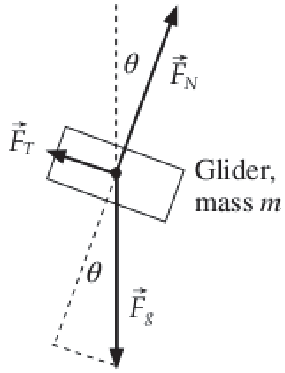
Unit: N
Chapter: N4
Problem Number: N4B.2

Return to User Page

N4B.2 A free-body diagram of the situation appears below.

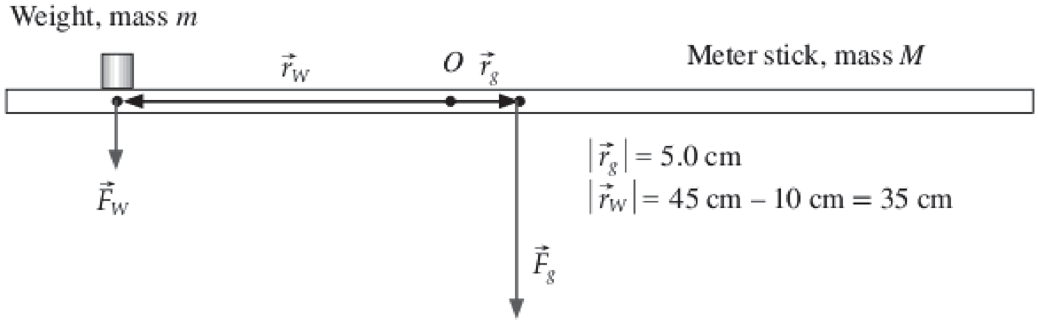


If the air track is tipped at an angle of θ from the vertical, then the normal force exerted by the air on the glider is tipped forward by the same angle. Since opposite angles are equal, this means that the angle the gravitational force makes with the direction perpendicular to the track is also θ. Since the string holds the glider at rest, the net force on the glider must be zero. The string force and the gravitational force are the only forces on the glider that have components parallel to the track, so the tension force must be equal in magnitude to the component of the gravitational force parallel to the track. By simple trigonometry, this implies that

$$\left|\vec{F}_T\right| = \left|\vec{F}_g\right| \sin\theta = m\left|\vec{g}\right| \sin\theta.$$

Unit: N Chapter: N4 Problem Number: N4B.3 Return to User Page

N4B.3 A torque diagram of the meter stick appears below.

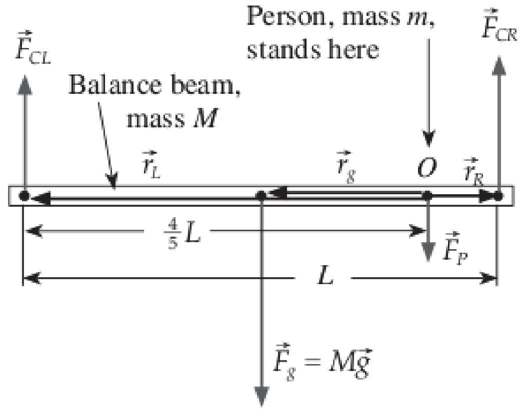


The problem is easiest if we put the origin at the balance point, as shown. If this meter stick is to balance at that point, the torque exerted by the gravitational force acting on the meter stick and the torque exerted by contact force $\vec{F}_W$ that the weight exerts meter stick must add up to zero, which will only be possible if the torque magnitudes are equal. Now, $\vec{F}_W$ is equal in magnitude to the contact force the meter stick exerts on the weight (by Newton's third law) which in turn is equal in magnitude to the gravitational force on the weight (since the weight does not accelerate vertically when everything is at rest). Therefore, $\vec{F}_W = m|\vec{g}|$. Note also that the position $\vec{r}_W$ of the point where $\vec{F}_W$ is applied is perpendicular to $\vec{F}_W$, and similarly, the position $\vec{r}_g$ of the stick's center of mass relative to O is perpendicular to the gravitational force acting on the stick $\vec{F}_g$. Therefore, if the torques contributed by these forces are to be equal we must have

$$|\vec{r}_W \times \vec{F}_W| = |\vec{r}_g \times \vec{F}_g| \quad \Rightarrow \quad |\vec{r}_W||\vec{F}_W|\sin 90° = |\vec{r}_g||\vec{F}_g|\sin 90° \quad \Rightarrow \quad |\vec{r}_W||\vec{F}_W| = |\vec{r}_g||\vec{F}_g|$$

$$\Rightarrow \quad |\vec{r}_W|m|\vec{g}| = |\vec{r}_g|M|\vec{g}| \quad \Rightarrow \quad M = \frac{|\vec{r}_W|m\cancel{|\vec{g}|}}{|\vec{r}_g|\cancel{|\vec{g}|}} = m\frac{|\vec{r}_W|}{|\vec{r}_g|} = (10\text{ g})\frac{35\ \cancel{\text{cm}}}{5\ \cancel{\text{cm}}} = 70\text{ g}.$$

N4B.5 A torque diagram of the balance beam appears below. Note that I am assuming that "the length of the beam" L technically refers to the distance between the points where the beam is supported.



