Unit: C ▼ Chapter: C12 ▼ Problem Number: C12B.2 ▼ Return to User Page

C12B.2 **(a)** According to the formula on the inside front cover,

$$T_{[C]} = \left(\frac{5°\text{C}}{9°\text{F}}\right)(T_{[F]} - 32°\text{F}) = \frac{5°\text{C}}{9°\text{F}}(98.6°\text{F} - 32°\text{F}) = 37.00°\text{C} \tag{1}$$

It is no accident that this is an nice integer. Historically, scientists using the Celsius temperature scale *defined* body temperature to be 37°C (a nice round number that embraced the range from 36.5°C to 37.5°C within which one's body temperature typically lies). This plausible round number became the over-precise value of 98.6°F when converted to Fahrenheit degrees. (Fahrenheit himself wanted 100°F to be body temperature when he originally designed the scale, but precisely because body temperature is so variable, his successors redefined the scale so that water's freezing and boiling points were 32°F and 212°F, respectively: this approximately reproduced his original scale but made its definition much more precise. The result, though, was that body temperature was no longer exactly 100°F.)

(b) According to the formula on the inside front cover,

$$T_{[F]} = \left(\frac{9°\text{F}}{5°\text{C}}\right)T_{[C]} + 32°\text{F} = \left(\frac{9°\text{F}}{5°\text{C}}\right)100°\text{C} + 32°\text{F} = 180°\text{F} + 32°\text{F} = 212°\text{F} \tag{2}$$

(c) According to the formulas on the inside front cover,

$$T_{[C]} = \left(\frac{5°\text{C}}{9°\text{F}}\right)(T_{[F]} - 32°\text{F}) = \frac{5°\text{C}}{9°\text{F}}(-40°\text{F} - 32°\text{F}) = -40°\text{C} \tag{3a}$$

$$T = \left(\frac{5\text{ K}}{9°\text{F}}\right)(T_{[F]} + 459.67°\text{F}) = \frac{5\text{ K}}{9°\text{F}}(-40°\text{F} + 459.67°\text{F}) = 233\text{ K} \tag{3b}$$

(The temperature corresponding to –40°F happens to be the single place on the Fahrenheit and Celsius scales where the numerical value of the temperatures agree.)

(d) The temperature of boiling liquid nitrogen is

$$T_{[C]} = \left(\frac{1°\text{C}}{1\text{ K}}\right)(T - 273.15\text{ K}) = \left(\frac{1°\text{C}}{1\text{ K}}\right)(77\text{ K} - 273.15\text{ K}) = -196°\text{C} \tag{4a}$$

$$T_{[F]} = \left(\frac{9°F}{5\,K}\right)T - 459.67°\text{F} = \left(\frac{9°\text{F}}{5\text{ K}}\right)(77\text{ K}) - 459.67°\text{F} = -321°\text{F} \tag{4b}$$

(e) According to the formula on the inside front cover,

$$T_{[F]} = \left(\frac{9°F}{5\,K}\right)T - 459.67°\text{F} = 0 - 459.67°\text{F} = -459.67°\text{F} \tag{5}$$

Unit: C
Chapter: C12
Problem Number: C12B.5

Return to User Page

C12B.5 **(a)** According to table C12.1, the specific "heat" of mammalian flesh is about 3400 J/(kg·K). The internal energy of person with a mass $m = 50$ kg whose temperature increases by $\Delta T = 2°\text{C} = 2$ K is thus

$$\Delta U^{\text{th}} \approx mc\Delta T = (50\ \cancel{\text{kg}})\left(3400\ \frac{\text{J}}{\cancel{\text{kg}}\cdot\cancel{\text{K}}}\right)(2\ \cancel{\text{K}}) = 340{,}000\ \text{J} \tag{1}$$

(b) In food calories, this is

$$340{,}000\ \cancel{\text{J}}\left(\frac{1\ \cancel{\text{cal}}}{4.186\ \cancel{\text{J}}}\right)\left(\frac{1\ \text{Cal}}{1000\ \cancel{\text{cal}}}\right) = 81\ \text{Cal}. \tag{2}$$

