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C11B.1 According to equation C11.3, an object's rotational kinetic energy is given by $U^{rot} = \frac{1}{2}I|\vec{\omega}|^2$, where I is the object's moment of inertia and $|\vec{\omega}|$ is its angular speed. Figure C11.6 tells us that for a cylinder, $I = \frac{1}{2}MR^2$, where M is the cylinder's mass and R its outer radius. Combining these equations and plugging in numbers yields

$$U^{rot} = \tfrac{1}{4}MR^2|\vec{\omega}|^2 = \tfrac{1}{4}(200\ \cancel{kg})(0.2\ \cancel{m})^2\left(\frac{10\ \cancel{rev}}{\cancel{s}}\right)^2\left(\frac{2\pi\ \text{rad}}{1\ \cancel{rev}}\right)^2\left(\frac{1\ \text{J}}{1\ \cancel{kg}\cdot\cancel{m}^2/\cancel{s}^2}\right) = 7900\ \text{J}$$

(Note how I dropped the "units" of rad^2 in the last step.)

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C11B.3 Recall that the answer for 10 pieces was 0.330. If we divide the rod up into 20 pieces, we will have 10 pieces on each side. So the first piece spans from $-R$ to $-0.9R$ and thus has a midpoint of $r_1 = 0.95$ R and a $u_1 = -0.95$. Similarly, $u_2 = -0.85$, $u_3 = -0.75$, … , $u_{20} = 0.95$. Note that $\Delta u = (2R/R)\,n = 1/10$ now. Equation C11.7 now becomes

$$I = MR^2\left[\frac{1}{2}\Delta u \sum_{j=1}^{20} u_j^2\right] = MR^2\left[\frac{1}{10}2(0.05^2 + 0.15^2 + \ldots + 0.95^2)\right] = 0.3325MR^2$$

where (as we did in equation C11.8), I have taken advantage of the fact that the squared u_js have the same set of values for the 10 pieces on the left as they do for the 10 pieces on the right. This final result is much closer to the idea value of 1/3 than the result for 10 pieces was.

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C11B.6 **(a)** If I curl my right fingers along with how the bowling ball rotates as it rolls away from me, I find that my thumb points to the left. Therefore, the ball's angular velocity vector $|\vec{\omega}|$ points to my left. Since the ball rolls without slipping, its angular speed $|\vec{\omega}|$ is related to its radius R and the speed of its center of mass $|\vec{v}_{CM}|$ by equation C11.15:

$$|\vec{\omega}| = \frac{|\vec{v}_{CM}|}{R} = \frac{4.0\ \cancel{m}/s}{0.12\ \cancel{m}} = \frac{33}{s} = 33\ \frac{rad}{s} \quad (1)$$

(b) So the bowling ball's angular velocity is 33 rad/s to my left.

(c) The ball's total energy under these circumstances is given by equation C11.12:

$$K + U^{rot} = \tfrac{1}{2}M|\vec{v}_{CM}|^2 + \tfrac{1}{2}I|\vec{\omega}|^2 \quad (2)$$

where M is the ball's mass and I is its moment of inertia. The moment of inertia of a sphere is (according to figure C11.6) $I = \frac{2}{5}MR^2$. Substituting this into the above and plugging in the numbers yields:

$$K + U^{rot} = \tfrac{1}{2}M|\vec{v}_{CM}|^2 + \tfrac{1}{2}(\tfrac{2}{5}MR^2)|\vec{\omega}|^2 = \tfrac{1}{2}M|\vec{v}_{CM}|^2 + \tfrac{1}{5}M\cancel{R^2}\left(\frac{|\vec{v}_{CM}|}{\cancel{R}}\right)^2 = \tfrac{1}{2}M|\vec{v}_{CM}|^2 + \tfrac{1}{5}M|\vec{v}_{CM}|^2$$

$$= \tfrac{7}{10}M|\vec{v}_{CM}|^2 = \tfrac{7}{10}(6\ \cancel{kg})(4\ \cancel{m/s})^2\left(\frac{1\ J}{1\ \cancel{kg \cdot m^2/s^2}}\right) = 67\ J. \quad (3)$$

