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C14B.2 Let glider 1 be the initially moving glider which we are told has mass $m_1 = 2m$ and an initial speed of $|\vec{v}_0| = 0.60$ m/s in the $+x$ direction. Let glider 2 be the glider originally at rest, which we are told has mass $m_2 = m$. In this one-dimensional collision, the motion will be confined to a line that we can define as the x axis. Elastic collisions where one object is initially at rest were explored in detail in example C14.2, so we can apply equations C14.9 to this situation. In this case, the mass ratio is $b = m_2/m_1 = m/2m = 1/2$. According to equation C14.9b, then, the x-velocity of the originally moving glider after the collision is

$$v_{1x} = \frac{1-b}{1+b}|\vec{v}_0| = \frac{1-\frac{1}{2}}{1+\frac{1}{2}}|\vec{v}_0| = \frac{\frac{1}{2}|\vec{v}_0|}{\frac{3}{2}} = \frac{|\vec{v}_0|}{3} = \frac{0.60\text{ m/s}}{3} = 0.20\ \frac{\text{m}}{\text{s}} \tag{1}$$

So this glider moves forward with speed 0.20 m/s after the collision. According to equation C14.9a, the final x-velocity of glider 2 is

$$v_{2x} = \frac{2}{1+b}|\vec{v}_0| = \frac{2}{\frac{3}{2}}|\vec{v}_0| = \frac{4(0.60\text{ m/s})}{3} = 0.80\ \frac{\text{m}}{\text{s}} \tag{2}$$

So this glider moves forward with a speed of 0.80 m/s.

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C14B.4 The system consisting of the bullet and block is momentarily isolated for motion in the $+x$ direction, the direction of the bullet's initial velocity (the rollers almost make it functionally isolated). Therefore its momentum and energy are conserved. Let $m = 16$ g $= 0.016$ kg be the bullet's mass, let $|\vec{v}_0| = 400$ m/s be the bullet's initial speed, let $M = 1.584$ kg be the block's mass , and let $\vec{v}$ be the block's unknown final velocity. The block's initial velocity is zero.

(a) Conservation of momentum in this case requires that

$$\begin{bmatrix} m|\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (m+M)v_x \\ (m+M)v_y \\ (m+M)v_z \end{bmatrix} \Rightarrow \begin{matrix} v_x = m|\vec{v}_0|/(M+m) \\ v_y = 0 \\ v_z = 0 \end{matrix} \Rightarrow v_x = \frac{(0.016 \cancel{\text{kg}})(400\text{ m/s})}{1.584 \cancel{\text{kg}} + 0.016 \cancel{\text{kg}}} = 4.0 \frac{\text{m}}{\text{s}} \tag{1}$$

So the combination moves at 4.0 m/s in the same direction as the bullet's initial velocity.

(b) The bullet's initial kinetic energy is $\frac{1}{2}m|\vec{v}_0|^2 = \frac{1}{2}(0.016\text{ kg})(400\text{ m/s})^2 = 1280\text{ kg}\cdot\text{m}^2/\text{s}^2 = 1280\text{ J}$.

(c) The kinetic energy of the bullet and the block is

$$\frac{1}{2}(M+m)|\vec{v}|^2 = \frac{1}{2}(1.600\text{ kg})(1.0\text{ m/s})^2 = 0.80\text{ kg}\cdot\text{m}^2/\text{s}^2 = 0.80\text{ J} \tag{2}$$

(d) This is a completely inelastic collision, so kinetic energy is not conserved. The difference in the kinetic energies has gone to internal energy in the bullet and the block, probably mostly thermal energy.