Unit: C ▼ Chapter: C12 ▼ Problem Number: C12B.6 ▼ Return to User Page

C12B.6 The specific "heat" of water is $c = 4186$ J/(kg·K), so energy released when the temperature of $\boldsymbol{m} = 1$ kg of water is decreased by $\Delta T = 1°\text{C} = 1$ K is simply

$$\Delta U^{\text{th}} \approx mc\,\Delta T = (1\ \text{kg})\left(4186\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-1\ \text{K}) = -4186\ \text{J}. \tag{1}$$

If we were to convert this energy into gravitational potential energy of the bottle interacting with the earth so that $m|\vec{g}\,|h = -\Delta U^{\text{th}}$, then we would increase the bottle's height by

$$h = \frac{-\Delta U^{\text{th}}}{m|\vec{g}\,|} = \frac{4186\ \text{J}}{(1\ \text{kg})(9.8\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m}^2/\text{s}^2}{1\ \text{J}}\right) = 427\ \frac{\text{m}^2}{\text{m}} = 427\ \text{m}. \quad (!) \tag{2}$$

Unit: C | Chapter: C12 | Problem Number: C12B.8 | Return to User Page

C12B.8 Assuming that the specified change in temperature $dT = T_f - T_i = 22°\text{C} - 95°\text{C} = -73°\text{C}$ is "sufficiently small" so that the specific "heat" of water is roughly constant then we can find the amount of energy that the cup loses using equation C12.10 and the fact that 1°C of temperature change = 1 K of temperature change:

$$dU^{\text{th}} = mc\,dT = (0.25\ \cancel{\text{kg}})(4186\ \text{J}\cdot\cancel{\text{kg}}^{-1}\cancel{\text{K}}^{-1})(-73\ \cancel{\text{K}}) = -76{,}000\ \text{J} \quad (1)$$

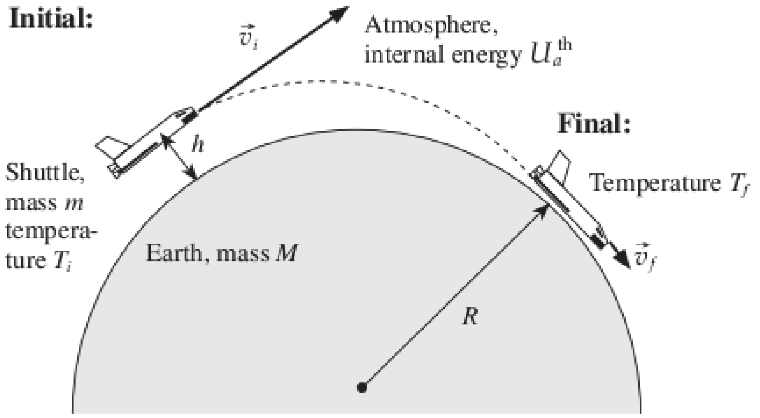
where the negative sign merely indicates that the cup loses internal energy. My mass is about 70 kg, so the kinetic energy that I would lose running into a brick wall at 5 m/s is more like:

$$\Delta K = K_f - K_i = 0 - \tfrac{1}{2}m|\vec{v}_i|^2 = -\tfrac{1}{2}(70\ \cancel{\text{kg}})(5\ \cancel{\text{m/s}})^2\left(\frac{1\ \text{J}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m}^2/\text{s}^2}}\right) = -880\ \text{J} \quad \text{(about 90 times smaller!)} \quad (2)$$

Even everyday thermal energy changes involve a lot of energy compared to everyday mechanical processes!

