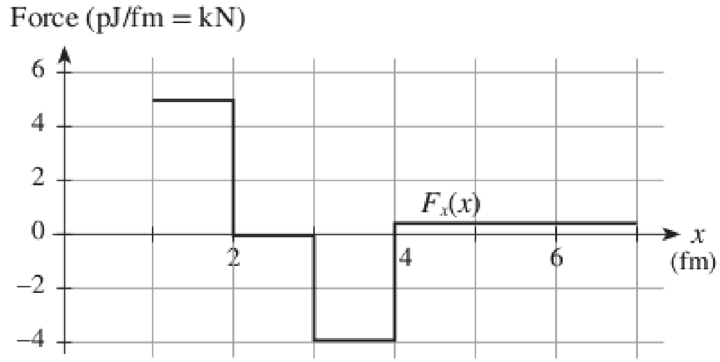
Unit: C
Chapter: C9
Problem Number: C9B.1

Return to User Page

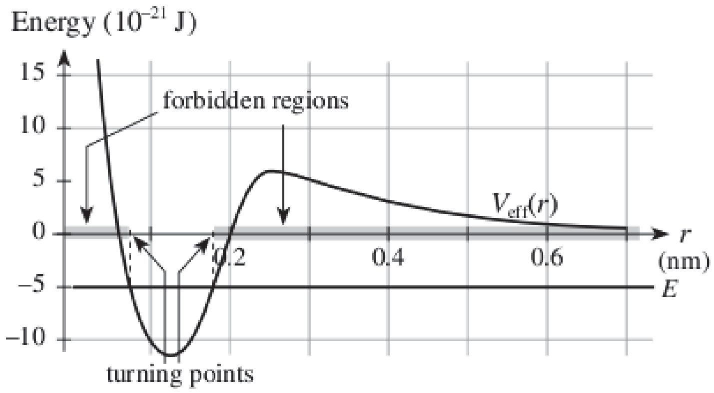
C9B.1 According to equation C9.2, the x component of the force is the negative slope of the potential energy graph, so the graph of the force as a function of position will look something like this:



Note that 1 pJ/fm = 10^{-12} J / 10^{-15} m = 1000 J/m = 1000 N = 1 kN. Note also that $F_x \approx +\frac{1}{3}$ kN for 4 fm to 7 fm.

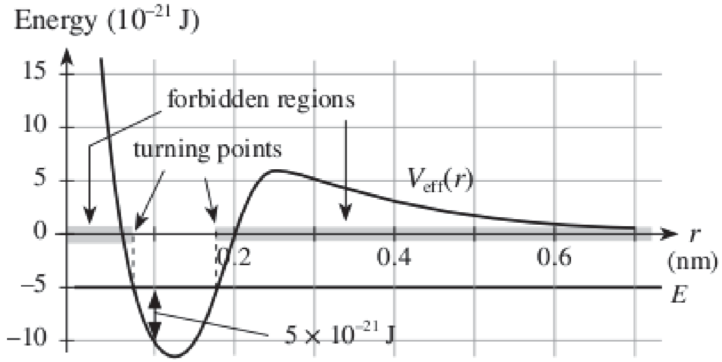
Unit: C
Chapter: C9
Problem Number: C9B.2
Return to User Page

C9B.2 The graph below shows the potential energy curve given in figure C9.13. The horizontal line corresponds to the system's total energy = $E = -5 \times 10^{-21}$ J. The forbidden regions are any values of r for which $V_{\text{eff}}(r) > E$. Allowed regions correspond to any other values of r, and turning points are at the boundaries between these regions. The graph below shows E, the forbidden regions, and the turning points in this particular case.



Unit: C Chapter: C9 Problem Number: C9B.4 Return to User Page

C9B.4 The system's effective potential energy at $r = 0.1$ nm is $V_{eff} \approx -10 \times 10^{-21}$ J. If the system's kinetic energy at this separation is 5×10^{-21} J, then the system's total energy must be -5×10^{-21} J, as shown below.

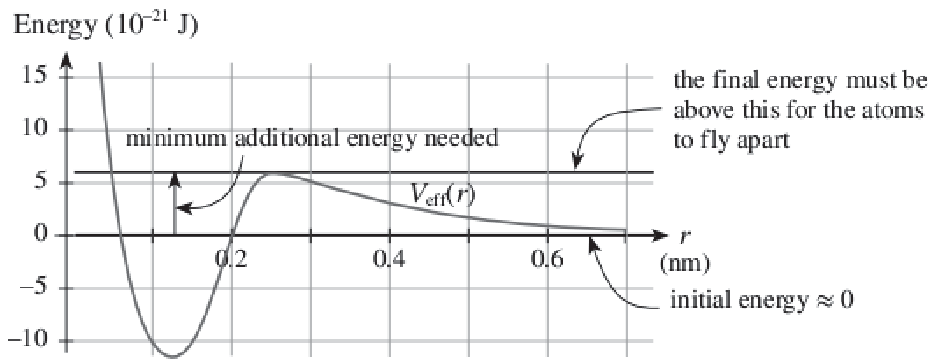


As the graph illustrates, the only allowed region for the system is the region where $0.07 \text{ nm} < r < 0.18$ nm. If the atoms are initially moving apart, their separation will increase to the turning point at 0.18 nm, then turn around and decrease until the separation reaches the turning point at 0.07 nm, turn around and increase, and so on. The inter-atomic separation will therefore oscillate back and forth between these two turning points.