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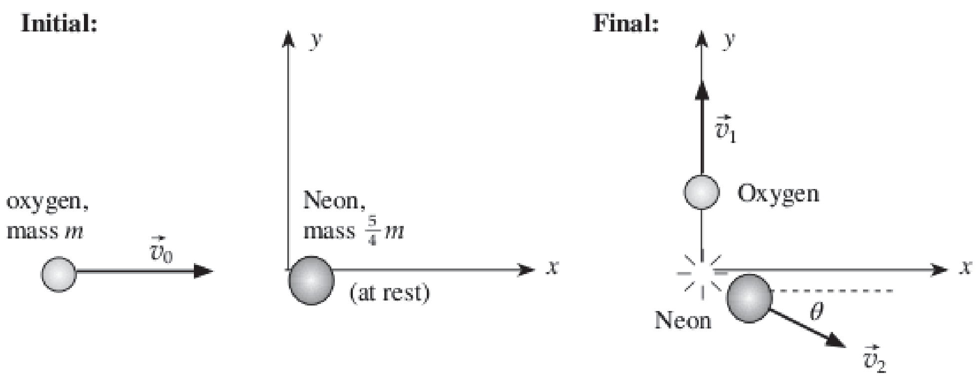
C14B.6 It would be better if the sail were reflective. If each photon carries momentum $\vec{p}$, then if the sail absorbs that photon, it would gain momentum $\vec{p}$. But if it reflects it, then by conservation of momentum the sail's momentum $\vec{P}$ must be such that $\vec{p} = \vec{P} - \vec{p} \Rightarrow \vec{P} = 2\vec{p}$ (assuming that the photon is reflected directly backward). Therefore, the sail gains more momentum from reflected photons than it does from absorbed photons.

C14M.7 Initial and final drawings for this situation appear below.



Known:
$\vec{v}_1$ in the $+y$ direction
$|\vec{v}_0|$, $|\vec{v}_1| = \frac{1}{3}|\vec{v}_0|$

Unknown:
$|\vec{v}_2|$, θ, elastic?

The system here consists of the two atoms. While these objects probably do participate in various external electromagnetic and gravitational interactions (certainly the latter), the whole process will take place over such a short interval of time that we can consider the process to be a collision that conserves energy and momentum. Conservation of momentum implies that

$$\begin{bmatrix} m|\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ m|\vec{v}_1| \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{5}{4}mv_{2x} \\ \frac{5}{4}mv_{2y} \\ 0 \end{bmatrix} \tag{1}$$

The two non-trivial components of equation 1 and equation 2 provide the two equations that we need to solve for the unknown components v_{2x}, and v_{2y}. Solving these components for v_{2x} and v_{2y} yields

$$v_{2x} = \frac{4}{5}|\vec{v}_0| \quad \text{and} \quad v_{2y} = -\frac{4}{5}|\vec{v}_1| = -\frac{1}{3}\frac{4}{5}|\vec{v}_0| \tag{2}$$

So the magnitude of $|\vec{v}_2|$ is

$$v_{2x}^2 + v_{2y}^2 = (\tfrac{4}{5}|\vec{v}_0|)^2 + (-\tfrac{1}{3}\tfrac{4}{5}|\vec{v}_0|)^2 = \tfrac{16}{25}(1+\tfrac{1}{9})|\vec{v}_0|^2 = \tfrac{16}{25}\tfrac{10}{9}|\vec{v}_0|^2 = \tfrac{32}{45}|\vec{v}_0|^2 \tag{3}$$

The system's final kinetic energy is therefore

$$\tfrac{1}{2}m|\vec{v}_1|^2 + \tfrac{1}{2}\tfrac{5}{4}m|\vec{v}_2|^2 = \tfrac{1}{2}m(\tfrac{1}{3}|\vec{v}_0|)^2 + \tfrac{1}{2}m\tfrac{5}{4}\tfrac{32}{45}|\vec{v}_0|^2 = \tfrac{1}{2}m|\vec{v}_0|^2(\tfrac{1}{9}+\tfrac{8}{9}) = \tfrac{1}{2}m|\vec{v}_0|^2 \tag{4}$$

As this is the same as the system's initial kinetic energy, the collision is indeed elastic. (One could go on to calculate the direction angle θ, which turns out to be 17°, but the problem did not ask for that.)

