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C13B.3 To melt a certain mass m of ice, we must increase its latent energy by $\Delta U^{\text{la}} = +mL$ (according to equation C13.1). The time T required will be this needed energy divided by the rate at which the hair dryer delivers thermal energy to do this. Let $dU/dt = 2500$ J/s $= 2.5$ kJ/s be this rate; then

$$\frac{mL}{T} = \left|\frac{dU}{dt}\right| \quad \Rightarrow \quad m = \frac{|dU/dt|T}{L} = \frac{(2.5\ \cancel{\text{kJ}}/\cancel{\text{s}})(60\ \cancel{\text{s}})}{333\ \cancel{\text{kJ}}/\text{kg}} = 0.45\ \text{kg}.$$

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C13B.5 In section C13.4 we learned that fusing 1 kg of a boron-hydrogen (H+B) mix releases about 70 TJ of thermal energy. If the U.S. uses energy at the rate of 10^{20} J / y, the rate at which we would consume the H+B mix would be

$$10^{20}\,\frac{\cancel{\text{J}}}{\text{y}}\left(\frac{1\ \cancel{\text{TJ}}}{10^{12}\ \cancel{\text{J}}}\right)\left(\frac{1\ \text{kg (H + B)}}{70\ \cancel{\text{TJ}}}\right) = 1.43\times 10^{6}\ \frac{\text{kg}}{\text{y}}$$

Since the fuel's weight is mostly boron, this means that the world's supply of boron could produce energy at this rate for more than 10,000 years.

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C13B.6 According to the Stefan-Boltzmann law, the rate at which an object emits energy $P = \varepsilon\sigma AT^4$, where ε is the object's emissivity, σ is the Stefan-Boltzmann constant, A is its surface area, and T is its temperature. We are given that ratio of the power P that this star emits to the power P_0 emitted by the sun is $P/P_0 = 8.7$ million. Both stars are spheres, so their surface areas are $A = 4\pi R^2$ and $A_0 = 4\pi R_0^2$, respectively. We also know that the star's surface temperature is $T = 53{,}000$ K and the sun's is $T_0 = 5800$ K. So if we assume (quite plausibly) that the stars have the same emissivity ε, then

$$\frac{P}{P_0} = \frac{\varepsilon\sigma(4\pi R^2)T^4}{\varepsilon\sigma(4\pi R_0^2)T_0^4} = \left(\frac{R}{R_0}\right)^2\left(\frac{T}{T_0}\right)^4 \quad \Rightarrow \quad \left(\frac{R}{R_0}\right)^2 = \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right)^4$$

$$\frac{R}{R_0} = \sqrt{\frac{P}{P_0}}\left(\frac{T_0}{T}\right)^2 = \sqrt{8.7\times 10^6}\left(\frac{5800\ \cancel{\text{K}}}{53{,}000\ \cancel{\text{K}}}\right)^2 = 35$$

So the radius of this star is 35 times that of the sun.

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C13B.9 The total energy E used by something at a constant rate P for time T is $E = |dE/dt|T = PT$. So

$$1\,\text{kW·hr} = (1\,\text{kW})(1\,\text{h}) = (1\,\cancel{\text{kW}})(1\,\cancel{\text{h}})\left(\frac{1000\,\cancel{\text{W}}}{1\,\cancel{\text{kW}}}\right)\left(\frac{\text{J}/\cancel{\text{s}}}{1\,\cancel{\text{W}}}\right)\left(\frac{3600\,\cancel{\text{s}}}{\cancel{\text{h}}}\right) = 3.6\times10^{6}\,\text{J} = 3.6\,\text{MJ}.$$