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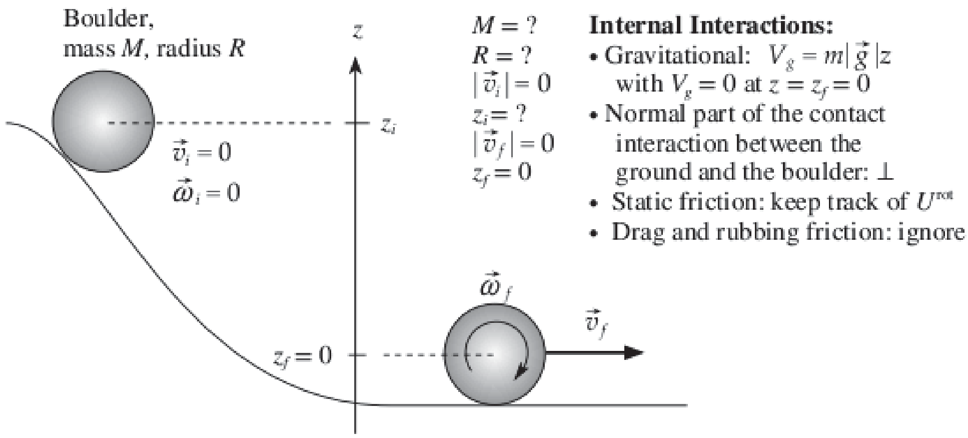
C11M.1 The total length of iron rod in this device is $2R+2R+2\pi R = 2(2+\pi)R$. Since the mass in any part of the wheel is proportional to the length of iron rod in that part, this means that each rod's mass m_r and the outer rim's mass m_o must be such that

$$\frac{m_r}{M} = \frac{2R}{2(2+\pi)R} = \frac{1}{2+\pi} \quad \text{and} \quad \frac{m_o}{M} = \frac{2\pi R}{2(2+\pi)R} = \frac{\pi}{2+\pi} \tag{1}$$

According to figure C11.6, the moment of inertia for a rod of length $2R$ and mass m_r rotating about its center is $I_r = \frac{1}{3}m_r R^2$. The mass of the outer rim is like that of a hoop, entirely at a distance of almost R from the axis, so its moment of inertia is $I_o = m_o R^2$. The wheel's total moment of inertia is therefore

$$I = 2I_r + I_o = 2\tfrac{1}{3}m_r R^2 + m_o R^2 = \frac{2}{3}\frac{M}{2+\pi}R^2 + \frac{\pi M}{2+\pi}R^2 = \frac{2+3\pi}{6+3\pi}MR^2 = 0.74MR^2. \tag{2}$$

C11M.4 Initial and final diagrams for this situation appear below:



The system is the rock and the earth, which is isolated because it floats in space. The internal interactions in this system are gravitational and the boulder's contact interactions with the ground and air. The normal part of the contact interaction with the ground exerts a force that is always perpendicular to the boulder's motion, so it has no energy implications for the boulder (and none for the earth either, because the earth is extremely massive compared even to a gigantic boulder). The friction part of the interaction that keeps the boulder rolling without slipping affects the boulder's rotational energy U^{rot}, so we will keep track of that. We will ignore any other kinds of friction, such as the boulder rubbing against the tunnel walls (probably a bad assumption, but what else can we do?). We will also ignore any drag with the air (a much better approximation). Let the rock's moment of inertia be $I = \alpha MR^2$, where $\alpha = 2/5$, if we assume (quite reasonably) that the rock is a uniform solid sphere. Finally, let's assume that the rock rolls without slipping from rest at $z = z_i$: this will mean that $|\vec{\omega}_f| = |\vec{v}_f|/R$. We will also ignore the earth's kinetic energy both initially and finally. Therefore, the conservation of energy master equation becomes

$$\tfrac{1}{2}M\overbrace{|\vec{v}_i|^2}^{0} + \tfrac{1}{2}I\overbrace{|\vec{\omega}_i|^2}^{0} + M|\vec{g}|z_i = \tfrac{1}{2}M|\vec{v}_f|^2 + \tfrac{1}{2}I|\vec{\omega}_f|^2 + M|\vec{g}|\overbrace{z_f}^{0} \tag{1}$$