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C14B.9 The change in the system's thermal energy ΔU^{th} is greater if the frisbee was initially spinning than if it was not (answer A). To see this, note that conservation of momentum requires the astronaut's final center-of-mass speed to be the same whether the frisbee is spinning or not. However, catching the frisbee will require the astronaut to rotate to conserve angular momentum. Suppose that the astronaut has moment of inertia I_1 and the frisbee has moment of inertia I_2 and is spinning with an initial angular velocity of $\vec{\omega}_0$. To conserve angular momentum, the combined system must have a final angular velocity $\vec{\omega}$ such that $(I_1 + I_2)\vec{\omega} = I_1\vec{\omega}_0$ (assuming the astronaut's and the frisbee's center of mass coincide after the collision. The rotational energy associated with this motion is

$$\tfrac{1}{2}(I_1 + I_2)|\vec{\omega}|^2 = \tfrac{1}{2}(I_1 + I_2)\left[\frac{I_2}{(I_1 + I_2)}|\vec{\omega}_0|\right]^2 = \tfrac{1}{2}I_2|\vec{\omega}_0|^2\left(\frac{I_2}{I_1 + I_2}\right) < \tfrac{1}{2}I_2|\vec{\omega}_0|^2$$

meaning that the system's final rotational energy is less than the rotational energy that the frisbee brought in. Therefore most of the frisbee's rotational energy must be converted to other forms of internal energy after the collision in addition to whatever kinetic energy the frisbee brought in. Assuming that all the internal energy increase goes to thermal energy, ΔU^{th} is greater if the frisbee was initially spinning than if it was not.

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C14M.1 The system here is the two carts, which are functionally isolated with regard to both energy and momentum if the carts have frictionless wheels. Let cart 1 be the one with mass $m_1 = 1.0$ kg and an initial speed of $|\vec{v}_1| = 1.5$ m/s in the $+x$ direction. Cart 2 then has mass $m_2 = 0.75$ kg and is moving at an initial speed of $|\vec{v}_2| = 2.0$ m/s in the $-x$ direction. Let their final velocities be $\vec{v}_3$ and $\vec{v}_4$ respectively, and assume that these velocities lie on the x axis. Conservation of momentum requires that

$$\begin{bmatrix} m_1|\vec{v}_1| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -m_2|\vec{v}_2| \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m_1 v_{3x} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} m_2 v_{3y} \\ 0 \\ 0 \end{bmatrix} \quad \Rightarrow \quad |\vec{v}_1| - b|\vec{v}_2| = v_{3x} + b v_{4x} \tag{1}$$

where $b \equiv m_1/m_2$. Since the collision is elastic, kinetic energy is also conserved:

$$\tfrac{1}{2}m_1|\vec{v}_1|^2 + \tfrac{1}{2}m_2|\vec{v}_2|^2 = \tfrac{1}{2}m_1(v_{3x})^2 + \tfrac{1}{2}m_2(v_{4x})^2 \quad \Rightarrow \quad |\vec{v}_1|^2 + b|\vec{v}_2|^2 = (v_{3x})^2 + b(v_{4x})^2 \tag{2}$$

This provides us with two equations in our two unknowns v_{3x} and v_{4x}, so we should be able to solve. We can actually simplify these two equations by converting the left side of each into a single quantity. We know that $|\vec{v}_2|/|\vec{v}_1| = (2.0$ m/s$)/(1.5$ m/s$) = 4/3$, and also that $b = (0.75$ kg$)/(1.0$ kg$) = 3/4$, so

$$|\vec{v}_1| - b|\vec{v}_2| = |\vec{v}_1| - \tfrac{3}{4}\tfrac{4}{3}|\vec{v}_1| = 0 \quad \text{and} \quad |\vec{v}_1|^2 + b|\vec{v}_2|^2 = |\vec{v}_1| + \tfrac{3}{4}(\tfrac{4}{3}|\vec{v}_1|)^2 = |\vec{v}_1|^2 + \tfrac{4}{3}|\vec{v}_1|^2 = \tfrac{7}{3}|\vec{v}_1|^2 \tag{3}$$

Equations 1 and 2 therefore become

$$0 = v_{3x} + \tfrac{3}{4}v_{4x} \quad \text{and} \quad \tfrac{7}{3}|\vec{v}_1|^2 = (v_{3x})^2 + \tfrac{3}{4}(v_{4x})^2 \tag{4}$$

Solving the first of these for v_{4x} and substituting the result into the second equation yields

$$v_{4x} = -\tfrac{4}{3}v_{3x} \quad \Rightarrow \quad \tfrac{7}{3}|\vec{v}_1|^2 = (v_{3x})^2 + \tfrac{3}{4}\left(\tfrac{4}{3}v_{3x}\right)^2 = \tfrac{7}{3}(v_{3x})^2 \quad \Rightarrow \quad (v_{3x})^2 = |\vec{v}_1|^2 \quad \Rightarrow \quad v_{3x} = \pm|\vec{v}_1| \tag{5a}$$

