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Problem Number: N6B.2

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N6B.2 **(a)** These forces are not third-law partners: both forces act on the same object (the plow) and they do not reflect the same contact interaction.

(b) This is a third-law pair; the two forces reflect the same contact interaction between the sled and the dog and act on different objects.

(c) These forces are not partners; the forces act on the same object (the box) and do not reflect the same interaction (the gravitational interaction between the box and the earth and the contact interaction between the earth and the box).

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N6B.4 Equation N6.12 gives the magnitude of the system's acceleration

$$a_z = \frac{|\vec{F}_T^{A(X)}|}{M} - |\vec{g}| = \frac{54\ \cancel{\text{N}}}{5.1\ \cancel{\text{kg}}}\left(\frac{1\ \cancel{\text{kg}}\cdot\text{m/s}^2}{1\ \cancel{\text{N}}}\right) - 9.8\ \text{m/s}^2 = 0.79\ \text{m/s}^2 \quad (1)$$

Given this acceleration, the magnitude of the tension force on block B is

$$|\vec{F}_T^{B(S)}| = m(|\vec{g}| + a_z) = (2.0\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2} + 0.79\ \cancel{\text{m/s}^2})\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m/s}^2}}\right) = 21.2\ \text{N} \quad (2)$$

Equation N6.14 gives the difference in the magnitudes of the tension forces exerted by each end of the string in this case:

$$|\vec{F}_T^{A(S)}| - |\vec{F}_T^{B(S)}| = m_S(|\vec{g}| + a_z) = (0.1\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2} + 0.79\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m/s}^2}}\right) = 1.1\ \text{N} \quad (3)$$

The ideal string approximation is a poor one in this case; the difference in the tension force magnitudes exerted by the string's ends is more than 5% of the magnitudes of the forces themselves.

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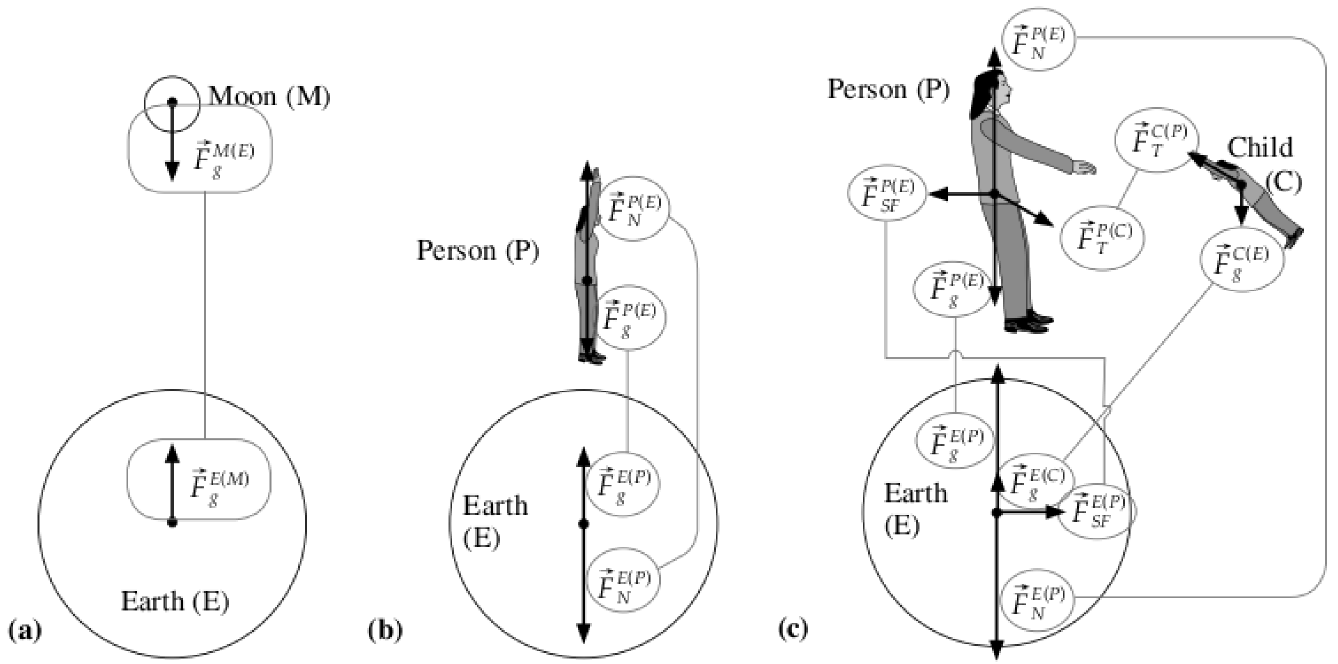
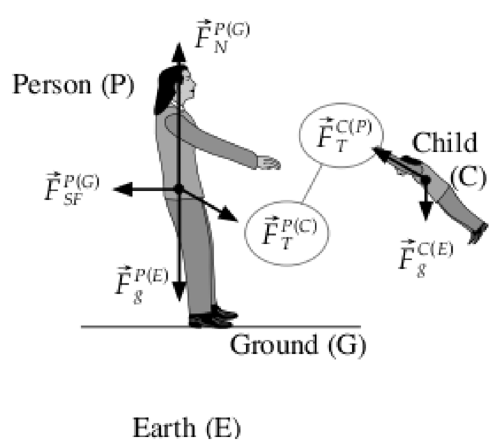
N6B.5 The free-body diagram sets for each of these three cases appear below. Please note that in parts (b) and (c), the objects are all touching each other, but have been drawn separated. Also please note the objects' sizes are not to scale (grossly so!) in any of the drawings.



