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C6B.2 Jupiter's radius is $r = 71{,}500$ km and its rate of rotation is $|\vec{\omega}| = 1$ revolution (2π rad) per 9.92 h. According to equation C6.6, the speed of a point on the Jupiter's equator is

$$|\vec{v}| = r|\vec{\omega}| = (71{,}500\ \cancel{\text{km}})\left(\frac{2\pi\ \text{rad}}{9.92\ \cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right) = 12{,}600\ \frac{\text{m}}{\text{s}}.$$

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C6B.4 If the Lazy Susan rotates once every 4 seconds, then its angular velocity is $|\vec{\omega}| = 2\pi$ rad/(4 s) = 1.57 rad/s. Its angular momentum is $\vec{L} = I|\vec{\omega}|$, where $I = \alpha MR^2$ is the disk's moment of inertia, $M = 5.0$ kg is its mass, $R = 0.5$ m is its radius, and (according to figure C6.7) $\alpha = \frac{1}{2}$ for a solid disk. Therefore, the disk's angular momentum has a magnitude of

$$|\vec{L}| = I|\vec{\omega}| = \alpha MR^2|\vec{\omega}| = \frac{1}{2}(5.0\text{ kg})(0.5\text{ m})^2\left(1.57\,\frac{\text{rad}}{\text{s}}\right) = 0.981\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

The common direction of $\vec{\omega}$ and $\vec{L} = I\vec{\omega}$ is (according to the right hand rule shown in figure C6.3) vertically downward if the disk rotates clockwise when viewed from above.

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C6B.6 The ball's angular momentum is $\vec{L} = I|\vec{\omega}|$, where $I = \alpha MR^2$ is the ball's moment of inertia, $M = 0.60$ kg is its mass, $R = 0.755$ m/2π is its radius, and (according to figure C6.7) $\alpha = 2/3$ for a hollow ball like a basketball. Therefore, the ball's initial angular momentum (when its angular speed is $|\vec{\omega}| = 4(2\pi \text{ rad})/\text{s} = 25.1 \text{ rad}/\text{s}$) has a magnitude of

$$|\vec{L}_0| = I|\vec{\omega}| = \alpha MR^2|\vec{\omega}| = \frac{2}{3}(0.60\,\text{kg})\left(\frac{0.755\,m}{2\pi}\right)^2\left(25.1\,\frac{\text{rad}}{\text{s}}\right) = 0.145\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}} \tag{1}$$

The torque is the rate at which an interaction contributes twirl to an object. If we assume that frictional torque is the only torque acting on the ball and that the torque is constant over the time $T = 5$ s that it takes the ball to rotate at half its original rate, then that torque is

$$\vec{\tau} = \frac{d\vec{L}}{dt} \approx \frac{L_f - \vec{L}_0}{T} = -\frac{\frac{1}{2}\vec{L}_0}{T} \quad \Rightarrow \quad |\vec{\tau}| = \frac{|\vec{L}_0|}{2T} = \frac{0.145\,\text{kg}\cdot\text{m}^2/\text{s}}{2(5.0\,\text{s})} = 0.0145\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \tag{2}$$

(The torque's direction is opposite to the ball's angular momentum direction.)

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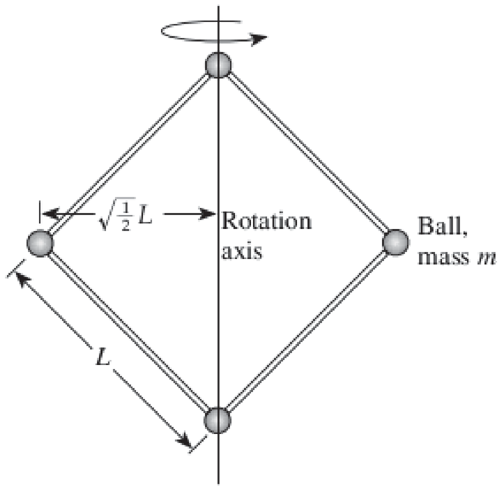
C6B.9 We can calculate the object's moment of inertia by summing the moments of inertia of its parts. Here is a cross-sectional view of the object and the rotation axis:



If we treat each of the balls as point particles, then the two balls that lie on the rotation axis have zero distance from that axis and thus contribute nothing to the object's moment of inertia. (Even if each such ball's radius is a finite value $r << L$, then its contribution $\sum_{\text{ball}} m_i r_i^2 << mL^2$ and so will be much smaller than what the other two balls contribute.) Therefore, this object's moment of inertia is approximately

$$I = m\left(\sqrt{\tfrac{1}{2}}L\right)^2 + m\left(\sqrt{\tfrac{1}{2}}L\right)^2 + \text{negligible} = \tfrac{1}{2}mL^2 + \tfrac{1}{2}mL^2 = mL^2 \quad (1)$$

In terms of the object's total mass $M = 4m$ and maximum radius $R = \sqrt{\tfrac{1}{2}}L$, we have

$$I = mL^2 = \tfrac{1}{4}(4m)2\left(\sqrt{\tfrac{1}{2}}L\right)^2 = \tfrac{1}{2}MR^2 \quad \Rightarrow \quad \alpha = \tfrac{1}{2} \quad (2)$$

Therefore α for this object is less than 1, as it must be.