Substituting this back into the first equation for v_{4x} yields $v_{4x} = -\tfrac{4}{3}(\pm|\vec{v}_1|) = \mp\tfrac{4}{3}|\vec{v}_1| = \mp|\vec{v}_2|$. Which of the two solutions applies? Since the positive solution for v_{3x} and the negative solution for v_{4x} mean that the carts have the same velocities after the collision that they did before, this is the unphysical solution where the carts simply pass like ghosts through each other. The physically realistic solution is that the carts simply rebound with the same speeds but in opposite directions to their original velocities: $v_{3x} = -|\vec{v}_1|$ and $v_{4x} = +|\vec{v}_2|$. So after the collision, cart 1 moves in the $-x$ direction at 1.5 m/s and the other cart moves in the $+x$ direction at 2.0 m/s.

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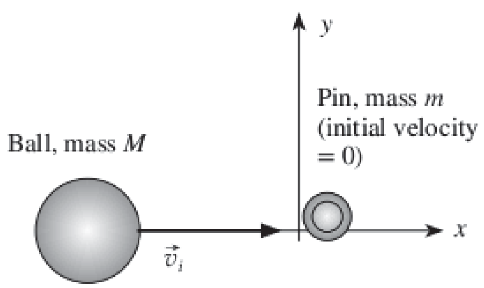
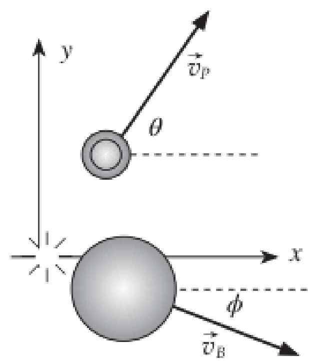
C14M.2 Let's set up our reference frame so that the motion takes place in the (horizontal) xy plane, and let the x axis coincide with the ball's original direction of motion. Initial and final sketches of this situation then look something like what is shown below (since we don't yet know what the ball's final velocity direction will be, the final sketch below is a guess):

Initial:
(Top View)



Final:
(Top View)



Known:

$$\vec{v}_i = \begin{bmatrix} +|\vec{v}_i| \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.5\ \text{m/s} \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{v}_P = \begin{bmatrix} +|\vec{v}_P|\cos\theta \\ +|\vec{v}_P|\sin\theta \\ 0 \end{bmatrix}$$

$|\vec{v}_P| = 3.0$ m/s, $\theta = 60°$
$M = 7.2$ kg, $m = 1.2$ kg

Unknown:
$\vec{v}_B = ?$ $\phi = ?$
elastic?

The system here consists of the ball and the pin. While these objects do participate in contact and gravitational interactions with the earth, we will treat the process as a *collision*: as long as we focus on the situation just before and just after the impact, the system's momentum and energy are conserved. The collision process may or may not convert kinetic energy to internal energy: we will address this later. Focusing on conservation of momentum first, we have:

$$m\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + M\begin{bmatrix} +|\vec{v}_i| \\ 0 \\ 0 \end{bmatrix} = m\begin{bmatrix} |\vec{v}_P|\cos\theta \\ |\vec{v}_P|\sin\theta \\ 0 \end{bmatrix} + M\begin{bmatrix} v_{Bx} \\ v_{By} \\ v_{Bz} \end{bmatrix} \tag{1}$$

The z component of this equation simply tells us that $v_{Bz} = 0$. Solving the x and y component equations for v_{Bx} and v_{By} yields

$$M|\vec{v}_i| = m|\vec{v}_P|\cos\theta + Mv_{Bx} \quad \Rightarrow \quad v_{Bx} = |\vec{v}_i| + (m/M)|\vec{v}_P|\cos\theta \tag{2a}$$

$$0 = m|\vec{v}_P|\sin\theta + Mv_{By} \quad \Rightarrow \quad v_{By} = -(m/M)|\vec{v}_P|\sin\theta \tag{2b}$$

