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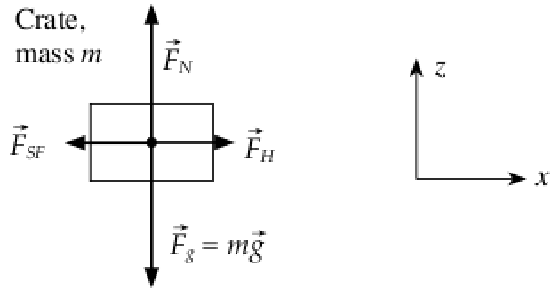
N5B.1 A free-body diagram of the crate appears below. $\vec{F}_H$ is the forward contact (normal) force exerted by the hand



Just before the crate starts moving, it is at rest, so we have $\vec{F}_{\text{net}} = 0$, or in column-vector form:

$$0 = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N| \end{bmatrix} + \begin{bmatrix} |\vec{F}_H| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Solving the x and z components of this equation for the unknown force magnitudes $|\vec{F}_{SF}|$ and $|\vec{F}_N|$ yields

$$|\vec{F}_{SF}| = |\vec{F}_H| \quad \text{and} \quad |\vec{F}_N| = m|\vec{g}| \tag{2}$$

The magnitude of the static friction force at it maximum value is $|\vec{F}_{SF,\,\max}| = \mu_s|\vec{F}_N| = \mu_s m|\vec{g}|$, so the coefficient of static friction must be

$$\mu_s = \frac{|\vec{F}_{SF,\max}|}{m|\vec{g}|} = \frac{|\vec{F}_{H,\max}|}{m|\vec{g}|} = \frac{200\ \cancel{\text{N}}}{(30\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})}\left(\frac{1\ \cancel{\text{kg}\cdot\text{m/s}^2}}{1\ \cancel{\text{N}}}\right) = 0.68 \tag{3}$$

(Note that coefficient comes out unitless, as it must.)

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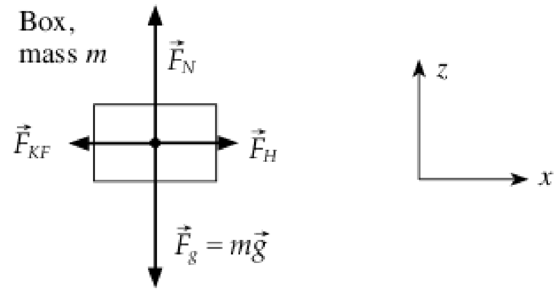
N5B.4 A free-body diagram of the box appears below. $\vec{F}_H$ is the forward contact (normal) force exerted by the hand



If the box is moving at a constant velocity (that is, with zero acceleration) we must have $\vec{F}_{\text{net}} = 0$, or in column-vector form:

$$0 = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N| \end{bmatrix} + \begin{bmatrix} |\vec{F}_H| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -|\vec{F}_{KF}| \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Solving the x and z components of this equation for the unknown force magnitudes $|\vec{F}_{KF}|$ and $|\vec{F}_N|$ yields

$$|\vec{F}_{KF}| = |\vec{F}_H| \quad \text{and} \quad |\vec{F}_N| = m|\vec{g}| \tag{2}$$

The magnitude of the kinetic friction force is $|\vec{F}_{KF}| = \mu_k|\vec{F}_N| = \mu_k m|\vec{g}|$, so the force that the hand exerts must have a magnitude of

$$|\vec{F}_H| = |\vec{F}_{KF}| = \mu_k m|\vec{g}| = (0.35)(15\,\cancel{\text{kg}})(9.8\,\cancel{\text{m/s}^2})\left(\frac{1\,\text{N}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m/s}^2}}\right) = 51\,\text{N} \tag{3}$$

(Note that the units come out correctly for a force.)