Unit: C ▼ Chapter: C12 ▼ Problem Number: C12M.2 ▼ Return to User Page

C12M.2 Here are initial and final diagrams of the situation:



Knowns and Unknowns:
$M = 6.0 \times 10^{24}$ kg
$m = 2 \times 10^6$ kg
$G = 6.67 \times 10^{-11}$ N·m²/kg²
$T_f \approx T_i$
$h = 100$ km
$R = 6380$ km
$\Delta U_a^{th} = ?$

Internal interactions:
- Gravitational: $V_g = -GMm/r$
- Contact with atmosphere: keep track of U_a^{th}.

The system the shuttle and the earth (we'll consider the atmosphere part of the latter). This system floats in space and should not radiate or absorb much energy (at least connected with this process) during the process. The earth is so massive that its kinetic energy K_e will remain essentially zero. Because the shuttle starts more than a few kilometers above the earth's surface, we will use the general gravitational potential energy formula $V_g = -GMm/r$ to describe the internal gravitational interaction here. The only thing that the shuttle touches during its descent is the atmosphere: we will handle the this interaction by keeping track of the atmosphere's initial and final thermal energies U_{ai}^{th} and U_{af}^{th} as well as the shuttle's thermal energy U_s^{th}. But the shuttle's initial and final temperatures are about the same, so the shuttle's thermal energy is constant for the process. The earth's thermal energy U_e^{th} is not affected by the gravitational interaction (the only interaction in which the earth directly takes part). The conservation of energy master equation therefore becomes:

$$\tfrac{1}{2}m|\vec{v}_i|^2 + \cancel{U_s^{th}} + \overbrace{K_e}^{0} + U_{ai}^{th} + \cancel{U_e^{th}} - \frac{GMm}{r+h} = \tfrac{1}{2}m|\vec{v}_f|^2 + \cancel{U_s^{th}} + \overbrace{K_e}^{\approx 0} + U_{af}^{th} + \cancel{U_e^{th}} - \frac{GMm}{r}$$

$$\Rightarrow\ U_{af}^{th} - U_{ai}^{th} = \tfrac{1}{2}m(|\vec{v}_i|^2 - |\vec{v}_f|^2) + \frac{GMm}{R} - \frac{GMm}{R+h} \tag{1}$$

It is interesting to calculate the kinetic and gravitational parts separately:

$$\tfrac{1}{2}m(|\vec{v}_i|^2 - |\vec{v}_f|^2) = \tfrac{1}{2}(2\times 10^6\ \cancel{\text{kg}})\left(\left[8000\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right]^2 - \left[100\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right]^2\right)\left(\frac{1\text{ J}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m}}^2/\cancel{\text{s}}^2}\right) = 6.4\times 10^{13}\text{ J} \tag{2a}$$

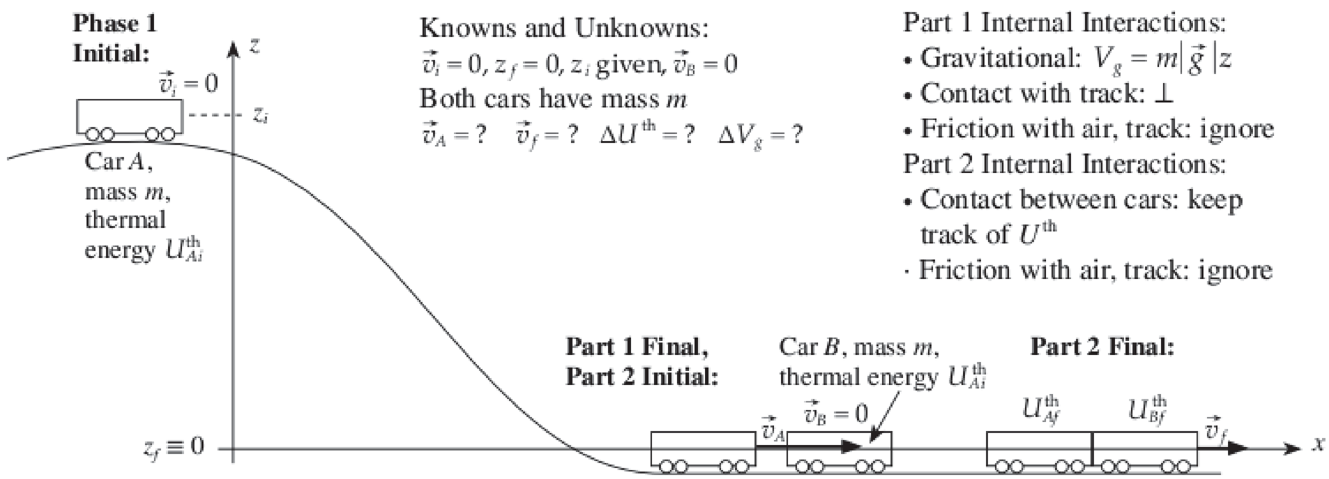
$$GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = \left(6.67\times 10^{-11}\ \frac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}}^2}\right)(6.0\times 10^{24}\ \cancel{\text{kg}})(2.0\times 10^6\ \cancel{\text{kg}})\left(\frac{1}{6380\ \cancel{\text{km}}} - \frac{1}{6480\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{km}}}{1000\text{ m}}\right)$$