Since we know everything on the right side of these equations, we can easily calculate v_{Bx} and v_{By} and compute the magnitude and direction of $\vec{v}_B$ from those components in the usual way:

$$v_{Bx} = 3.5\,\frac{\text{m}}{\text{s}} - \frac{1.2\ \cancel{\text{kg}}}{7.2\ \cancel{\text{kg}}}\left(3.0\,\frac{\text{m}}{\text{s}}\right)\cos 60° = 3.25\,\frac{\text{m}}{\text{s}}, \quad v_{By} = -\frac{1.2\ \cancel{\text{kg}}}{7.2\ \cancel{\text{kg}}}\left(3.0\,\frac{\text{m}}{\text{s}}\right)\sin 60° = -0.43\,\frac{\text{m}}{\text{s}} \tag{3a}$$

$$\Rightarrow \quad |\vec{v}_B| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(3.25\ \text{m/s})^2 + (-0.43\ \text{m/s})^2 + 0^2} = 3.28\ \text{m/s} \tag{3b}$$

$$\phi = \tan^{-1}\left|\frac{v_{By}}{v_{Bx}}\right| = \tan^{-1}\left|\frac{-0.43\ \cancel{\text{m/s}}}{3.25\ \cancel{\text{m/s}}}\right| = \tan^{-1} 0.133 = 7.5° \tag{3c}$$

So the ball comes out of the collision with a velocity of 3.28 m/s 7.5° in the $-y$ direction from the x axis. Now, a collision between two objects is elastic if the total kinetic energy of the objects is conserved in the collision. Since we now know $|\vec{v}_B|$, we can simply compare the kinetic energy K_i before the collision to the kinetic energy K_f after the collision:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}m|\vec{v}_P|^2 + \frac{1}{2}M|\vec{v}_B|^2}{\frac{1}{2}M|\vec{v}_i|^2 + 0} = \frac{(m/M)|\vec{v}_P|^2 + |\vec{v}_B|^2}{|\vec{v}_i|^2} = \frac{\frac{1}{6}(3.0\ \text{m/s})^2 + (3.28\ \text{m/s})^2}{(3.5\ \text{m/s})^2} = 1.00 \tag{4}$$

Since the initial and final energies are equal, the collision is elastic. Note that the units work out in all the calculations, the signs of both components seem reasonable (clearly v_{By} has to be negative to conserve y-momentum here), and the magnitude is plausible (a bit less than the ball's original speed). Everything looks plausible!

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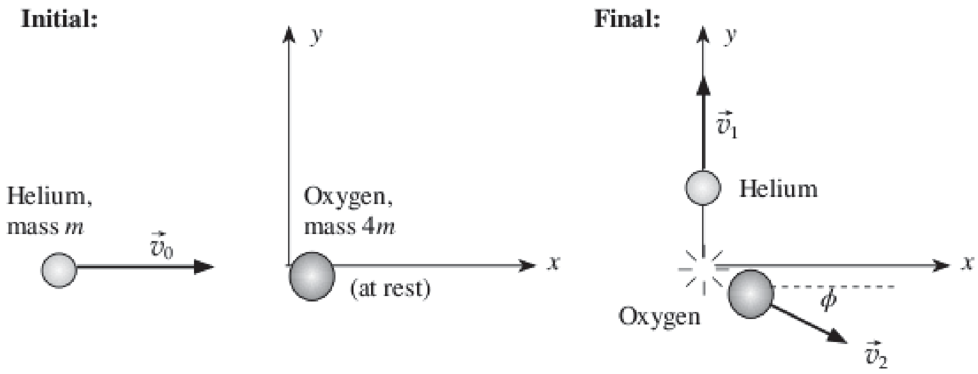
C14M.5 Initial and final drawings for this situation appear below.