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N5B.6 For identical cars moving in identical conditions, the only thing that causes the drag forces acting on these cars to be different will be their respective speeds. Let car 1 travel at speed $|\vec{v}_1| = 65$ mi/h and car 2 travel at $|\vec{v}_2| = 45$ mi/h. The ratio of the drag force magnitudes on the two cars will therefore be

$$\frac{|\vec{F}_{D1}|}{|\vec{F}_{D2}|} = \frac{\cancel{\frac{1}{2}C\rho A}|\vec{v}_1|^2}{\cancel{\frac{1}{2}C\rho A}|\vec{v}_2|^2} = \left(\frac{65\ \cancel{\text{mi/h}}}{45\ \cancel{\text{mi/h}}}\right)^2 = 2.1$$

(Note that we don't even have to convert the units of the speeds, because they cancel out whatever the units might be.)

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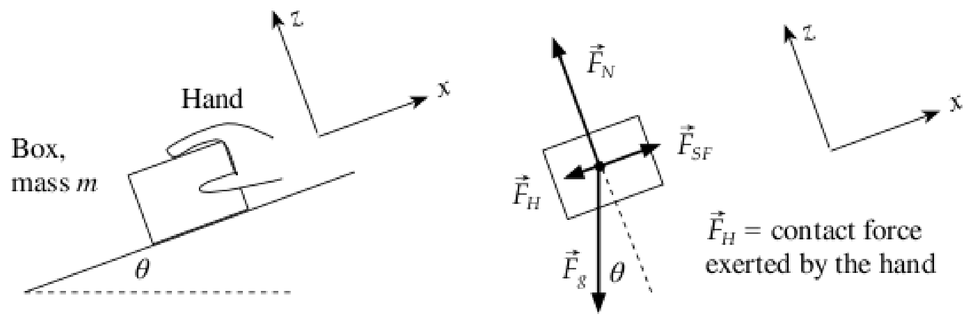
N5M.5 A diagram of the situation and a free-body diagram for the box appear below:



Known: $m = 12$ kg, $\theta = 15°$, $\mu_s = 0.30$, $|\vec{F}_g| = m|\vec{g}|$, $\vec{v} = 0$, $\vec{a} = 0$.
Unknown: $|\vec{F}_N| = ?$ $|\vec{F}_{SF}| = ?$ $|\vec{F}_H| = ?$

The box only touches the incline, the hand, and the air, and we will ignore the last. The contact interaction with the incline exerts both a normal force and a static friction force on the box, as shown in the diagram. Let's assume that the box is at rest and see what constraints this puts on the magnitude of the applied force from the hand $\vec{F}_H$. Newton's second law for the box when we are applying the hand force but the box is not yet moving tells us that

$$0 = \begin{bmatrix} -m|\vec{g}|\sin\theta \\ 0 \\ -m|\vec{g}|\cos\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ +|\vec{F}_N| \end{bmatrix} + \begin{bmatrix} +|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -|\vec{F}_H| \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Note that the magnitude of the static friction force is constrained to be such that $|\vec{F}_{SF}| \le |\vec{F}_{SF,\max}| = \mu_s|\vec{F}_N|$. This provides an equation that, in conjunction with the two nonzero rows of Newton's second law, give three equations for our three unknowns. The x and z components of Newton's second law imply that

$$|\vec{F}_H| = |\vec{F}_{SF}| - m|\vec{g}|\sin\theta \quad \text{and} \quad |\vec{F}_N| = m|\vec{g}|\cos\theta \tag{2}$$

respectively. Since $|\vec{F}_{SF,\max}| = \mu_s|\vec{F}_N|$, we have:

$$|\vec{F}_H| \le |\vec{F}_{SF,\max}| - m|\vec{g}|\sin\theta = \mu_s|\vec{F}_N| - m|\vec{g}|\sin\theta = \mu_s m|\vec{g}|\cos\theta - m|\vec{g}|\sin\theta = (\mu_s\cos\theta - \sin\theta)m|\vec{g}|$$

$$= (0.30\cdot\cos 15° - \sin 15°)(12\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m/s}^2}}\right) = 3.6\ \text{N} \tag{3}$$

It follows then that if we apply a force any greater than 3.6 N, the box will begin to slide. The magnitude (about 0.8 lb) doesn't seem unreasonable. Note also that the units and the sign are both correct for a force magnitude.