Note that by Newton's third law, the contact force $\vec{F}_P$ that the person exerts on the beam is equal in magnitude to the upward force that the beam exerts on the person, which (by Newton's *second* law) must be equal in magnitude to the gravitational force $m\vec{g}$ exerted on the person (assuming the person its at rest).

(a) The torque exerted by the right support seeks to turn the beam counterclockwise about a horizontal axis through O that is perpendicular to the plane of the drawing. By the right-hand rule, this torque is therefore directed horizontally toward the viewer. Since the position vector $\vec{r}_R$ (relative to O) of the point where $\vec{F}_{CR}$ acts is perpendicular to that force and $|\vec{r}_R| = \frac{1}{5}L$, this torque has a magnitude of

$$|\vec{\tau}_R| = |\vec{r}_R \times \vec{F}_{CR}| = |\vec{r}_R||\vec{F}_{CR}|\sin 90° = \tfrac{1}{5}L|\vec{F}_{CR}| \tag{1}$$

(b) The torque exerted by the left support seeks to turn the beam clockwise about a horizontal axis through O that is perpendicular to the plane of the drawing. By the right-hand rule, this torque is therefore directed horizontally away from the viewer. Since the position vector $\vec{r}_L$ (relative to O) of the point where $\vec{F}_{CL}$ acts is perpendicular to that force and $|\vec{r}_L| = \frac{4}{5}L$, this torque has a magnitude of

$$|\vec{\tau}_L| = |\vec{r}_L \times \vec{F}_{CL}| = |\vec{r}_L||\vec{F}_{CL}|\sin 90° = \tfrac{4}{5}L|\vec{F}_{CL}| \tag{2}$$

(c) The torque exerted by beam's weight seeks to turn the beam counterclockwise about a horizontal axis through O that is perpendicular to the plane of the drawing. By the right-hand rule, this torque therefore points horizontally toward the viewer. Since the position vector $\vec{r}_g$ (relative to O) of the beam's center of mass (where the beam's weight $\vec{F}_g$ effectively acts) is perpendicular to $\vec{F}_g$ and $|\vec{r}_g| = \frac{1}{2}L - \frac{1}{5}L = \frac{3}{10}L$, this torque has a magnitude of

$$|\vec{\tau}_g| = |\vec{r}_g \times \vec{F}_g| = |\vec{r}_g||\vec{F}_g|\sin 90° = \tfrac{3}{10}ML|\vec{g}| \tag{3}$$

Extra: This problem only asks for the torques, not for the values of $|\vec{F}_{CR}|$ and $|\vec{F}_{CL}|$ in terms of M and m. But one can pretty easily go on from here to find those values. Requiring the clockwise torque to balance the counterclockwise torques requires that

$$|\vec{\tau}_L| = |\vec{\tau}_R| + |\vec{\tau}_g| \;\Rightarrow\; \tfrac{4}{5}L|\vec{F}_{CL}| = \tfrac{1}{5}L|\vec{F}_{CR}| + \tfrac{3}{10}LM|\vec{g}| \;\Rightarrow\; \tfrac{4}{5}|\vec{F}_{CL}| = \tfrac{1}{5}|\vec{F}_{CR}| + \tfrac{3}{10}M|\vec{g}| \tag{4}$$

Requiring that the net force on the beam be zero requires that

$$|\vec{F}_{CL}| + |\vec{F}_{CR}| = M|\vec{g}| + |\vec{F}_P| = M|\vec{g}| + m|\vec{g}| \tag{5}$$

Multiplying equation 5 by 4/5 and subtracting it from equation 4 yields

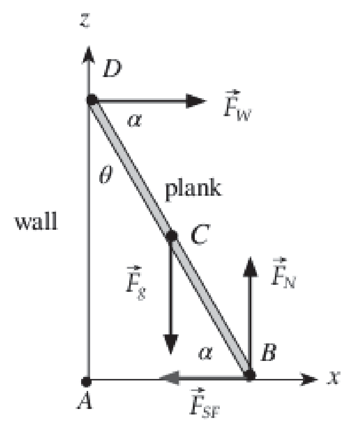
$$-\tfrac{4}{5}|\vec{F}_{CR}| = \tfrac{1}{5}|\vec{F}_{CR}| + \tfrac{3}{10}M|\vec{g}| - \tfrac{4}{5}M|\vec{g}| - \tfrac{4}{5}m|\vec{g}| \;\Rightarrow\; -|\vec{F}_{CR}| = -\tfrac{5}{10}M|\vec{g}| - \tfrac{4}{5}m|\vec{g}| \;\Rightarrow\; |\vec{F}_{CR}| = (\tfrac{1}{2}M + \tfrac{4}{5}m)|\vec{g}| \tag{6}$$