Known:
$\vec{v}_1$ in the $+y$ direction
$|\vec{v}_0| = 240$ m/s

Unknown:
$|\vec{v}_1|$, $\vec{v}_2$, m

The system here consists of the two atoms. While these objects probably do participate in various external electromagnetic and gravitational interactions (certainly the latter), the whole process will take place over such a short interval of time that we can consider the process to be a collision that conserves energy and momentum. Conservation of momentum implies that

$$\begin{bmatrix} m|\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ m|\vec{v}_1| \\ 0 \end{bmatrix} + \begin{bmatrix} 4mv_{2x} \\ 4mv_{2y} \\ 0 \end{bmatrix} \tag{1}$$

(It turns out to be *much* easier to work with the components of $\vec{v}_2$ rather than $|\vec{v}_2|$ and ϕ, as you can easily find out by doing it the other way.) We are told that the collision is elastic, so kinetic energy is conserved:

$$\tfrac{1}{2}m|\vec{v}_0|^2 + 0 = \tfrac{1}{2}m|\vec{v}_1|^2 + \tfrac{1}{2}4m|\vec{v}_2|^2 \quad \Rightarrow \quad |\vec{v}_0|^2 = |\vec{v}_1|^2 + 4(v_{2x}^2 + v_{2y}^2) \tag{2}$$

The two non-trivial components of equation 1 and equation 2 provide three equations in the three unknown quantities $|\vec{v}_1|$, v_{2x}, and v_{2y}. We can eliminate the last two from equation 2 by solving the two lines of equation 1 for v_{2x} and v_{2y}:

$$v_{2x} = \tfrac{1}{4}|\vec{v}_0| \qquad \text{and} \qquad v_{2y} = -\tfrac{1}{4}|\vec{v}_1| \tag{3}$$

Substituting the results into equation 2 yields

$$|\vec{v}_0|^2 = |\vec{v}_1|^2 + 4([\tfrac{1}{4}|\vec{v}_0|]^2 + [-\tfrac{1}{4}|\vec{v}_1|]^2) = |\vec{v}_1|^2 + \tfrac{1}{4}|\vec{v}_0|^2 + \tfrac{1}{4}|\vec{v}_1|^2 \quad \Rightarrow \quad \tfrac{3}{4}|\vec{v}_0|^2 = \tfrac{5}{4}|\vec{v}_1|^2 \quad \Rightarrow \quad |\vec{v}_1| = \sqrt{\tfrac{3}{5}}|\vec{v}_0| \tag{4a}$$

$$|\vec{v}_1| = \sqrt{\tfrac{3}{5}}(240\text{ m/s}) = 187\text{ m/s} \tag{4b}$$

Substituting this into the second sub-equation of equation 3 yields

$$v_{2y} = -\tfrac{1}{4}|\vec{v}_1| = -\tfrac{1}{4}\sqrt{\tfrac{3}{5}}|\vec{v}_0| = -47\text{ m/s} \tag{5}$$

and the first sub-equation of equation 3 yields $v_{2x} = \frac{1}{4}(240\text{ m/s}) = 60\text{ m/s}$. From these results we can determine the magnitude and direction of $\vec{v}_2$:

$$|\vec{v}_2| = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{(\tfrac{1}{4}|\vec{v}_0|)^2 + (-\tfrac{1}{4}\sqrt{\tfrac{3}{5}}|\vec{v}_0|)^2} = \tfrac{1}{4}|\vec{v}_0|\sqrt{1+\tfrac{3}{5}} = \tfrac{1}{4}\sqrt{\tfrac{8}{5}}(240\text{ m/s}) = 76\text{ m/s} \tag{6}$$

$$\phi = \tan^{-1}\left|\frac{v_{2y}}{v_{2x}}\right| = \tan^{-1}\left|\frac{-\frac{1}{4}\sqrt{\frac{3}{5}}|\vec{v}_0|}{\frac{1}{4}|\vec{v}_0|}\right| = \tan^{-1}\sqrt{\tfrac{3}{5}} = 38° \tag{7}$$

So we see that the oxygen atom will recoil from this collision with a speed of 76 m/s in a direction 38° in the $-y$ direction from the x axis. and the helium atom will rebound with a speed of 187 m/s. The units all work out and this seems reasonable.

