friction, Tarzan and the earth participate in two important internal interactions: a gravitational interaction and a contact interaction mediated by the vine. We can *ignore* the latter if we treat the vine as an inextensible string (see section C10.5). We can handle the gravitational interaction by using the near-earth potential energy formula $V_g = m|\vec{g}|z$. Neither interaction will affect Tarzan's internal energy U_T or the earth's internal energy U_e, and the earth's kinetic energy K_e will be negligible throughout because it is very massive compared to Tarzan. Our conservation of energy master equation in this case becomes

$$\tfrac{1}{2}m\overbrace{|\vec{v}_i|^2}^{0} + \cancel{U_T} + \overbrace{K_e}^{0} + \cancel{U_e} + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_T} + \overbrace{K_e}^{0} + \cancel{U_e} + m|\vec{g}|z_f$$

$$\Rightarrow \quad \tfrac{1}{2}m|\vec{v}_f|^2 = m|\vec{g}|(z_i - z_f) \quad \Rightarrow \quad |\vec{v}_f|^2 = 2|\vec{g}|(z_i - z_f)$$

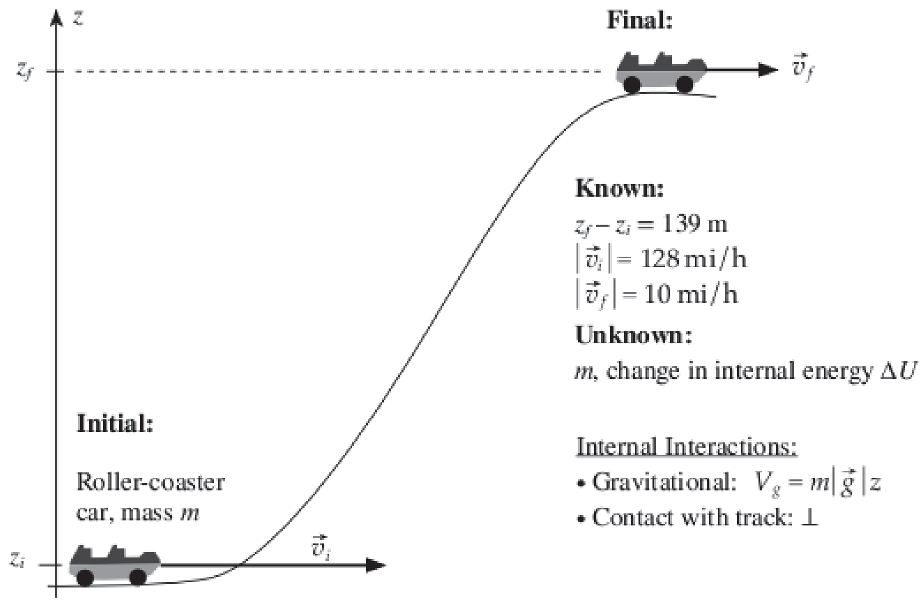
$$\Rightarrow \quad |\vec{v}_f| = \sqrt{2|\vec{g}|(z_i - z_f)} = \sqrt{2(9.8\,\text{m/s}^2)(3.0\,\text{m} - 1.0\,\text{m})} = 6.3\,\frac{\text{m}}{\text{s}} \tag{1}$$

Note that the units work out and the magnitude (≈ 14 mi/h) is credible.

Unit: C
Chapter: C10
Problem Number: C10M.7

Return to User Page

C10M.7 Initial and final drawings for this situation appear below.



The system here is the car and the earth, which is isolated because it floats in space. The car and the earth participate in three significant internal interactions: a gravitational interaction (which we will handle using the near-earth gravitational potential energy formula $V_g = m|\vec{g}|z$), a normal contact interaction between the car and the rails (which we will ignore, because the force it exerts is always perpendicular to the car's motion), and a friction interaction with the rails and/or the air that channels energy to internal energy. Because we don't know how much goes to the cart or the air or the rails, let's just say that the system's total internal energy is U_i before and some possibly different value U_f afterward. The earth's kinetic energy K_e remains negligible because the earth is much more massive than the car. The conservation of energy master equation for this situation is therefore

$$\tfrac{1}{2}m|\vec{v}_i|^2 + \overbrace{K_e}^{0} + U_i + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_f|^2 + \overbrace{K_e}^{0} + U_f + m|\vec{g}|z_f$$

$$\Rightarrow \quad \Delta U = U_f - U_i = \tfrac{1}{2}m\left(|\vec{v}_i|^2 - |\vec{v}_f|^2\right) - m|\vec{g}|(z_f - z_i) \tag{1}$$

The fraction of the car's initial kinetic energy that has gone to internal energy is therefore

$$\frac{\Delta U}{\frac{1}{2}m|\vec{v}_i|^2} = 1 - \frac{|\vec{v}_f|^2}{|\vec{v}_i|^2} - \frac{2|\vec{g}|(z_f - z_i)}{|\vec{v}_i|^2} = 1 - \left(\frac{10\ \cancel{\text{mi/h}}}{128\ \cancel{\text{mi/h}}}\right)^2 - \frac{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}}^2)(139\ \cancel{\text{m}})}{(128\ \cancel{\text{mi/h}})^2}\left(\frac{2.24\ \cancel{\text{mi/h}}}{1\ \cancel{\text{m}}/\cancel{\text{s}}}\right)^2$$

$$= 1 - 0.006 - 0.834 = 0.16 \tag{2}$$

Therefore, friction has removed only about 16% of the car's initial kinetic energy.