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C7B.2 Since $|\vec{L}| = |\vec{r}||\vec{p}||\sin\theta|$, a particle's angular momentum around a point O will be zero if $\vec{r}$ is zero (the particle is at the origin), $\vec{p}$ is zero (the particle is not moving), or $|\sin\theta|$ is zero (the particle is either moving directly away from the origin or directly toward the origin). The particle's angular momentum is not zero if none of these three conditions apply.

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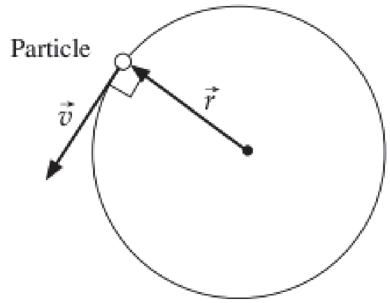
C7B.6 As the diagram below shows, the velocity of any particle moving in a circular path is perpendicular to its position vector from the circle's center.



This means that if the particle has mass m and speed $|\vec{v}|$, and the radius of its orbit is r, then the magnitude of its angular momentum must be $|\vec{L}| = |\vec{r} \times \vec{p}| = rm|\vec{v}|\sin 90° = rm|\vec{v}|$. In this case, the particle is an electron (whose mass is on the inside front cover) and we are given r and $|\vec{v}|$, so

$$|\vec{L}| = (9.11 \times 10^{-31}\text{ kg})(0.053\ \cancel{\text{nm}})\left(\frac{3.0 \times 10^{8}\text{ m/s}}{137}\right)\left(\frac{10^{-9}\text{ m}}{1\ \cancel{\text{nm}}}\right) = 1.06 \times 10^{-34}\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}} \tag{1}$$

The value of $h/2\pi$ is

$$\frac{h}{2\pi} = \frac{6.63 \times 10^{-34}\ \cancel{\text{J}}\cdot\text{s}}{2\pi}\left(\frac{1\text{ kg}\cdot\text{m}^2/\text{s}^2}{1\ \cancel{\text{J}}}\right) = 1.06 \times 10^{-34}\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}} \tag{2}$$

We see that the units and magnitude of these quantities are the same (indeed, quantum mechanics *requires* these quantities to be the same).

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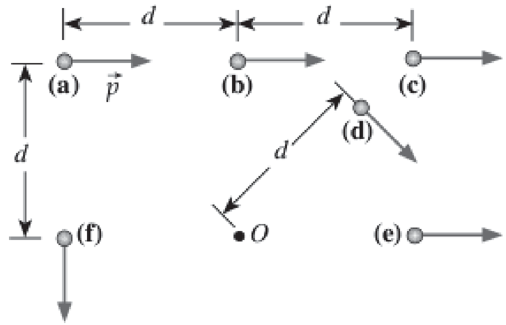
C7B.7 Here is the drawing in the problem, repeated here for convenience:



We saw in section C7.3 that we can express the magnitude of a particle's angular momentum about a given origin O as $|\vec{L}| = |\vec{r}_\perp||\vec{p}|$ where $|\vec{r}_\perp|$ is the length of the component vector of the particle's position relative to O that is perpendicular to the particle's momentum, which is also the distance of the closest point to O along the line along which $\vec{p}$ lies. In all of the cases (a) through (d) and also (f), this distance is simply d, so $|\vec{L}| = |\vec{r}_\perp||\vec{p}| = |\vec{p}|d = p_0 d$. In case (e), a line along which $\vec{p}$ lies goes through the origin, so $|\vec{r}_\perp| = 0$, meaning that $|\vec{L}| = 0$. (Another way to see the last result is to note that $|\vec{L}| = |\vec{r}||\vec{p}||\sin 0°| = 0$, because $\vec{r}$ and $\vec{p}$ are parallel on this case.) As for directions, you can see with your right hand that $\vec{L}$ points into the drawing for cases (a) through (d) but out of the drawing for case (f). Therefore, the values of L_z are:

(a) through (d) $L_z = -p_0 d$ **(e)** $L_z = 0$ **(f)** $L_z = +p_0 d$

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C7B.9 **(a)** The rotational angular momentum of a rotating symmetrical rigid object with moment of inertia I and angular velocity $\vec{\omega}$ is given by $\vec{L} = I\vec{\omega}$. Figure C6.7 tells us that the moment of inertia for a solid sphere is $I = \frac{2}{5}MR^2$, where M is the mass of the sphere and R is its radius. So the magnitude of the rotational angular momentum of this particular ball is

$$|\vec{L}| = I|\vec{\omega}| = \tfrac{2}{5}MR^2|\vec{\omega}| = \tfrac{2}{5}(7\text{ kg})(0.13\text{ m})^2\left(\frac{3\cdot 2\pi}{\text{s}}\right) = 0.89\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}} \tag{1}$$

If I curl my right fingers along with the rotation of the ball as it rolls toward me, my right thumb points to the right. So the ball's rotational angular momentum is $\vec{L} = 0.89$ kg·m^2/s to my right.

(b) Because the ball's center of mass is moving (approximately) directly toward my feet, its center-of-mass velocity $\vec{v}_{CM}$ is opposite to position vector $\vec{r}_{CM}$ from my feet to the ball's center of mass. So the angular momentum of the ball's center of mass about my feet is zero, because $|\vec{L}_{CM}| = |\vec{r}_{CM} \times M\vec{v}_{CM}| = |\vec{r}_{CM}|M|\vec{v}_{CM}||\sin 180°| = 0$.

This approximate answer is acceptable (and this approximation was assumed in arriving at the answer given in the back of the book). But how *good* the approximation is depends on exactly where the origin O represented by "my feet" is. If O is about 13 centimeters off the floor (roughly the same as the bowling ball's radius), then the answer above is correct. But if O is on the floor (at the *bottom* of the feet), then $\vec{r}_{CM}$ and $\vec{v}_{CM}$ are not exactly opposite, so the ball's center of mass angular momentum is not zero. If we treat the ball as a particle located at its center of mass, then according to the argument in section C7.3, the ball's center-of-mass angular momentum about the origin has a magnitude of $|\vec{L}_{CM}| = bM|\vec{v}_{CM}|$, where b is the perpendicular distance between the origin and the line representing the linear trajectory of the ball's center of mass. If the origin is on the floor, then $b \approx$ 13 cm (the radius of the bowling ball), so

$$|\vec{L}_{CM}| \approx bM|\vec{v}_{CM}| = (0.13\text{ m})(7\text{ kg})(2.5\text{ m/s}) \approx 0.23\text{ kg}\cdot\text{m}^2/\text{s} \tag{2}$$

This is significantly smaller than the result given in equation 1, but is arguably not negligible.

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C7B.11 The cord and your hand apply opposing torques to the wheel. If the wheel remains at rest, the two torques must cancel, so their magnitudes must be equal. The cord will exert a force $\vec{F}_c$ tangential to the axle and therefore perpendicular to the radius vector $\vec{r}_c$ from the axle's center to the place where the cord exerts the force. If we also assume that your hand exerts a force $\vec{F}_h$ that is also tangential to the wheel's rim, then this force will also be perpendicular to the radius vector $\vec{r}_h$ from the wheel's axle to its rim. Therefore, for the two torques to cancel, we must have

$$|\vec{r}_c \times \vec{F}_c| = |\vec{r}_h \times \vec{F}_h| \quad \Rightarrow \quad |\vec{r}_c||\vec{F}_c|\sin 90° = |\vec{r}_h||\vec{F}_h|\sin 90° \quad \Rightarrow \quad |\vec{F}_h| = \frac{|\vec{r}_c|}{|\vec{r}_h|}|\vec{F}_c| = \frac{1.5\ \cancel{\text{cm}}}{60\ \cancel{\text{cm}}} 50\ \text{N} = 1.25\ \text{N}$$

The units work out, and a smaller force on the rim seems plausible.