Substituting this back into equation 4 yields

$$|\vec{F}_{CL}| = M|\vec{g}| + m|\vec{g}| - \tfrac{1}{2}M|\vec{g}| - \tfrac{4}{5}m|\vec{g}| = (\tfrac{1}{2}M + \tfrac{1}{5}m)|\vec{g}| \tag{7}$$

Unit: N | Chapter: N4 | Problem Number: N4B.7 | Return to User Page

N4B.7 **(a)** The finished diagram looks like this (the added arrow is $\vec{F}_{SF}$):



(The static friction force points in the –x direction from point B.)

(b) The missing component is the leftward component of the static friction force:

$$0 = \begin{bmatrix} |\vec{F}_W| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N| \end{bmatrix} \quad \Rightarrow \quad \begin{matrix} |\vec{F}_W| = |\vec{F}_{SF}| \\ |\vec{F}_N| = m|\vec{g}| \end{matrix} \tag{1}$$

(c) Point B has two unknown forces acting on it ($\vec{F}_{SF}$ and $\vec{F}_N$). This makes it a convenient place to define the origin, because the torques exerted by these forces become automatically zero.

(d) The torques exerted by $\vec{F}_g$ and $\vec{F}_W$ about this origin are (by the right-hand rule) toward us and away from us in the diagram above, so they point in the –y and +y directions respectively.

(e) So the y component of $\vec{\tau}_{\text{net}} = 0$ requires that

$$0 = -\tfrac{1}{2}|\vec{F}_g|\sin\theta + |\vec{F}_W|\sin\alpha \tag{2}$$

Solving this for $|\vec{F}_W|$ yields

$$|\vec{F}_W| = \frac{\frac{1}{2}|\vec{F}_g|\sin\theta}{\sin\alpha} = \frac{m|\vec{g}|\sin\theta}{2\sin\alpha} \tag{3}$$

(f) Substituting $\sin\alpha = \cos\theta$ into the above yields

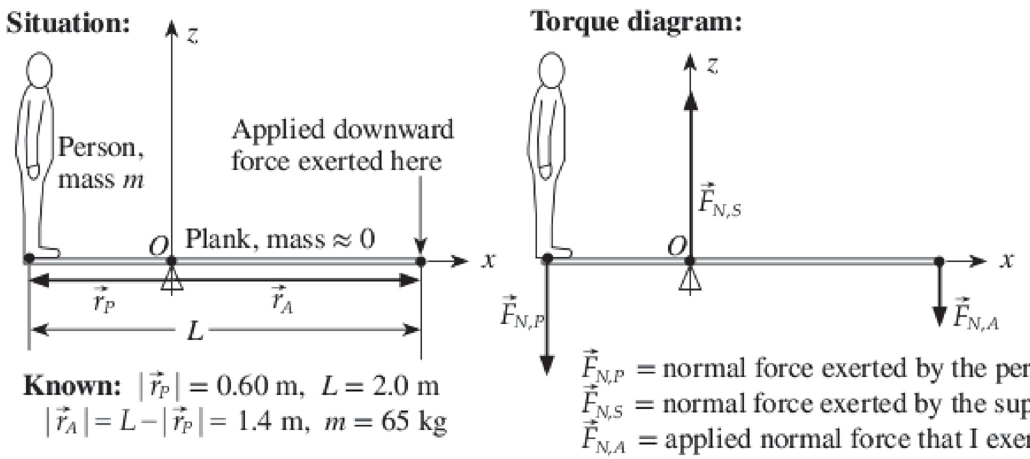
$$|\vec{F}_W| = \frac{m|\vec{g}|\sin\theta}{2\cos\theta} = \tfrac{1}{2}m|\vec{g}|\tan\theta \tag{4}$$

(g) Equation N4.21 tells us that $|\vec{F}_N| = |\vec{F}_g| = m|\vec{g}|$ and also that $|\vec{F}_{SF}| = |\vec{F}_W|$. Therefore, we must have

$$\mu_s|\vec{F}_N| = \mu_s m|\vec{g}| \geq |\vec{F}_{SF}| = |\vec{F}_W| = \tfrac{1}{2}m|\vec{g}|\tan\theta \quad \Rightarrow \quad \tan\theta = 2\mu_s \quad \Rightarrow \quad \theta = \tan^{-1}(2\mu_s) \tag{5}$$

Unit: N Chapter: N4 Problem Number: N4M.3 Return to User Page

N4M.3 A drawing of the situation and a torque diagram for the plank appear below:



$\vec{F}_{N,P}$ = normal force exerted by the person
$\vec{F}_{N,S}$ = normal force exerted by the support
$\vec{F}_{N,A}$ = applied normal force that I exert by hand

Note that the plank only touches the support, the person's feet, my hand, and the surrounding air. If we ignore any forces exerted by the air, the other forces exerted on the plank are all normal forces. I've set the origin at the support, so the torque about that point is zero. Since the plank's mass is not given, I am going to assume that it is negligible. I am going to assume that the push that I apply keeps everything balanced and at rest. I will get the most torque out of the push if I push on the plank at an angle perpendicular to the plank's surface (as shown). $\vec{F}_{N,P}$ is the third-law partner to the upward normal force that the plank exerts on the person, which in turn (assuming the person is at rest) must cancel the person's weight $m\vec{g}$ by Newton's second law. Therefore, we can conclude that $|\vec{F}_{N,P}| = m|\vec{g}|$. Newton's second law applied to the motionless plank then requires that $\vec{F}_{N,P} + \vec{F}_{N,S} + \vec{F}_{N,A} = 0$, the z-component of which requires that

$$-\left|\vec{F}_{N,P}\right| + \left|\vec{F}_{N,S}\right| - \left|\vec{F}_{N,A}\right| = 0 \quad \Rightarrow \quad \left|\vec{F}_{N,S}\right| - \left|\vec{F}_{N,A}\right| = \left|\vec{F}_{N,P}\right| = m|\vec{g}| \tag{1}$$

Now, $\vec{F}_{N,P}$ exerts a counterclockwise torque around the origin (the vector $\vec{\tau}_P$ points toward the viewer in the $-y$ direction), while $\vec{F}_{N,A}$ exerts a clockwise torque ($\vec{\tau}_A$ points in the $+y$ direction). The torque due to $\vec{F}_{N,S}$ is zero because it acts at the point that we are taking to be the origin O, so $\vec{r} = 0$. Therefore the requirement that the clockwise and counterclockwise torques on the plank cancel means that

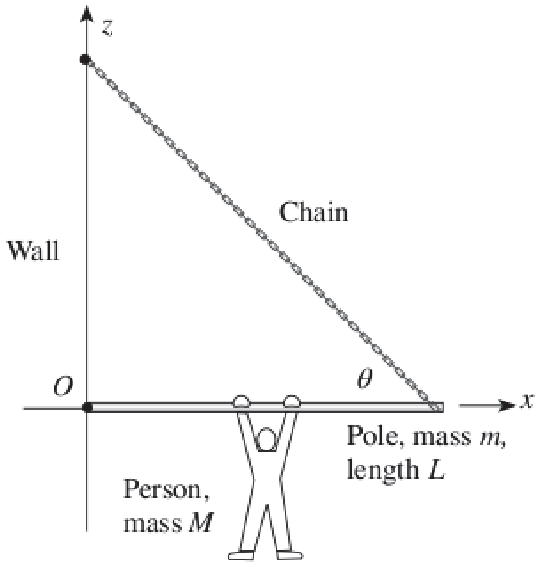
$$|\vec{\tau}_A| = |\vec{\tau}_P| \quad \Rightarrow \quad \left|\vec{r}_A \times \vec{F}_{N,A}\right| = \left|\vec{r}_P \times \vec{F}_{N,P}\right| \quad \Rightarrow \quad |\vec{r}_A|\left|\vec{F}_{N,A}\right|\sin 90° = |\vec{r}_P|\left|\vec{F}_{N,P}\right|\sin 90° = |\vec{r}_P|m|\vec{g}| \tag{2}$$

Equations 1 and 2 provide two equations in our remaining unknowns $\left|\vec{F}_{N,A}\right|$ and $\left|\vec{F}_{N,S}\right|$, so we are in a position to completely solve the problem. But here, we are only interested in $\left|\vec{F}_{N,A}\right|$, which we can get by solving equation 2 for that force magnitude:

$$\left|\vec{F}_{N,A}\right| = \frac{|\vec{r}_P|m|\vec{g}|}{|\vec{r}_A|} = \left(\frac{0.60\ \text{m}}{1.4\ \text{m}}\right)(65\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 270\ \text{N} \tag{3}$$

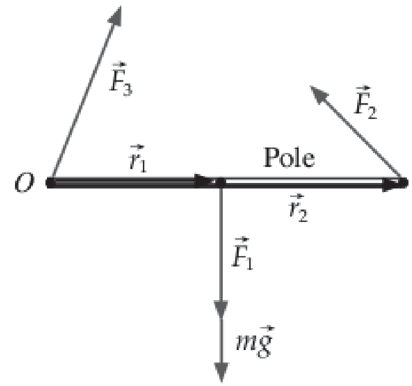
So I have to apply a force perpendicular to the plank having a magnitude of at least 270 N (≈ 61 lbs). The units and sign of this result are correct for a force magnitude, and the magnitude itself seems reasonable. It makes sense that the force that I would exert on this lever would be somewhat less than my friend's weight of (65 kg)(9.8 m/s^2) = 640 N. (For the sake of completeness, equation 1 then implies that $\left|\vec{F}_{N,S}\right| = 640\ \text{N} + 270\ \text{N} = 910\ \text{N}$.