Unit: C
Chapter: C9
Problem Number: C9B.8

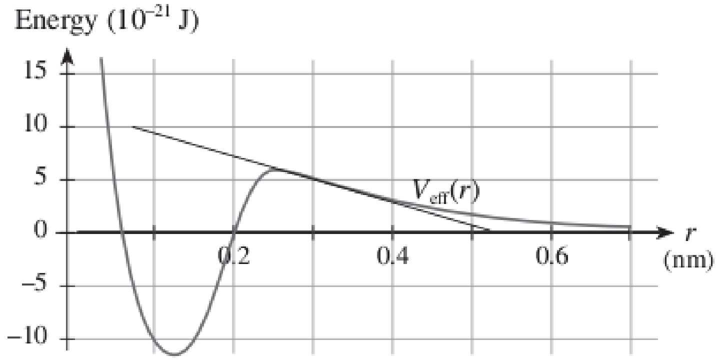
Return to User Page

C9B.8 In the two-atom system shown in figure C9.13, the atoms will be able to move to infinite separation only if the system's total energy E is greater than the interaction's potential energy at the peak of the "hill" at $x \approx 0.25$ nm; only then will the system separation have an allowed region flanked by two forbidden regions that prevent its escape. The potential energy of this peak looks to be about 6×10^{-21} J. Now, the effective potential energy of the system at a separation of $r = 0.10$ nm is about -10×10^{-21} J, so if the system has a kinetic energy of 10×10^{-21} J at that separation then the system's total energy is about equal to 0 J. For the atoms to come apart, then, the system's energy must be increased from 0 J to above 6×10^{-21} J, so we need to add at least 6×10^{-21} J to the system. The diagram below illustrates this:



Unit: C
Chapter: C9
Problem Number: C9B.9
Return to User Page

C9B.9 The diagram below shows that at $r = 0.30$ nm, the effective potential energy's curve seems to have a negative slope of about -5×10^{-21} J per 0.20 nm.



According to equation C9.2, the x component of the force exerted by one atom on the other is therefore

$$F_x = -\frac{dV}{dx} \approx \frac{5 \times 10^{-21}\ \text{J}}{0.22\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right)\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left(\frac{1\ \text{pN}}{10^{-12}\ \text{N}}\right) \approx 23\ \text{pN}$$

if we define the x axis to coincide with the r axis.

Unit: C
Chapter: C9
Problem Number: C9B.10
Return to User Page

C9B.10 According to equation C9.7, we have $V_s(x) = \frac{1}{2}k_s(x - x_0)^2 + C$, where x_0 is the spring's relaxed length and C is a constant. Therefore, the difference in the energy that the spring contains at $x = 12$ cm compared to what it stores at $x = x_0 = 10$ cm is

$$V_s(x) - V_s(x_0) = \tfrac{1}{2}k_s(x - x_0)^2 + \cancel{C} - 0 - \cancel{C} = \tfrac{1}{2}(1000\text{ J/m}^2)(0.020\text{ m})^2 = 0.20\text{ J}.$$

Unit: C
Chapter: C9
Problem Number: C9M.4
Return to User Page

C9M.4 **(a)** Note that if each star goes once around its orbit in time T, then each star's orbital speed must be $|\vec{v}_0| = 2\pi R/T$. Therefore, the system's total energy is

$$E = \tfrac{1}{2}M|\vec{v}_0|^2 + \tfrac{1}{2}M|\vec{v}_0|^2 - \frac{GM^2}{2R} = M\frac{4\pi^2 R^2}{T^2} - \frac{GM^2}{2R} \tag{1}$$

(b) If we consider one star to be at rest, while the other orbits around it at a distance of $2R$ in time T, then the moving star's speed $|\vec{v}|$ relative to the other is $|\vec{v}| = 2\pi(2R)/T = 4\pi R/T$. The reduced mass for the moving star in this case is