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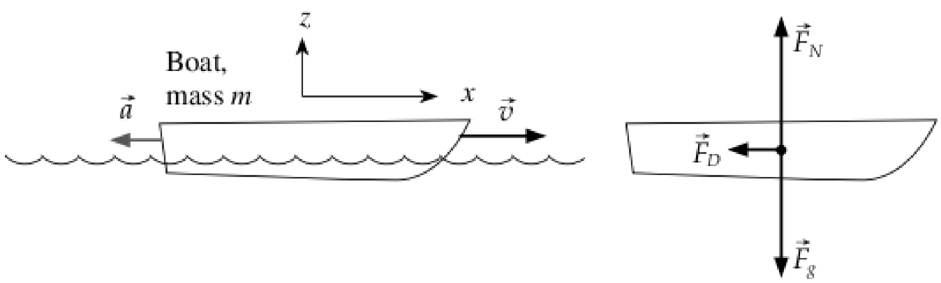
N5M.7 A drawing of the situation and a free-body diagram for the boat appear below. Note that we are defining the $+x$ direction to be the boat's direction of motion, so (because the boat is slowing down), its acceleration must be in the $-x$ direction, as shown.



Known:
$v_x(t) = |\vec{v}_0| e^{-qt}$
$q = 0.50\ \text{s}^{-1}$
$m = 200$ kg

Unknown:
$|\vec{a}| = ?$
Does $|\vec{F}_D| = b|\vec{v}|$?
with $b = ?$

Assume that the boat moves entirely in the $+x$ direction after the motor dies (this is not explicitly required by the problem statement). If $v_x(t) = |\vec{v}_0| e^{-qt}$ and $v_y(t) = v_z(t) = 0$ describes the boat's motion after the motor dies, then we have (by the definition of acceleration)

$$a_x(t) = \frac{dv_x}{dt} = \frac{d}{dt}[|\vec{v}_0| e^{-qt}] = -q(|\vec{v}_0| e^{-qt}) = -qv_x(t), \quad a_y(t) = \frac{dv_y}{dt} = 0, \quad a_z = \frac{dv_z}{dt} = 0 \tag{1}$$

Newton's second law in this situation therefore reads

$$m\vec{a} = \begin{bmatrix} -mq|\vec{v}_0| e^{-qt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -|\vec{F}_D| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -|\vec{F}_g| \end{bmatrix} \tag{2}$$

Since the boat's motion is entirely in the x direction, $v_x(t) = |\vec{v}_0| e^{-qt} = |\vec{v}(t)|$, so the x component of this equation does indeed seem to be consistent with the relation $|\vec{F}_D| = b|\vec{v}(t)|$ as long as we identify $b = mq$. Therefore, the value of b in this case is

$$b = mq = (200\ \text{kg})\left(\frac{0.50}{\text{s}}\right) = 100\ \frac{\text{kg}}{\text{s}} \tag{3}$$

Note that if b has units of kg/s, then $b|\vec{v}|$ has units of

$$\left(\frac{\cancel{\text{kg}}}{\cancel{\text{s}}}\right)\left(\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}} \cdot \cancel{\text{m}} / \cancel{\text{s}^2}}\right) = \text{N} \tag{4}$$

so the units make sense here. (Even so, there is something strange about this boat, because an object this large moving at reasonable speeds through water *should* experience a drag force of $|\vec{F}_D| \propto |\vec{v}|^2$, as discussed in the text. Perhaps the motor actually continued to push on the boat a bit after we thought it had died, or maybe the wake of another boat was pushing the boat forward initially to create the observed motion. Or perhaps the uncertainties were so large that we really could not distinguish between the hypothesis of exponentially-decaying velocity and whatever the velocity dependence would be in the $|\vec{F}_D| \propto |\vec{v}|^2$ case.)