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C13M.4 The system here is the water and the liquid nitrogen. Since everything is at rest, external forces can do no work on the system, but we will have to assume that the process goes quickly enough that the "insulating container" is able to prevent a significant amount of heat from flowing through its walls. We will handle the contact interaction between the ice and water by keeping track of internal energies. We can usefully break this process into three steps: (1) the water cools to 0°C, (2) the water at 0°C converts to ice at 0°C, and (3) the newly formed ice cools from there to the boiling point of liquid nitrogen. Note that the nitrogen's temperature will always be at that boiling point (T_f = 77 K) throughout the process, so its *thermal* energy does not change, though its latent energy increases as it vaporizes. Assume that the gravitational potential energy released as the water is poured into the nitrogen is negligible. As neither object has initial or final kinetic energy, the contact interaction has no potential energy, most terms in the conservation of energy master equation are zero: but since the change in the total energy of this (allegedly) isolated system must be zero, we can write the remaining terms for the first step this way:

$$\Delta U_w^{\text{th}} + \Delta U_{LN}^{\text{la}} = 0 \tag{1}$$

where U_w^{th} is the water's thermal energy, U_{LN}^{la} is the latent energy of the liquid nitrogen. Let m = 5 g be the mass of the water, $\Delta T_1 =$ 273 K – 295 K = –22 K be the change in the water's temperature during the first step, and m_1 be the mass of the liquid nitrogen boiled during that step. The latent "heat" for boiling liquid nitrogen is L = 199 kJ/kg (from table C13.1) and the specific heat of water is c_w = 4186 J(kg·K) (from table C12.1). If we assume that the water's specific heat is constant over the temperature range it is going to experience, we have $U_w^{\text{th}} = mc_w\Delta T_1$ and $U_{LN}^{\text{la}} = +m_1 L$, so

$$0 = +m_1 L + mc_w\Delta T \;\Rightarrow\; m_1 L = -mc_w\Delta T$$

$$\Rightarrow\; m_1 = -\frac{mc_w\Delta T}{L} = -\frac{(5.0\text{ g})(4.186\ \cancel{\text{kJ}}\cdot\cancel{\text{kg}}^{-1}\cancel{\text{K}}^{-1})(-22\ \cancel{\text{K}})}{199\ \cancel{\text{kJ}}\cdot\cancel{\text{kg}}^{-1}} = +2.3\text{ g} \tag{2}$$

During the second step, the water is freezing to ice at a temperature of 0°C. Let m_2 be the mass of liquid nitrogen boiled in this step, and let L_i be the latent "heat" of ice, which is 333 kJ/kg. The latent energy change of the ice as it freezes is $\Delta U_i^{\text{la}} = -mL_i$, so conservation of energy for this step states that

$$\Delta U_i^{\text{la}} + \Delta U_{LN}^{\text{la}} = 0 \;\Rightarrow\; -mL_i + m_2 L = 0 \;\Rightarrow\; m_2 = m\frac{L_i}{L} = (5.0\text{ g})\frac{333\ \cancel{\text{kJ}\cdot\text{kg}}}{199\ \cancel{\text{kJ}\cdot\text{kg}}} = +8.4\text{ g} \tag{3}$$

In the final step, the ice cools down to the temperature of the liquid nitrogen. Let c_i = 2100 J/kg= 2.1 kJ/kg be the specific "heat" of ice (see table C12.1), and assume that it remains constant during its temperature change of ΔT_3 = 77 K – 273 K = –196 K: its thermal energy change will therefore be $\Delta U_i^{\text{th}} = mc_i\Delta T_3$. Let m_3 be the mass of liquid nitrogen boiled in this step. Conservation of energy for this step then requires that

$$\Delta U_i^{\text{th}} + \Delta U_{LN}^{\text{la}} = 0 \quad\Rightarrow\quad mc_i\Delta T_3 + m_3 L = 0 \quad\Rightarrow\quad m_3 L = -mc_i\Delta T_3$$

$$\Rightarrow\; m_3 = -\frac{mc_i\Delta T_3}{L} = -\frac{(5.0\text{ g})(2.1\ \cancel{\text{kJ}}\cdot\cancel{\text{kg}}^{-1}\cancel{\text{K}}^{-1})(-196\ \cancel{\text{K}})}{199\ \cancel{\text{kJ}}\cdot\cancel{\text{kg}}^{-1}} = +10.3\text{ g} \tag{4}$$

The total mass of liquid nitrogen boiled is therefore $m_1 + m_2 + m_3$ = 2.3 g + 8.4 g + 10.3 g = 21.0 g.