Substituting $I = \alpha MR^2$ and $|\vec{\omega}_f| = |\vec{v}_f|/R$ into this expression yields

$$M|\vec{g}|z_i = \tfrac{1}{2}M|\vec{v}_f|^2 + \tfrac{1}{2}\alpha M\cancel{R^2}\frac{|\vec{v}_f|^2}{\cancel{R^2}} = \tfrac{1}{2}(1+\alpha)M|\vec{v}_f|^2 = (\tfrac{1}{2}+\tfrac{2}{10})M|\vec{v}_f|^2 = \tfrac{7}{10}M|\vec{v}_f|^2$$

$$\Rightarrow \quad z_i = \frac{7|\vec{v}_f|^2}{10|\vec{g}|} = \frac{7(15\ \cancel{\text{mi/h}})^2}{10(9.8\ \text{m}/\cancel{\text{s}}^2)}\left(\frac{1\ \text{m}/\cancel{\text{s}}}{2.24\ \cancel{\text{mi/h}}}\right)^2 = 3.2\ \text{m} \tag{2}$$

where I found the conversion factor from mi/h to m/s in the inside front cover of the book. With this conversion factor the units work out nicely and the result (about 10 ft) is plausible. Note how the boulder's unknown mass and radius cancelled out very nicely!

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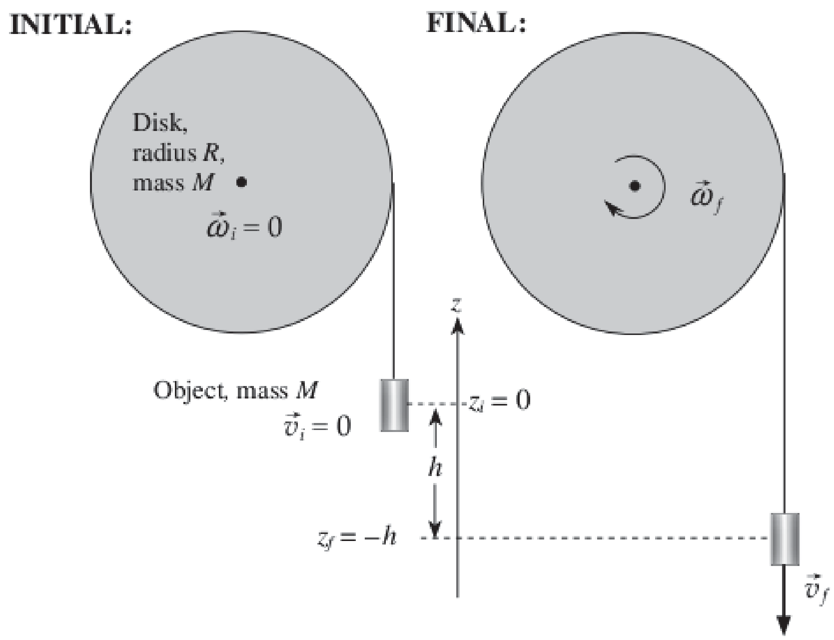
C11M.7 Let us orient our reference frame so that the z axis is vertically upward and $z = 0$ corresponds to the position of the small object's center of mass as it begins to fall. Initial and final sketches then look as shown:



Known:
$\vec{v}_i = 0$
$z_i = 0$, $z_f = -h$
(I will treat M, R, m, $|\vec{g}|$ and h as knowns)

Unknown:
$|\vec{v}_f| = ?$

Internal Interactions:
- Gravitatational: $V_g = m|\vec{g}|z$ with $V_g = 0$ at $z = z_i = 0$
- Contact interaction with thread: Keep track of rotational energy
- Air and thread friction: ignore.