$$= 1.9\times 10^{12}\ \frac{\cancel{\text{N}}\cdot\cancel{\text{m}}^2}{\cancel{\text{m}}}\left(\frac{1\text{ J}}{1\ \cancel{\text{N}}\cdot\cancel{\text{m}}}\right) = 1.9\times 10^{12}\text{ J} \tag{2b}$$

(a) The total kinetic + potential energy converted to other forms is about 6.6×10^{13} J, almost all of which is the shuttle's kinetic energy.
(b) As we can see from the analysis above, this must go essentially entirely to thermal energy in the atmosphere.

Unit: C ▼ Chapter: C12 ▼ Problem Number: C12M.6 ▼ Return to User Page

C12M.6 Initial, intermediate, and final diagrams for this situation appear below. Note that I have defined the reference frame so that all motion takes place in the *xz* plane.



We can divide the process into two parts: (1) the first car rolling down the hill, and (2) the collision. In the first part, the system is the rolling car and the earth. This system floats in space and should not radiate or absorb much energy (at least associated with this process) during the time interval of interest. The earth is so massive that its kinetic energy K_e should remain essentially zero. Since the rolling car is always close to the earth's surface, we can use the near-earth gravitational potential energy formula $V_g = m|\vec{g}|z$ to handle the internal gravitational interaction. Assuming that friction is negligible, the internal contact interaction between the car and the track can be ignored, because it exerts a force on the car that is always perpendicular to their motion. In both parts, I will ignore the contact interaction between the cars and the air, and also the comparatively negligible rotational energy of the car's wheels (negligible because the wheels are only a small fraction of the attached car's mass).

In the second part, the two cars are functionally isolated from the earth, so this system's energy and momentum are conserved separately. The sole internal interaction for the two-car system during this phase is the contact interaction between the two cars, which we will handle by keep track of the cars' thermal energies U^{th}_A and U^{th}_B.

During the first part, nothing is significantly affecting the car's internal energy U_A or the earth's internal energy U_e, so the conservation of energy master equation becomes

$$\tfrac{1}{2}m\overbrace{|\vec{v}_i|^2}^{0} + \cancel{U_A} + \overbrace{K_e}^{0} + \cancel{U_e} + m|\vec{g}|z_i = \tfrac{1}{2}m|\vec{v}_A|^2 + \cancel{U_A} + \overbrace{K_e}^{\approx 0} + \cancel{U_e} + m|\vec{g}|\overbrace{z_f}^{0}$$

$$\Rightarrow \quad \tfrac{1}{2}m|\vec{v}_A|^2 = m|\vec{g}|z_i \quad \Rightarrow \quad |\vec{v}_A| = \sqrt{2|\vec{g}|z_i} \tag{1}$$