N4M.4 A diagram of the situation and a torque diagram for the pole appear below.



$\vec{F}_1$ = contact force exerted by the person
$m\vec{g}$ = the pole's weight
$\vec{F}_2$ = tension force exerted by the chain
$\vec{F}_3$ = contact force exerted by the wall

Torque diagram:



Known: $|\vec{r}_2| = L = 1.8\text{ m},\ |\vec{r}_1| = \frac{1}{2}L,\ M = 65\text{ kg},\ m = 8\text{ kg},\ \theta = 45°$
Unknown: $|\vec{F}_1| = ?\ \ |\vec{F}_2| = ?\ \ \vec{F}_3 = ?$

The pole touches the wall, the person, the chain, and the air: we'll ignore any effects of the air. Since the person is at rest, the net force on the person must be zero, meaning that the person's weight $M\vec{g}$ must cancel the upward force that the pole exerts on the person. The latter force must have the same magnitude as the force the person exerts on the pole by Newton's third law. Therefore, $|\vec{F}_1| = M|\vec{g}\,|$. This eliminates one of the unknowns. Newton's second law applied to the pole tells us that

$$0 = \begin{bmatrix} 0 \\ 0 \\ -|\vec{F}_1| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}\,| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_2|\cos\theta \\ 0 \\ +|\vec{F}_2|\sin\theta \end{bmatrix} + \begin{bmatrix} F_{3x} \\ 0 \\ F_{3z} \end{bmatrix} \tag{1}$$

We know that $|\vec{F}_1| = M|\vec{g}\,|$, but this equation makes it clear that we still have three unknowns: $|\vec{F}_2|$, F_{3x}, and F_{3z}, and equation 1 provides only two component equations in these three unknowns. Fortunately, we also know that the pole is not rotating, so the net torque on the pole must be zero: this will provide the remaining equation that we need. Note that since $\vec{F}_3$ acts directly on our chosen origin point O, it exerts zero torque about that point. The forces $\vec{F}_1$ and $m\vec{g}$ exert clockwise torques about O, while force $\vec{F}_2$ exerts a counterclockwise torque. For the net torque to be zero, these torques must cancel, meaning that

$$|\vec{r}_1 \times (\vec{F}_1 + m\vec{g})| = |\vec{r}_2 \times \vec{F}_2| \quad \Rightarrow \quad |\vec{r}_1|(M+m)|\vec{g}\,|\sin 90° = |\vec{r}_2||\vec{F}_2|\sin\theta \tag{2}$$

Because we know everything in this expression except $|\vec{F}_2|$, it happens (though it was not perhaps initially obvious) that we need only this equation to solve for $|\vec{F}_2|$ (which is what the problem asks for):

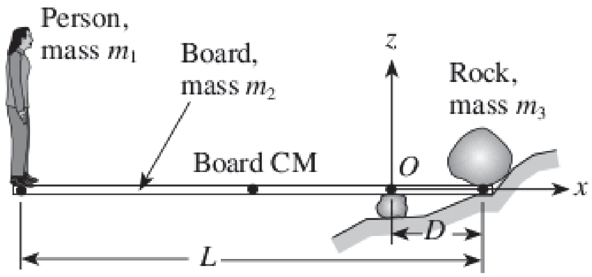
$$|\vec{F}_2| = \frac{|\vec{r}_1|}{|\vec{r}_2|}\frac{(M+m)|\vec{g}\,|}{\sin\theta} = \frac{1}{2}\,\frac{(65\,\cancel{\text{kg}} + 8\,\cancel{\text{kg}})(9.8\,\cancel{\text{m/s}^2})}{\sin 45°}\left(\frac{1\text{ N}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m/s}^2}}\right) = 510\text{ N} \tag{3}$$

(to two significant digits). Note that the units work out, and the magnitude is comparable to the person's weight (640 N), so this looks plausible. (Note also that one could put this result back into equation 1 to compute both F_{3x} and F_{3z} if the problem had asked us for $\vec{F}_3$.)

Unit: N Chapter: N4 Problem Number: N4M.6 Return to User Page

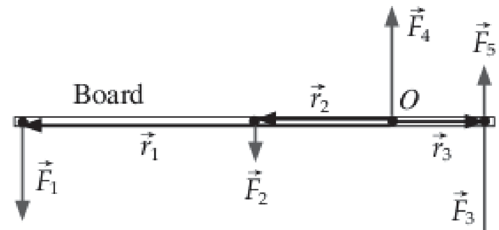
N4M.6 A drawing of the situation and a torque diagram of the board appear below.