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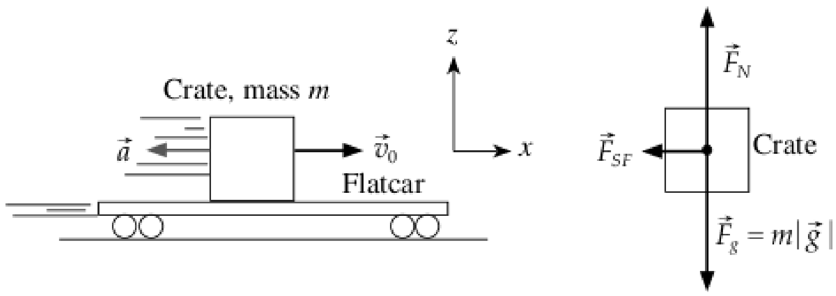
N5M.10 A drawing of the situation and a free-body diagram of the crate appear below. Note that the crate touches the flatcar: this contact interaction (in general) exerts both a normal force and a static friction force on the crate if the crate is not sliding. Note also that even though the crate (along with the flatcar) are moving to the right, if the crate is slowing down with the car, the crate's acceleration is to the left. If we choose the $+x$ direction to be the direction that the flatcar is moving, then the acceleration will be entirely in the $-x$ direction, as shown. Since drag would actually help slow the crate down, ignoring drag actually puts the most stringent limit on the safe braking distance, so this is what we will assume.



Known:
$v_{0x} = |\vec{v}_0| = 22$ m/s
$\mu_s = 0.5$

Unknown:
stopping distance $D = ?$
$|\vec{F}_N| = ?$
$|\vec{F}_{SF}| = ?$

Newton's second law then requires that

$$m\begin{bmatrix} -|\vec{a}| \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ +|\vec{F}_N| \end{bmatrix} + \begin{bmatrix} +|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

The x component of this equation implies that $|\vec{a}| = |\vec{F}_{SF}|/m$, which means that the maximum acceleration that the crate can tolerate without slipping is proportional to the maximum value that $|\vec{F}_{SF}|$ can have, which is $|\vec{F}_{SF,\max}| = \mu_s|\vec{F}_N|$. The z component of Newton's second law implies that $|\vec{F}_N| = m|\vec{g}|$. Therefore, we see that we must have

$$|\vec{a}| = \frac{|\vec{F}_{SF}|}{m} \le \frac{|\vec{F}_{SF,\max}|}{m} = \frac{\mu_s|\vec{F}_N|}{m} = \frac{\mu_s \cancel{m}|\vec{g}|}{\cancel{m}} = \mu_s|\vec{g}| \tag{2}$$

Now, how can we find the safe braking distance D? If we assume that the braking acceleration is in the $-x$ direction and has a constant magnitude, then equation N3.6 tells us that $x(t) = \frac{1}{2}a_x t^2 + v_{0x}t + x_0$, where $a_x = -|\vec{a}|$. Let T be the time required to come to rest. Then the braking distance $D = x(T) - x_0$, meaning that

$$D = x(T) - x_0 = -\tfrac{1}{2}|\vec{a}|T^2 + v_{0x}T = |\vec{v}_0|T - \tfrac{1}{2}|\vec{a}|T^2 \tag{3}$$

Since we know $|\vec{v}_0|$ and the maximum value of $|\vec{a}|$, we can use equation 3 to find the minimum value of D if we know T. Equation N3.8 also tells us that $v_x(t) = a_x t + v_{0x} = -|\vec{a}|t + |\vec{v}_0|$. We know that the car must finally be at rest, so we can solve $v_x(T) = 0$ for T:

$$0 = v_x(T) = -|\vec{a}|T + |\vec{v}_0| \quad \Rightarrow \quad |\vec{a}|T = |\vec{v}_0| \quad \Rightarrow \quad T = \frac{|\vec{v}_0|}{|\vec{a}|} \tag{4}$$

Substituting this into equation 3 yields

$$D = |\vec{v}_0|T - \tfrac{1}{2}|\vec{a}|T^2 = |\vec{v}_0|\frac{|\vec{v}_0|}{|\vec{a}|} - \tfrac{1}{2}|\vec{a}|\left(\frac{|\vec{v}_0|}{|\vec{a}|}\right)^2 = \frac{|\vec{v}_0|^2}{|\vec{a}|} - \frac{|\vec{v}_0|^2}{2|\vec{a}|} = \frac{|\vec{v}_0|^2}{2|\vec{a}|} \tag{5}$$