The system here consists of the earth, the disk, the object, and the thread (the threads mediates a contact interaction between the disk and the object). This system floats in space, so it is isolated and its energy is conserved. The big disk interacts via a gravitational and contact interaction with the earth, but its center of mass does not move, so the only way that its energy changes is through changes in its rotational rate. If we ignore friction of all types the the system's total internal energy U (other than the disk's rotational energy) will not change. We will also assume that the thread is essentially massless (so that its kinetic energy and the potential energy of its gravitational interaction with the earth is always negligible). Since the object remains close to the earth's surface we can use the near-earth potential energy formula $V_g = m|\vec{g}|z$. The moment of inertia of a disk is $I = \frac{1}{2}MR^2$. The change in the earth's kinetic energy K_e is negligible because of its great mass. The conservation of energy master equation therefore becomes:

$$\tfrac{1}{2}m|\overbrace{\vec{v}_i}^{0}|^2 + \tfrac{1}{2}I|\overbrace{\vec{\omega}_i}^{0}|^2 + \overbrace{K_e}^{0} + \cancel{U} + m|\vec{g}|\overbrace{z_i}^{0} = \tfrac{1}{2}m|\vec{v}_f|^2 + \tfrac{1}{2}I|\vec{\omega}_f|^2 + \overbrace{K_e}^{\approx 0} + \cancel{U} + m|\vec{g}|\overbrace{z_f}^{-h} \qquad (1)$$

We want to solve for $|\vec{v}_f|$ in terms of $|\vec{g}|$, h, m, M and R, so we need to eliminate $|\vec{\omega}_f|$. Note that if the object goes down a distance ds in a tiny time interval dt, the disk has to turn at such rate such that its outer rim rotates through a distant ds also. Thus the disk will have to rotate through an angle $d\theta$ *(in radians) such that* $ds = R|d\theta|$. *This is turn means that the disk's rotational speed* $|\vec{\omega}|$ *is linked to the object's downward speed* $|\vec{v}|$ *as follows:*

$$|\vec{v}| = \left|\frac{ds}{dt}\right| = \frac{R|d\theta|}{dt} = R\left|\frac{d\theta}{dt}\right| = R|\vec{\omega}| \;\Rightarrow\; |\vec{\omega}| = \frac{|\vec{v}|}{R} \qquad (2)$$

Substituting this information into equation 1 will eliminate $|\vec{\omega}_f|$:

$$0 = \tfrac{1}{2}m|\vec{v}_f|^2 + \tfrac{1}{4}MR^2\left(\frac{|\vec{v}_f|}{R}\right)^2 - m|\vec{g}|h \;\Rightarrow\; \left(1 + \frac{M}{2m}\right)|\vec{v}_f|^2 = +2|\vec{g}|h \;\Rightarrow\; |\vec{v}_f| = \sqrt{\frac{2|\vec{g}|h}{\left(1 + \dfrac{M}{2m}\right)}} \qquad (3)$$

This gives the spool's final speed in terms of of $|\vec{g}|$, h, m and M, which is what we wanted. (Note that the disk's radius turns out to be irrelevant!) But is this answer plausible? First of all note that the quantity in the denominator is unitless and always positive. Since the top is always positive as well, we will always have a real result, which is good. The units of $|\vec{g}|h$ are (m/s^2)(m)= m^2/s^2, so $|\vec{v}_f|$ will have the right units. Note also that $|\vec{v}_f|$ increases with increasing h and decreases with increasing disk's mass M as one would expect intuitively. On the other hand, if $m >> M$, then we can essentially ignore the disk, and $|\vec{v}_f|$ approaches $\sqrt{2|\vec{g}|h}$, which is what one would get if the little mass were freely falling. So this formula seems plausible in every way.