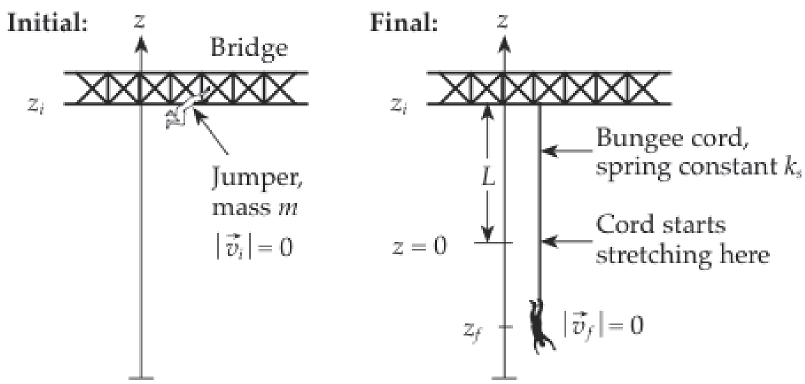
$$\mu = \frac{M^2}{M+M} = \tfrac{1}{2}M \tag{2}$$

Therefore, equation C9.6 predicts that in this situation, the system's energy is

$$E = \tfrac{1}{2}\mu|\vec{v}|^2 - \frac{GM^2}{2R} = \tfrac{1}{2}(\tfrac{1}{2}M)\left(\frac{4\pi R}{T}\right)^2 - \frac{GM^2}{2R} = \frac{M}{4}\frac{16\pi^2 R^2}{T^2} - \frac{GM^2}{2R} = M\frac{4\pi^2 R^2}{T^2} - \frac{GM^2}{2R} \tag{3}$$

Unit: C | Chapter: C9 | Problem Number: C9M.7 | Return to User Page

C9M.7 Initial and final drawings for this situation appear below.



Known:
m, k_s, L, $|\vec{g}|$
$|\vec{v}_i| = |\vec{v}_f| = 0$
$z_i = L$

Unknown:
$z_f = ?$

Internal interactions:
- Gravitation $V_g = m|\vec{g}|z$
- Bungee cord $V_s = \frac{1}{2}k_s z^2$ when $z < 0$

The system here is the jumper and the earth, which is isolated because it floats in space. The jumper and the earth participate in a gravitational interaction, which we will handle using the near-earth potential energy function $V_g = m|\vec{g}|z$, and a contact interaction through the bungee cord, which we will handle using the ideal-spring model where we treat the cord as if it were an interaction with a potential energy $V_s = \frac{1}{2}k_s(L - L_0)^2$, where L_0 is the bungee cord's relaxed length and $L > L_0$ its actual length ($V_s = 0$ if $L < L_0$). If we set $z = 0$ to be the jumper's position when the cord begins to stretch, then $L - L_0 = |z|$ (when $z < 0$) and V_s is simply $V_s = \frac{1}{2}k_s z^2$ We'll ignore friction, which is the same as assuming that the system's total internal energy U does not change, and we'll assume that the earth's kinetic energy K_e remains negligible. The jumper is at rest both at the beginning and at the bottom of her trajectory. The conservation of energy master equation in this situation therefore becomes

$$\tfrac{1}{2}m\overbrace{|\vec{v}_i|^2}^{0} + \overbrace{K_e}^{0} + \cancel{U} + m|\vec{g}|z_i = \tfrac{1}{2}m\overbrace{|\vec{v}_f|^2}^{0} + \overbrace{K_e}^{\approx 0} + \cancel{U} + m|\vec{g}|z_f + \tfrac{1}{2}k_s z_f^2$$

$$\Rightarrow \quad m|\vec{g}|z_i = m|\vec{g}|z_f + \tfrac{1}{2}k_s z_f^2 \quad \Rightarrow \quad 0 = z_f^2 + 2z_0 z_f - 2z_0 z_i \tag{1}$$

where $z_0 \equiv m|\vec{g}|/k_s$. Note that the units of z_0 are:

$$\frac{\cancel{\text{kg}}\cdot\text{m}/\cancel{\text{s}^2}}{\cancel{\text{J}}/\cancel{\text{m}^2}}\left(\frac{1\,\cancel{\text{J}}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m}^2}/\cancel{\text{s}^2}}\right) = \text{m} \tag{2}$$

which is why I am giving it a symbol we might associate with a length. Solving the quadratic for z_f *yields*

$$z_f = \frac{-2z_0 \pm \sqrt{4z_0^2 + 8z_0 z_i}}{2} = -z_0 \pm \sqrt{z_0^2 + 2z_0 z_i} \tag{3}$$

One of these solutions is positive, which is absurd, since the cord does not even start pulling until z becomes negative. So we want the negative solution. Noting that the bridge is at $z_i = L$, the jumper's maximum distance below the bridge is

$$L + |z_f| = L + z_0 + \sqrt{z_0^2 + 2z_0 L} = L + \frac{m|\vec{g}|}{k_s} + \sqrt{\left(\frac{m|\vec{g}|}{k_s}\right)\left(\frac{m|\vec{g}|}{k_s} + 2L\right)} \tag{4}$$