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C13M.8 The system here is the snow and my body, which participate in a contact interaction. I will assume that energy exchanges between my body and the external environment are minimal, so that this system is approximately isolated. The snow undergoes a phase change, and my body uses up chemical energy U^{ch}_{me}. But my body's internal temperature will remain roughly constant, so the change in my thermal energy will be negligible. The snow undergoes a phase change so its latent energy increases: $\Delta U^{la}_{s} = +mL$, where m = 1 kg is the mass of the snow and L = 333 kJ/kg is the latent "heat" of melting ice. The snow must also be warmed up to body temperature, so its thermal energy must increase by $\Delta U^{th}_{s} = mc\Delta T$, where c = 4186 J/kg is the specific "heat" of water and ΔT = 37°C – 0°C = +37°C = +37 K is how much the snow's temperature must increase. Nothing is moving very rapidly here at all, so kinetic energies will be negligible compared to other forms of energy. Conservation of energy requires that the change in the system's total energy must be zero, so in this case, this all reduces to

$$0 = \Delta U^{la}_{s} + \Delta U^{th}_{s} + \Delta U^{ch}_{me} = +mL + mc\Delta T + \Delta U^{ch}_{me}$$

$$\Rightarrow \Delta U^{ch}_{me} = -m(L + c\Delta T) = -(1\ \cancel{\text{kg}})\left(333\frac{\cancel{\text{kJ}}}{\cancel{\text{kg}}} + 4.186\frac{\cancel{\text{kJ}}}{\text{kg}\cdot\cancel{\text{K}}}(37\ \cancel{\text{K}})\right)\left(\frac{1\ \text{Cal}}{4.186\ \cancel{\text{kJ}}}\right) = 117\ \text{Cal}$$

So this "meal" costs my body about 120 Cal of chemical energy. The units work out, and the magnitude seems reasonable (a small fraction of a whole day's food energy intake ≈ 2000 Cal).

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C13M.10 The system here is the grendel. Suppose that a grendel has mass m, and can accelerate from rest ($\vec{v}_i = 0$) to a final speed of $|\vec{v}_f| = 30$ m/s in a time $t = 3$ s. This corresponds to conversion of chemical energy to the grendel's kinetic energy K at a rate of

$$P_0 = \frac{dK}{dt} = \frac{\frac{1}{2}m|\vec{v}_f|^2 - 0}{t} = \frac{m|\vec{v}_f|^2}{2t} \tag{1}$$

But because of its muscles' inefficiency, the grendel must burn chemical energy at 5 times this rate, so $P = |\Delta U_g^{ch}/\Delta t| = 5P_0$, where ΔU_g^{ch} is the change in the grendel's chemical energy. Now the grendel cannot sweat, and direct energy loss to the air via conduction or convection will be slow, and even radiation will not count for much until the grendel's surface temperature is significantly hotter than that of its surroundings, so (especially initially), the grendel's chemical energy that does *not* go to mechanical energy must go to its internal energy U_g^{th}: therefore $\Delta U_g^{th} \approx \frac{4}{5}\Delta U_g^{ch}$. But as long as the grendel's specific "heat" $c = 3400$ J/kg, does not change much, $\Delta U_g^{th} = mc\,\Delta T$. Therefore $\Delta U_g^{th}/\Delta t = mc(\Delta T/\Delta t) \approx \frac{4}{5}(5P_0) = 4P_0$. Thus

$$\frac{\Delta T}{\Delta t} \approx \frac{4P_0}{mc} = \frac{1}{\cancel{m}c}\left(\frac{4\cancel{m}|\vec{v}_f|^2}{2t}\right) = \frac{2(30\,\cancel{\text{m}/\text{s}})^2}{(3400\,\cancel{\text{J}}\cdot\cancel{\text{kg}}^{-1}\text{K}^{-1})(3\,\text{s})}\left(\frac{1\,\cancel{\text{J}}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m}^2/\text{s}^2}}\right) = \frac{0.18\,\text{K}}{\text{s}} \tag{2}$$

(Note that the grendel's unknown mass cancels out.) If the grendel can tolerate about a 3-K temperature increase (an increase that would make a person very uncomfortable), the grendel has 17 seconds to wreak havoc before desperately needing a cooling bath. (To understand why this creature is called a grendel, look up "Beowulf" online.)