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C6M.2 According to figure C6.7, a solid sphere of mass M and radius R has a moment of inertia I given by $I = \frac{2}{5}MR^2$. Such a sphere's angular momentum is $\vec{L} = I\vec{\omega}$, and (according to equation C6.6) the speed of a point on the sphere's equator is $|\vec{v}| = R|\vec{\omega}|$. Therefore, if the magnitude of the electron's spin angular momentum must be $|\vec{L}| = h/4\pi$, then

$$|\vec{v}| = R|\vec{\omega}| = R\frac{|\vec{L}|}{I} = R\frac{(h/4\pi)}{\frac{2}{5}MR^2} = \frac{5h}{8\pi MR} = \frac{5(6.63\times 10^{-34}\ \cancel{\text{J}}\cdot\text{s})}{8\pi(9.11\times 10^{-31}\ \cancel{\text{kg}})(10^{-18}\ \text{m})}\left(\frac{1\ \cancel{\text{kg}}\cdot\text{m}^2/\text{s}^2}{1\ \cancel{\text{J}}}\right)$$

$$= 1.45\times 10^{14}\ \frac{\text{s}\cdot\text{m}^2}{\text{m}\cdot\text{s}^2} = 1.45\times 10^{14}\ \frac{\text{m}}{\text{s}} \qquad (1)$$

Since the speed of light is about 3.0×10^8 m/s, this is almost 500,000 times the speed of light. As nothing can travel faster than light, this model can't be right.

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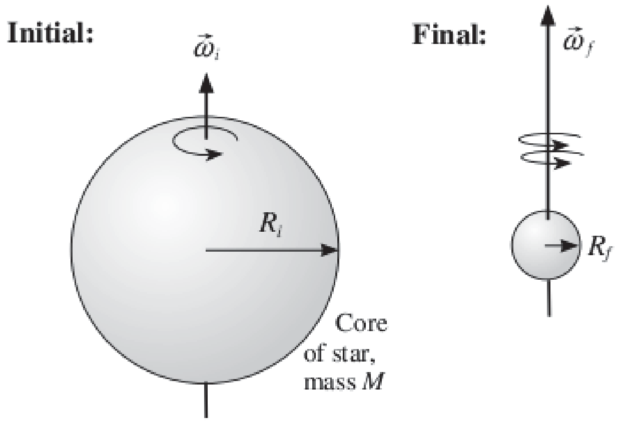
C6M.3 Initial and final diagrams for this problem look like this:



Known:
$R_i \approx 700{,}000$ km
$R_f = 12$ km
Initial rotation period $T_i = 2.4$ Ms

Unknown:
$|\vec{\omega}_i| = ?$ $|\vec{\omega}_f| = ?$ $M = ?$
Final rotation period $T_f = ?$

The system here is the star's core, which is isolated because it floats in space. The core's rotation rate must change because its angular momentum must be conserved:

$$I_i\vec{\omega}_i = I_f\vec{\omega}_f \tag{1}$$

where $I_i = \alpha_i M R_i^2$ and $I_f = \alpha_f M R_f^2$ are the star's initial and final moments of inertia respectively. The values of α_i and α_f in depend on the distribution of mass within the core (that is, the core's density profile). For example, if the density of the core were constant, then $\alpha = 2/5$. However, the core is more likely to be more dense in the center and less dense at its surface, yielding an α an unknown amount smaller than 2/5. The only way forward with this problem is to assume that the density profile remains the same as the core collapses: $\alpha_i = \alpha_f$. It still seems like we have too many unknowns and not enough equations, but perhaps some things will cancel out. If we take the magnitude of both sides of equation 1, divide both sides by I_f, and substitute in the expressions for the moments of inertia, we get:

$$\frac{|\vec{\omega}_f|}{|\vec{\omega}_i|} = \frac{I_i}{I_f} = \frac{\cancel{\alpha_i}\cancel{M}R_i^2}{\cancel{\alpha_f}\cancel{M}R_f^2} = \left(\frac{R_i}{R_f}\right)^2 \tag{4}$$

Note that the core mass M and the value of α cancel out, yielding the ratio of the final and initial angular velocities in terms of known quantities. But note that $|\vec{\omega}_i| = 2\pi/T_i$ and $|\vec{\omega}_f| = 2\pi/T_f$, so substituting these equations into the expression above yields

$$\frac{\cancel{2\pi}/T_f}{\cancel{2\pi}/T_i} = \left(\frac{R_i}{R_f}\right)^2 \Rightarrow \frac{T_i}{T_f} = \left(\frac{R_i}{R_f}\right)^2 \Rightarrow T_f = T_i\left(\frac{R_f}{R_i}\right)^2 = 2.6\times10^6\ \text{s}\left(\frac{12\ \cancel{\text{km}}}{700{,}000\ \cancel{\text{km}}}\right)^2 = 7.6\times10^{-4}\ \text{s} \approx 0.8\ \text{ms} \tag{5}$$

The units work out nicely. Note also that this period is consistent with the observational data reported in the problem statement.