In the second part, conservation of the two-car system's momentum requires that

$$m\begin{bmatrix}|\vec{v}_A|\\0\\0\end{bmatrix} + m\begin{bmatrix}0\\0\\0\end{bmatrix} = m\begin{bmatrix}|\vec{v}_f|\\0\\0\end{bmatrix} + m\begin{bmatrix}|\vec{v}_f|\\0\\0\end{bmatrix} = 2m\begin{bmatrix}|\vec{v}_f|\\0\\0\end{bmatrix} \quad \Rightarrow \quad |\vec{v}_f| = \frac{m}{2m}|\vec{v}_A| = \tfrac{1}{2}|\vec{v}_A| \tag{2}$$

Conservation of the two-car system's energy requires that

$$\tfrac{1}{2}m|\vec{v}_A|^2 + U^{th}_{Ai} + \tfrac{1}{2}m\overbrace{|\vec{v}_B|^2}^{0} + U^{th}_{Bi} = \tfrac{1}{2}(2m)|\vec{v}_f|^2 + U^{th}_{Af} + U^{th}_{Bf}$$

$$\Rightarrow \quad \Delta U^{th} \equiv U^{th}_{Af} + U^{th}_{Bf} - U^{th}_{Ai} - U^{th}_{Bi} = \tfrac{1}{2}m|\vec{v}_A|^2 - m|\vec{v}_f|^2 = \tfrac{1}{2}m|\vec{v}_A|^2 - \tfrac{1}{4}m|\vec{v}_A|^2 = \tfrac{1}{4}m|\vec{v}_A|^2 \tag{3}$$

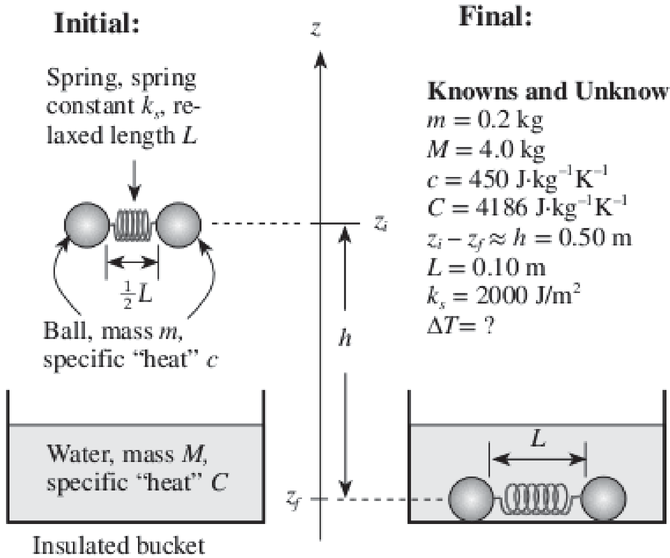
The change in gravitational potential energy during the first part is $m|\vec{g}|(z_f - z_i) = -m|\vec{g}|z_i$, since $z_f \equiv 0$. Therefore, ΔU^{th} during the second part as a fraction of the absolute value of this change is

$$\frac{\Delta U^{th}}{m|\vec{g}|z_i} = \frac{\tfrac{1}{4}m|\vec{v}_A|^2}{m|\vec{g}|z_i} = \frac{\tfrac{1}{4}\cancel{m}(2\cancel{|\vec{g}|z_i})}{\cancel{m}\cancel{|\vec{g}|z_i}} = \frac{1}{2} \tag{4}$$

Therefore, half of the change in potential energy has been converted to thermal energy in this process.