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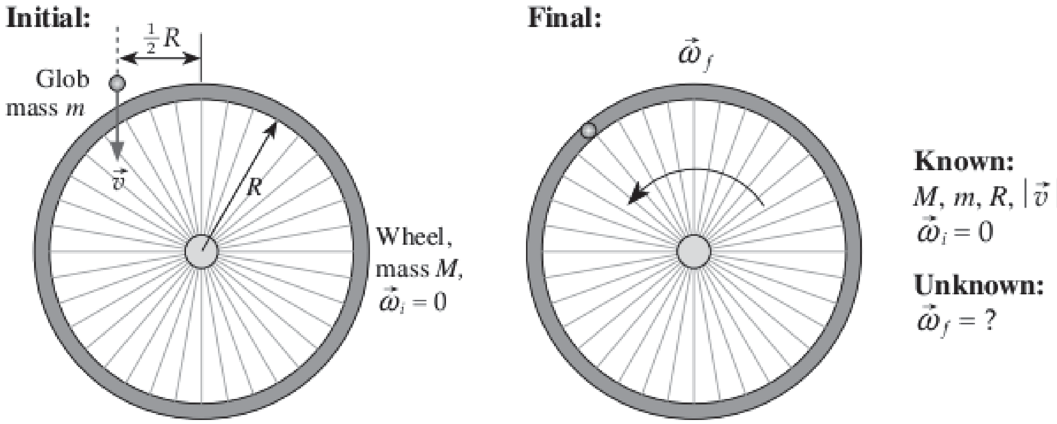
C7M.2 Initial and final diagrams of the situation look like this:



The system here is the glob and the wheel. The problem statement says that the wheel is "free to rotate" around a horizontal axis, so this implies that friction will be small. On the other hand, the force of gravity on the glob after it sticks to the wheel will exert an unbalanced torque on the wheel, causing its angular momentum to change. Therefore, we can apply conservation of angular momentum only if we treat the interaction between the glob and wheel as being a "collision" and focus on the system's state just before and just after the glob hits the wheel. We will take the wheel's center as the origin O around which we will calculate the system's center of mass.

Now, we saw in section C7.3 that if an object is moving in a straight line, its angular momentum has a magnitude equal to its momentum times the distance between the origin and the nearest point on the object's trajectory. Since the glob's would miss the wheel's center by $\frac{1}{2}R$ if it were to continue to drop, its angular momentum around the wheel's center must have a magnitude of

$$|\vec{L}_g| = \tfrac{1}{2}Rm|\vec{v}| \tag{1}$$

The wheel is initially at rest, so its initial angular momentum is zero. As angular momentum is conserved in this collision, the wheel's angular momentum immediately after the glob sticks must be $|\vec{L}_w| = |\vec{L}_g|$. Because $m << M$, we will assume that the glob does not significantly affect the wheel's moment of inertia, which was given to be $I = \frac{3}{4}MR^2$. Therefore, conservation of angular momentum in this case means that

$$|\vec{L}_w| = \tfrac{3}{4}MR^2|\vec{\omega}_f| = |\vec{L}_g| = \tfrac{1}{2}mR|\vec{v}| \quad \Rightarrow \quad |\vec{\omega}_f| = \frac{\frac{1}{2}mR|\vec{v}|}{\frac{3}{4}MR^2} = \frac{2}{3}\left(\frac{m}{M}\right)\left(\frac{|\vec{v}|}{R}\right) \tag{2}$$

Note that the ratio m/M is unitless, and $|\vec{v}|/R$ has units of inverse seconds, which are the correct units for angular velocity. It also makes sense that $|\vec{\omega}_f|$ increases as m increases relative to M and also as the glob's speed increases. So this expression looks credible.

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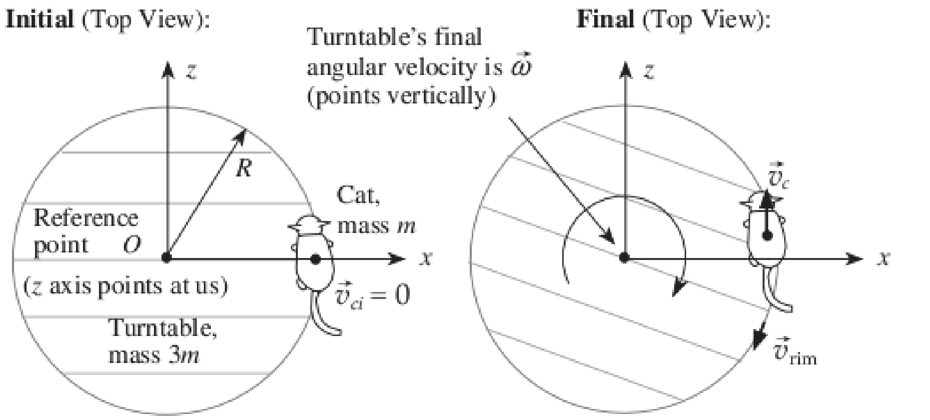
C7M.6 We will define the z axis of our reference frame to coincide with the turntable's axis of rotation and point vertically upward. Top-view initial and final diagrams of this situation then look like this:



Known:
$\vec{\omega}_i = 0$ (Turntable's initial angular velocity)
$\vec{v}_{ci} = 0$
$|\vec{v}_c| = 0.6\ \text{m/s}$
Turntable mass $M = 3m$

Unknown:
m, R, $|\vec{\omega}_f|$, $|\vec{v}_{\text{rim}}|$

The system here is the cat and the turntable. I will consider the system to be functionally isolated for rotations around the x and y axes (that is, rotations around those axes are forbidden, but nothing transfers net external angular momentum to the system with components along these axes either). Even if the turntable is not perfectly "free to rotate" around the vertical axis, we can take the interaction between the turntable and the cat to be like a collision: as long as we focus on the angular velocities just before and just after the cat starts walking, we can consider angular momentum around the vertical axis to be conserved. I will model the turntable by a disk, so that its moment of inertia is $I = \frac{1}{2}MR^2 = \frac{3}{2}mR^2$. I will also model the cat by a point particle (what else can we do?) and *assume* that its center of mass is roughly R from the turntable's center. *We* will take the reference point O to be the center of the turntable. Both the turntable and the cat are initially at rest, so conservation of angular momentum implies that $0 = \vec{L}_T + \vec{L}_c$, where $\vec{L}_T$ and $\vec{L}_c$ are the final angular momenta of the turntable and cat, respectively. The magnitude of the cat's angular momentum after it starts walking (if we model the cat as a particle) is (note that $\vec{v}_c$ is always perpendicular to its position vector $\vec{r}$ from the turntable's center, so $\sin\theta = 1$). Since $\vec{v}_c$ and $\vec{r}$ both lie in the xy plane, $\vec{L}_c = \vec{r} \times \vec{v}_c$ must point along the z axis: the right-hand rule indicates the *positive* z direction. The turntable's final angular momentum is given by $\vec{L}_T = I\vec{\omega} = \frac{3}{2}mR^2\vec{\omega}$, and $\vec{\omega}$ points either up or down the z axis. So conservation of angular momentum implies that

$$|\vec{L}_c| = |\vec{r} \times m\vec{v}_c| = |\vec{r}\,||m\vec{v}_c|\sin 90° = mR|\vec{v}_c|$$

$$0 = \vec{L}_T + \vec{L}_c = \begin{bmatrix} 0 \\ 0 \\ +mR|\vec{v}_c| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{3}{2}mR^2\omega_z \end{bmatrix} \quad (1)$$

Now, according to equation C6.6, we also have $|\vec{v}_{\text{rim}}| = R|\vec{\omega}| = R|\omega_z|$ here. This and the z component of equation 1 provide two equations in the four unknowns m, R, ω_z, and $|\vec{v}_{\text{rim}}|$, but the m and R will cancel out, so we can solve. Dividing the z component of equation 1 by m and R implies that

$$0 = |\vec{v}_c| + \tfrac{3}{2}R\omega_z \;\Rightarrow\; R\omega_z = -\tfrac{2}{3}|\vec{v}_c| \;\Rightarrow\; |\vec{v}_{\text{rim}}| = R|\omega_z| = \tfrac{2}{3}|\vec{v}_c| = \tfrac{2}{3}(0.6\ \text{m/s}) = 0.4\ \text{m/s} \quad (2)$$

Since this means that $\vec{\omega}$ points in the $-z$ direction, the right hand rule implies that the turntable rotates clockwise, as shown in the picture. (Since the cat is walking forward at 0.6 m/s relative to the ground, and the rim is moving backward at 0.4 m/s, the cat must be moving at 0.6 m/s + 0.4 m/s = 1.0 m/s forward relative to the rim). Note that the direction of the table's rotation *should* be clockwise to conserve $\vec{L}$, the units are right, and 1.0 m/s seems reasonable, so this all looks credible.