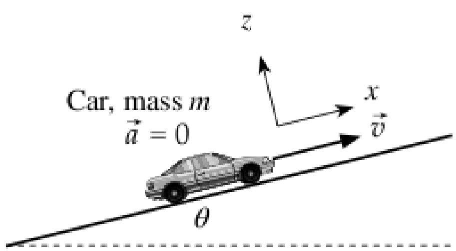
Since the maximum value of $|\vec{a}|$ is $\mu_s|\vec{g}|$, D must be larger than the minimum distance $D_{\min}$ that one would slide at that maximum acceleration:

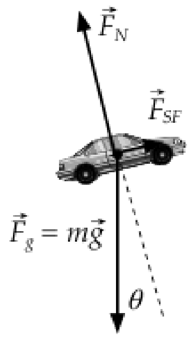
$$D \ge D_{\min} = \frac{|\vec{v}_0|^2}{2\mu_s|\vec{g}|} = \frac{(22\text{ m}/\cancel{s})^2}{2(0.5)(9.8\text{ m}/\cancel{s^2})} = 49\,\frac{\text{m}^2}{\text{m}} = 49\text{ m} \tag{6}$$

So we should allow at least 49 m of stopping distance to be assured that the crate will not slide. Since drag will actually help slow the crate down, this should provide a safe margin. Note that the units work out, the sign is positive as expected, and the magnitude is credible.

expected, and the magnitude is credible.

N5M.13 A drawing of the situation and a free-body diagram of the car appear below. Note that if we ignore interactions with the air, then the car touches only the road, which exerts a normal force and a static friction force on the car (the latter to keep it moving up the road, as opposed to sliding down it). Note also that the car is constrained to move along the road and therefore in the $+x$ direction as we have defined it in the diagrams. The drawings also assume that the car is traveling at a constant speed up the road.





Known:
$m = 1800$ kg, four-wheel drive
$\theta = 12°$, $\vec{a} = 0$
$v_{0x} = |\vec{v}| = 15$ m/s, $v_{0y} = v_{0z} = 0$

Unknown: $|\vec{F}_{SF}| = ?$ $|\vec{F}_N| = ?$ $\mu_s = ?$

In this case, Newton's second law implies that

$$0 = \begin{bmatrix} -m|\vec{g}|\sin\theta \\ 0 \\ -m|\vec{g}|\cos\theta \end{bmatrix} + \begin{bmatrix} +|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ +|\vec{F}_N| \end{bmatrix} \tag{1}$$

The x and z components provide two equations in two unknowns $|\vec{F}_{SF}|$ and $|\vec{F}_N|$, but we need a third equation to find the static friction coefficient μ_s. Because the car has four-wheel drive, all four wheels can exert a forward static friction force. This means that the *total* normal force exerted by the road on all four wheels counts when computing the total static friction force that the wheels can exert (if we had two-wheel drive, only the normal force on the two driving wheels would count). Therefore, $|\vec{F}_{SF}| \le \mu_s |\vec{F}_N|$, where $|\vec{F}_N|$ is the total normal force on the car. Solving this for μ_s and using the x and z components of Newton's second law yields

$$\mu_s \ge \frac{|\vec{F}_{SF}|}{|\vec{F}_N|} = \frac{\cancel{m|\vec{g}|}\sin\theta}{\cancel{m|\vec{g}|}\cos\theta} = \tan\theta = \tan 12° = 0.21 \tag{2}$$

Therefore, we can conclude that the coefficient of static friction is at least 0.21. Note that the answer is unitless and positive as it should be. The magnitude is in the low end of the range of values given in Table N5.1, but since this is only a lower limit on the coefficient, this is not a problem. Note also that the only information necessary to solve the problem was the angle of the incline, the fact that the car had four-wheel drive, and the assumption that the car travels at a constant rate; everything else is superfluous.