Unit: C ▼ Chapter: C12 ▼ Problem Number: C12M.9 ▼ Return to User Page

C12M.9 Here are initial and final diagrams of the situation:



Knowns and Unknowns:
$m = 0.2$ kg
$M = 4.0$ kg
$c = 450\ \text{J}\cdot\text{kg}^{-1}\text{K}^{-1}$
$C = 4186\ \text{J}\cdot\text{kg}^{-1}\text{K}^{-1}$
$z_i - z_f \approx h = 0.50$ m
$L = 0.10$ m
$k_s = 2000\ \text{J/m}^2$
$\Delta T = ?$

Internal interactions:
- Gravitational: $V_g = m|\vec{g}|z$
- Contact with water: keep track of internal energies.
- Other contact: ignore

The system is the two balls, the spring, the water, and the earth (the container is part of the latter). This system floats in space and should not radiate or absorb energy (at least connected with this process) during the process. The earth is so massive that its kinetic energy K_e will remain essentially zero. Since the ball + spring is always close to the earth's surface, we can use $V_g = m|\vec{g}|z$ to describe the internal gravitational interaction. The contact interaction between the balls and the water is complicated, but we can handle it by keeping track of the system's initial and final internal energies. Specifically, let each ball's initial and final thermal energies be U^{th}_{Bi} and U^{th}_{Bf} and the water's initial and final thermal energies be U^{th}_{wi} and U^{th}_{wf}. The spring has an initial internal energy $U_{Si} = \frac{1}{2}k_s(\frac{1}{2}L - L)^2 = \frac{1}{8}k_sL^2$ due to its compression, but its final internal energy $U_{Sf} = 0$ because the spring is relaxed. We will ignore its thermal energy (its mass will likely be much smaller than that of the other objects). We can ignore the water's contact interaction with the container because the container is at rest (so the contact forces do no work) and we are told that the process happens rapidly enough that little heat flows to the person or the surroundings. The problem statement discusses why we can ignore the balls' contact interaction with the container. The balls have zero kinetic energy $K_B = 0$ both initially and finally. Therefore, the conservation of energy master equation therefore becomes:

$$\overbrace{K_B}^{0} + U^{\text{th}}_{Bi} + \tfrac{1}{8}k_sL^2 + \overbrace{K_e}^{0} + U^{\text{th}}_{wi} + 2m|\vec{g}|z_i = \overbrace{K_B}^{0} + U^{\text{th}}_{Bf} + \overbrace{U_{Sf}}^{0} + \overbrace{K_e}^{\approx 0} + U^{\text{th}}_{wf} + 2m|\vec{g}|z_f$$

$$\Rightarrow \quad \tfrac{1}{8}k_sL^2 + 2m|\vec{g}|(z_i - z_f) = U^{\text{th}}_{Bf} - U^{\text{th}}_{Bi} + U^{\text{th}}_{wf} - U^{\text{th}}_{wi} = \Delta U^{\text{th}}_B + \Delta U^{\text{th}}_w \qquad (1)$$

(a) If we can assume that the temperature change ΔT is small enough so that the specific "heats" of the iron balls and the water remain constant, then $\Delta U^{\text{th}}_B \approx 2mc\,\Delta T$ and $\Delta U^{\text{th}}_w \approx MC\,\Delta T$. We also note that we are given that $z_f - z_i = h$. Substituting these things into equation 1 above yields

$$\tfrac{1}{8}k_sL^2 + 2m|\vec{g}|h = (mc + MC)\Delta T \quad \Rightarrow \quad \Delta T = \frac{\frac{1}{8}k_sL^2 + 2m|\vec{g}|h}{mc + MC} \qquad (2)$$

(b) Substituting in the numbers yields

$$\Delta T = \frac{\frac{1}{8}(2000\ \text{J/m}^2)(0.10\ \text{m})^2 + 2(0.20\ \text{kg})(9.8\ \text{m/s}^2)(0.5\ \text{m})\left(\dfrac{1\ \text{J}}{1\ \text{kg}\cdot\text{m}^2/\text{s}^2}\right)}{2(0.2\ \text{kg})(450\ \text{J}\cdot\text{kg}^{-1}\text{K}^{-1}) + (4.0\ \text{kg})(4186\ \text{J}\cdot\text{kg}^{-1}\text{K}^{-1})} = 2.6\times10^{-4}\ \text{K} = 0.26\ \text{mK} \qquad (3)$$

Note that the units work out, and the result is small, as expected (indeed, small enough that we can consider the temperature change truly negligible).