Diagram of the situation:



Known:
$m_1 = 62$ kg, $m_2 = 22$ kg, $m_3 = 380$ kg
$L = 4.5$ m, $D = 0.85$ m $= |\vec{r}_3|$
$|\vec{r}_1| = L - D = 3.65$ m, $|\vec{r}_2| = \frac{1}{2}L - D = 1.40$ m
Unknown:
$|\vec{F}_1| = ?$ $|\vec{F}_2| = ?$ $|\vec{F}_3| = ?$ $|\vec{F}_4| = ?$ $|\vec{F}_5| = ?$

Torque diagram:



$\vec{F}_1$ = contact force from person
$\vec{F}_2$ = weight of board ($= m_2\vec{g}$)
$\vec{F}_3$ = contact force from large rock
$\vec{F}_4$ = contact force from pivot
$\vec{F}_5$ = contact force from ground

The board touches the person, the supporting pivot, the ground, the rock, and the air. I will assume that forces exerted by the board's contact with the air (for example, buoyancy) are negligible and that the board is uniform, so that its center of mass (where the board's weight $\vec{F}_2 = m_2\vec{g}$ effectively acts) is halfway along the board's length. By Newton's third law, force $\vec{F}_1$ has the same magnitude as the force that the board exerts on the person that holds that person at rest against gravity. By Newton's second law, that force must have the same magnitude as the person's weight so that the net force on the person is zero. This means that $|\vec{F}_1| = m_1|\vec{g}|$. A similar argument applies to force $\vec{F}_3$: it is the third-law partner to the upward force that supports the rock against gravity, and the latter must cancel the force of gravity by Newton's second law. Therefore, $|\vec{F}_3| = m_3|\vec{g}|$.

Intuitively, whether the board can or cannot support the person will depend on how the board is balanced (that is, what the torques on the board are). Therefore we will look at torque issues first. Note that $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_5$ exert counterclockwise torques on the board, while $\vec{F}_3$ alone exerts a clockwise torque. Zero net torque on the board requires that these opposing torques balance:

$$|\vec{r}_3 \times \vec{F}_3| = |\vec{r}_1 \times \vec{F}_1| + |\vec{r}_2 \times \vec{F}_2| + |\vec{r}_3 \times \vec{F}_5| \;\Rightarrow\; |\vec{r}_3||\vec{F}_3|\sin 90° = |\vec{r}_1||\vec{F}_1|\sin 90° + |\vec{r}_2||\vec{F}_2|\sin 90° + |\vec{r}_3||F_5|\sin 90° \quad (1)$$

since all the forces act perpendicular to the position vectors from the origin to the point where the force is applied. From the arguments above, we know the fixed magnitudes of $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. The remaining unknown forces are $\vec{F}_4$ and $\vec{F}_5$, forces that the pivot and the ground below the rock exert on the board respectively. Both of these forces can and will vary as the person moves toward the end of the board. But since $\vec{F}_4$ acts at our chosen origin O, it exerts zero torque on the board and therefore is irrelevant to our discussion of torques. Since $\vec{F}_2$ and $\vec{F}_3$ also have fixed locations, the torques they contribute will also be fixed. $\vec{F}_1$ has a fixed magnitude, but as the person moves further out the board, the counterclockwise torque it contributes will increase. This means that for the net torque to remain zero, $|\vec{F}_5|$ must decrease so that the counterclockwise torque that it contributes decreases as that from $\vec{F}_1$ increases. But $|\vec{F}_5|$ cannot decrease below zero: if this force were to go to zero, it would mean that the ground is no longer exerting a contact force on the board, which in turn would mean that the board must have left the ground. The force $\vec{F}_5$ also cannot switch direction to point downward, as that would mean that the ground would be exerting some kind of tension force on the board instead of a compression force. So if we substitute $\sin 90° = 1$ into equation 1 and solve for $|\vec{F}_5|$, we see that the condition for the board remaining balanced is

$$0 \le |\vec{F}_5| = |\vec{F}_3| - \frac{|\vec{r}_1|}{|\vec{r}_3|}|\vec{F}_1| - \frac{|\vec{r}_2|}{|\vec{r}_3|}|\vec{F}_2| = \left(m_3 - \frac{|\vec{r}_1|}{D}m_1 - \frac{\frac{1}{2}L - D}{D}m_2\right)|\vec{g}| \quad (2)$$

One could solve this for $|\vec{r}_1|$ to see how far out the person can go, but it is a bit easier to simply test whether going out to the end (where $|\vec{r}_1| = L - D = 3.65$ m is too far. In that case, we have