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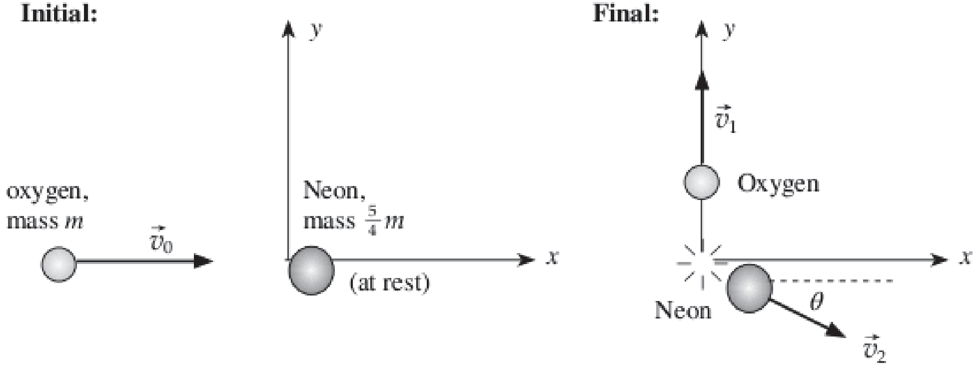
C14M.7 Initial and final drawings for this situation appear below.



Known:
$\vec{v}_1$ in the $+y$ direction
$|\vec{v}_0|$, $|\vec{v}_1| = \frac{1}{3}|\vec{v}_0|$

Unknown:
$|\vec{v}_2|$, θ, elastic?

The system here consists of the two atoms. While these objects probably do participate in various external electromagnetic and gravitational interactions (certainly the latter), the whole process will take place over such a short interval of time that we can consider the process to be a collision that conserves energy and momentum. Conservation of momentum implies that

$$\begin{bmatrix} m|\vec{v}_0| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ m|\vec{v}_1| \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{5}{4}mv_{2x} \\ \frac{5}{4}mv_{2y} \\ 0 \end{bmatrix} \tag{1}$$

The two non-trivial components of equation 1 and equation 2 provide the two equations that we need to solve for the unknown components v_{2x}, and v_{2y}. Solving these components for v_{2x} and v_{2y} yields

$$v_{2x} = \tfrac{4}{5}|\vec{v}_0| \qquad \text{and} \qquad v_{2y} = -\tfrac{4}{5}|\vec{v}_1| = -\tfrac{1}{3}\tfrac{4}{5}|\vec{v}_0| \tag{2}$$

So the magnitude of $|\vec{v}_2|$ is

$$v_{2x}^2 + v_{2y}^2 = (\tfrac{4}{5}|\vec{v}_0|)^2 + (-\tfrac{1}{3}\tfrac{4}{5}|\vec{v}_0|)^2 = \tfrac{16}{25}(1+\tfrac{1}{9})|\vec{v}_0|^2 = \tfrac{16}{25}\tfrac{10}{9}|\vec{v}_0|^2 = \tfrac{32}{45}|\vec{v}_0|^2 \tag{3}$$

The system's final kinetic energy is therefore

$$\tfrac{1}{2}m|\vec{v}_1|^2 + \tfrac{1}{2}\tfrac{5}{4}m|\vec{v}_2|^2 = \tfrac{1}{2}m(\tfrac{1}{3}|\vec{v}_0|)^2 + \tfrac{1}{2}m\tfrac{5}{4}\tfrac{32}{45}|\vec{v}_0|^2 = \tfrac{1}{2}m|\vec{v}_0|^2(\tfrac{1}{9}+\tfrac{8}{9}) = \tfrac{1}{2}m|\vec{v}_0|^2 \tag{4}$$

As this is the same as the system's initial kinetic energy, the collision is indeed elastic. (One could go on to calculate the direction angle θ, which turns out to be 17°, but the problem did not ask for that.)

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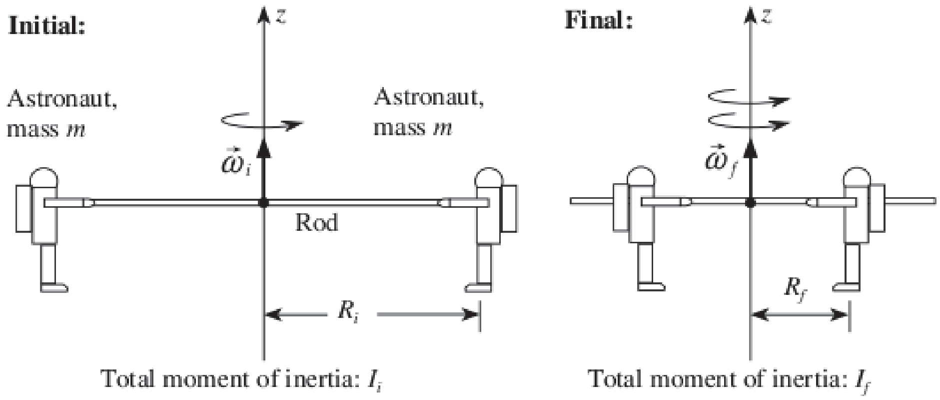
C14M.11 We will define the z axis of our reference frame to coincide with the rod's. Initial and final sketches (side views) of this situation then look like this:



Known:
L, $R_i = \frac{1}{2}L$, $R_f = \frac{1}{4}L$

Unknown:
Factor by which the system's rotational energy has increased = ?
m = ? mass of rod m_{rod} = ?
$|\vec{\omega}_i|$ = ? $|\vec{\omega}_f|$ = ?

The system in this case is the rod and the two astronauts. This system floats in space, so it is isolated. I will consider all angular momenta to be measured around the origin (the center of the rod). Conservation of this system's angular momentum then tells us that

$$I_i\vec{\omega}_i = I_f\vec{\omega}_f \tag{1}$$

Now, the total moment of inertia of a composite object is just the sum of the moments of inertia of each of its parts. In this case, the system consists of a rod (with moment of inertia $= \frac{1}{12}m_{\text{rod}}L^2 = \frac{1}{12}m_{\text{rod}}(2R_i)^2$) and two astronauts, which we treat as particles (each located at the astronaut's center of mass). Each of the astronauts has a moment of inertia of mR_i^2 initially and mR_f^2 finally around the center of the rod. Therefore:

$$I_i = \tfrac{1}{3}m_{\text{rod}}R_i^2 + 2mR_i^2, \qquad I_f = \tfrac{1}{3}m_{\text{rod}}R_i^2 + 2mR_f^2 \tag{2}$$

Note that because of the factor of $\frac{1}{3}$ that even if the rod is a significant fraction of an astronaut's mass, it is going to contribute little to the system's overall moment of inertia compared to the astronauts. Since we are given no information about the rod except that it is "light," let's assume that it is so light that its contribution to I_i and I_f is negligible. Equations 1 and 2 then provide three equations in our three unknowns I_i, I_f, and $|\vec{\omega}_f$. If we take the magnitude of both sides of equation 1, divide both sides by I_f, and plug in the results of equation 2 (with $m_{\text{rod}} \approx 0$), we get:

$$|\vec{\omega}_f| = \frac{2mR_i^2}{2mR_f^2}|\vec{\omega}_i| = \left(\frac{R_i}{R_f}\right)^2|\vec{\omega}_i| = \left(\frac{10\text{ m}}{5\text{ m}}\right)^2\left(\frac{1\text{ rev}}{30\text{s}}\right) = \frac{4\text{ rev}}{30\text{s}} = \frac{1\text{ rev}}{7.5\text{s}} \tag{3}$$

The system's rotational energy has therefore increased from

$$\tfrac{1}{2}I_i|\vec{\omega}_i|^2 = \tfrac{1}{2}2mR_i^2|\vec{\omega}_i|^2 \quad \text{to} \quad \tfrac{1}{2}I_f|\vec{\omega}_f|^2 = \tfrac{1}{2}2mR_f^2\left(\frac{R_i}{R_f}\right)^4|\vec{\omega}_i|^2 = \tfrac{1}{2}2mR_i^2\left(\frac{R_i}{R_f}\right)^2|\vec{\omega}_i|^2 \tag{4}$$

Note that the units work out, and the signs are right (all positive). The magnitudes also seem credible.
Comment: Since $R_i/R_f = 2$, the system's rotational energy has increased by a factor of four in this process! The only interactions in this problem (and thus the only interactions that could be the source of this kinetic energy) are the contact interactions between the astronauts' hands and the rod. The astronauts must spend chemical energy to clamber closer together. This in turn means that the astronauts must expend more effort doing this than they would if the rod were not rotating; they will thus feel a resistance due to the extra energy they will have to supply. (People usually interpret this "extra resistance" as being due to an outward "centrifugal" force, but this is not the most useful way to look at this. Chapter N8 discusses this issue in more depth.)