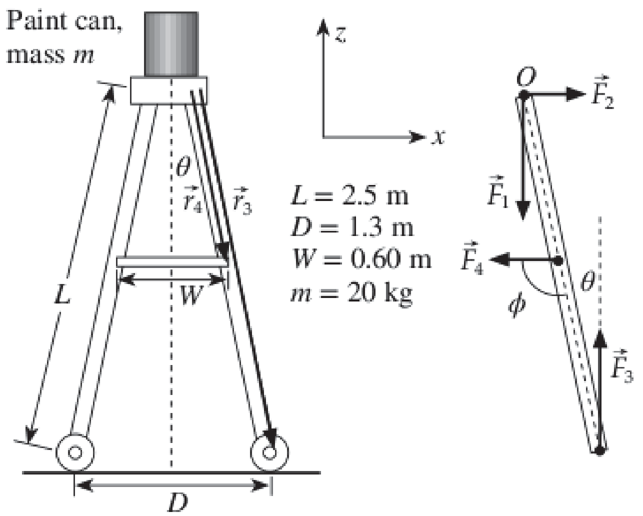
$$\frac{|\vec{F}_5|}{|\vec{g}|} = \left(380\text{ kg} - \frac{3.65\text{ m}}{0.85\text{ m}}62\text{ kg} - \frac{1.40\text{ m}}{0.85\text{ m}}22\text{ kg}\right) = +77.5\text{ kg} \tag{3}$$

This comes out positive, so the condition is satisfied: the person *can* safely go all the way to the end. (Note that the units work out, and the magnitude is plausible.)

Unit: N ▼ Chapter: N4 ▼ Problem Number: N4M.7 ▼ Return to User Page

N4M.7 A drawing of the situation and an enhanced free-body diagram (which shows where the forces act) of one leg of this ladder are shown below.

Enhanced free-body diagram
(for one ladder leg):



$\vec{F}_1$ = normal force exerted by the paint can
$\vec{F}_2$ = contact force exerted by the hinge
$\vec{F}_3$ = normal force exerted by the floor
$\vec{F}_4$ = tension force exerted by the tie
Origin O is at the hinge (at the leg's top)

We are told that the ladder is "much lighter" than the can, so I will assume that its mass is essentially zero. Assuming that the wheels at the bottom of each leg are frictionless, they will experience no static friction force from the floor, so the force $\vec{F}_3$ that the floor exerts on each leg will be entirely vertical. Note also that the third-law partner to $\vec{F}_1$ (and the third-law partner to the equivalent force on the top of the other leg) are what hold the can up, so if the can is to remain at rest, then we must have $|\vec{F}_1| = \frac{1}{2}m|\vec{g}|$.

So, if everything is to be stable, we must have $\vec{F}_{\text{net}} = 0$ and $\vec{\tau}_{\text{net}} = 0$. Note that if we define the origin to be at the leg's upper end, forces $\vec{F}_1$ and $\vec{F}_2$ will exert zero torque on the ladder leg, $\vec{F}_3$ will exert a counterclockwise torque (that is, a torque vector in the $+y$ direction, away from the viewer), and $\vec{F}_4$ will exert a clockwise torque (a torque vector in the $-y$ direction, toward the viewer). So the y component of the torque equation will be simply

$$0 = |\vec{r}_3 \times \vec{F}_3| - |\vec{r}_4 \times \vec{F}_4| = |\vec{r}_3||\vec{F}_3|\sin\theta - |\vec{r}_4||\vec{F}_4|\sin\phi \tag{1}$$

Note that $|\vec{r}_3| = L$, the length of the leg. We aren't given $|\vec{r}_4|$, θ, or ϕ directly, but we can puzzle them out from given data. Note that $|\vec{r}_4|$ is to W what L is to D, so $|\vec{r}_4|/W = L/D \Rightarrow |\vec{r}_4| = WL/D$. Note also that

$$\sin\theta = \frac{\frac{1}{2}D}{L} = \frac{0.65\ \cancel{\text{m}}}{2.5\ \cancel{\text{m}}} = 0.26 \ \Rightarrow\ \theta = \sin^{-1}(0.26) = 15° \ \Rightarrow\ \phi = 90° + \theta = 105° \tag{2}$$

This gives us all that we need to solve equation (1) for $|\vec{F}_4|$ if we know $|\vec{F}_3|$. We can get this from Newton's second law, which in this case reads

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -|\vec{F}_1| \end{bmatrix} + \begin{bmatrix} +|\vec{F}_2| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ +|\vec{F}_3| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_4| \\ 0 \\ 0 \end{bmatrix} \ \Rightarrow\ |\vec{F}_3| = |\vec{F}_1| = \tfrac{1}{2}m|\vec{g}| \tag{3}$$

Substituting all these results into equation 1 yields

$$|\vec{r}_4||\vec{F}_4|\sin\phi = |\vec{r}_3||\vec{F}_3|\sin\theta = L(\tfrac{1}{2}m|\vec{g}|)\left(\frac{\frac{1}{2}D}{L}\right) = \tfrac{1}{4}m|\vec{g}|D$$

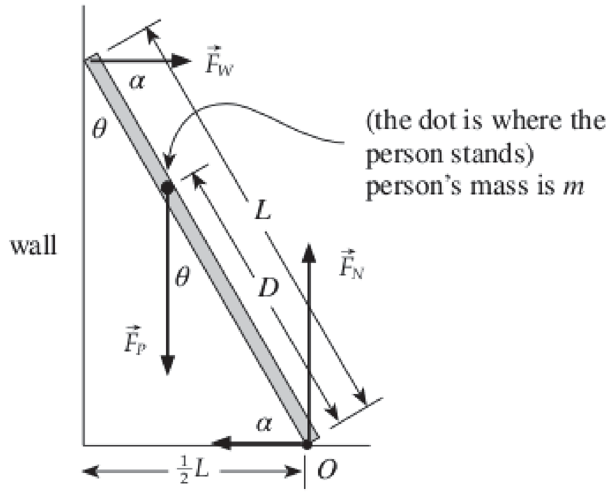
$$\Rightarrow |\vec{F}_4| = \frac{\frac{1}{4}m|\vec{g}|D}{|\vec{r}_4|\sin\phi} = \frac{\frac{1}{4}m|\vec{g}|D}{(WL/D)\sin\phi} = \frac{m|\vec{g}|D^2}{4WL\sin\phi} = \frac{(20\ \cancel{\text{kg}})(9.8\ \cancel{\text{m}}/\cancel{\text{s}}^2)(1.3\ \cancel{\text{m}})^2}{4(0.60\ \cancel{\text{m}})(2.5\ \cancel{\text{m}})\sin 105°}\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m}}/\cancel{\text{s}}^2}\right) = 57\ \text{N} \tag{4}$$

The units work out, and the magnitude is clearly positive and seems reasonable.

Unit: N
Chapter: N4
Problem Number: N4M.8

Return to User Page

N4M.8 The drawing for this situation looks like this:



The force $\vec{F}_P$ that the person exerts on the ladder must by Newton's third law be equal in magnitude to the force that the ladder exerts upward on the person. If the person is not moving, then this upward force must cancel the downward force of gravity on the person. This chain of reasoning therefore implies that $|\vec{F}_P| = m|\vec{g}|$, where m is the person's mass. If we ignore the ladder's mass, then Newton's second law for the ladder states that

$$0 = \begin{bmatrix} |\vec{F}_W| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N| \end{bmatrix} \tag{1}$$

This implies that

$$|\vec{F}_W| = |\vec{F}_{SF}| \quad \text{and} \quad |\vec{F}_N| = m|\vec{g}| \tag{2}$$

Choose the origin O marked above is convenient, because two unknown forces act on this point, so setting the origin so these forces exert zero torque makes things easier. The torques exerted by $\vec{F}_P = m\vec{g}$ and the wall force $\vec{F}_W$ point (by the right hand rule for the cross products) toward us and away from us, respectively, meaning that they point in the $-y$ and $+y$ directions, respectively. The y component of the torque equation with our choice of origin therefore implies that

$$0 = -m|\vec{g}|D\sin\theta + |\vec{F}_W|L\sin\alpha \quad \Rightarrow \quad |\vec{F}_W| = \frac{m|\vec{g}|D\sin\theta}{L\sin\alpha} \tag{3}$$

But note that

$$\sin\theta = \frac{\frac{1}{2}L}{L} = \frac{1}{2}, \quad \sin\alpha = \frac{\sqrt{L^2 - (\frac{1}{2}L)^2}}{L} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \tag{4}$$

Therefore, equation 3 becomes

$$|\vec{F}_W| = \frac{m|\vec{g}|D(1/2)}{L(\sqrt{3}/2)} = \frac{m|\vec{g}|D}{\sqrt{3}L} \quad \Rightarrow \quad D = \frac{\sqrt{3}|\vec{F}_W|L}{m|\vec{g}|} \tag{5}$$

But we know from equation 2 and the problem statement that $|\vec{F}_W| = |\vec{F}_{SF}| \le \mu_s|\vec{F}_N| = \mu_s m|\vec{g}|$, so the final expression for D is

$$D \le \frac{\sqrt{3}L\mu_s \cancel{m|\vec{g}|}}{\cancel{m|\vec{g}|}} = \sqrt{3}\mu_s L \tag{6}$$

(b) If $L = 5$ m and $\mu_s = 0.4$, then the maximum safe distance up the ladder that a person can stand is

$$D = \sqrt{3}\,(5\text{ m})(0.4) = 3.46\text{ m} \tag{7}$$

This seems credible.