Note that in the diagrams for part (b), the person is accelerating upward as he or she leaps off the ground, net force on that person must be upward. In the diagrams for part (c) the child is accelerating horizontally to the left at the instant shown, but the person is essentially at rest, so the force vectors on the person must add up to zero, which in this case means that the normal force is a bit larger than the gravitational force. We see that in this situation, the earth must experience a net force to the right, but the earth's acceleration will be negligible because the earth's mass is so large. (Note: One might quite reasonably consider the person and the child to be "the objects involved" in the situation in part (c). If so, then we would treat the forces exerted by the earth and ground to be external forces applied to these objects, and the diagram would look as shown below.)



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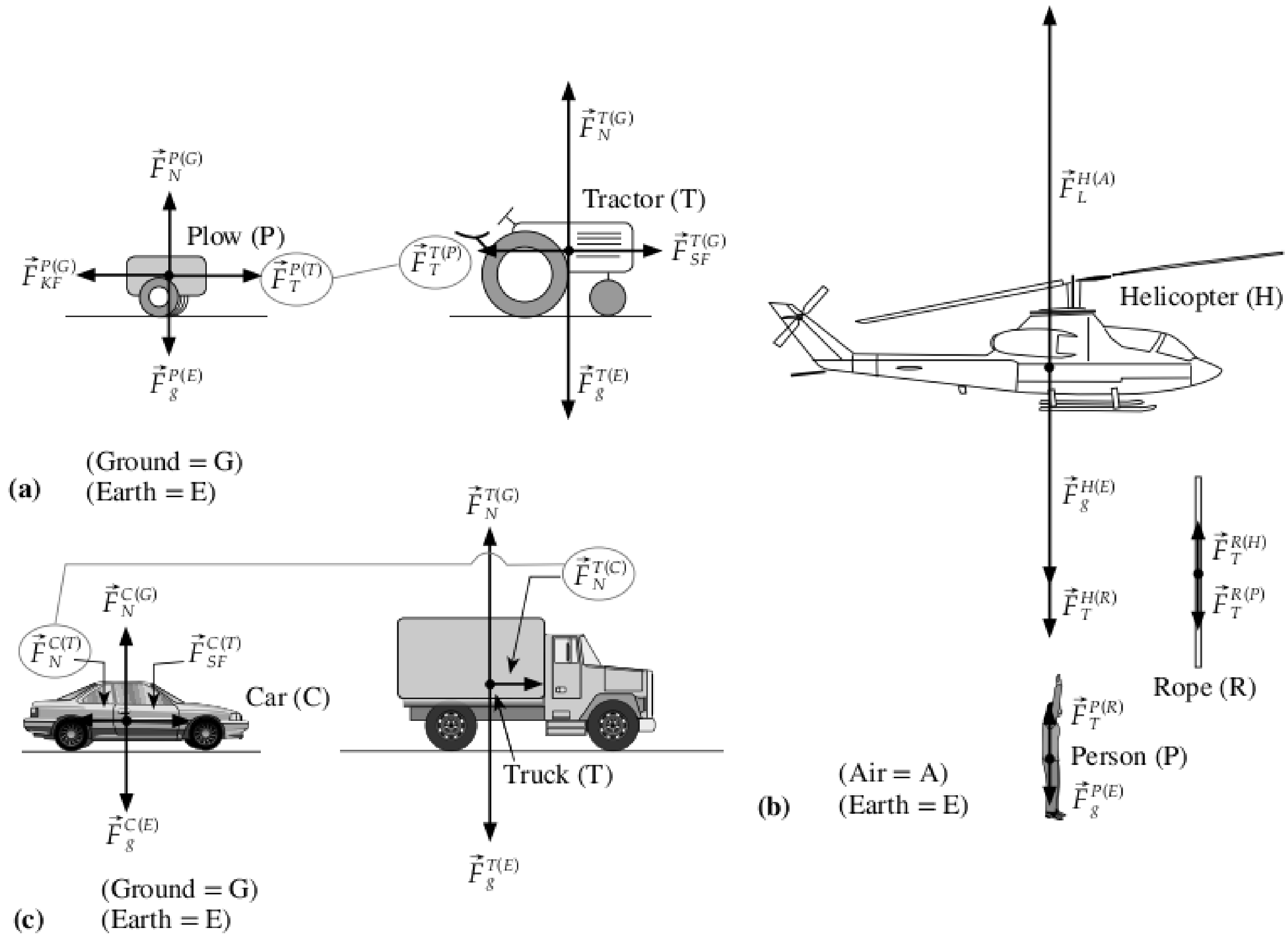
N6B.7 The free-body diagram sets for each of these three cases appear below. In each case, I have not drawn a diagram for the earth (even though one could certainly consider it to be one of the "objects involved" in the first and last cases), instead focusing on the specific diagrams requested in the problem statement. Also note that the objects are actually touching (or connected) in all three cases, even though we have separated the free-body diagrams.



In case (a), the objects are not accelerating, so the forces on each individual object must add to zero. Note that the forward friction force on the tractor is a static friction force (because the tractor wheels are not slipping), but the backward force on the plow is a kinetic friction force, because the plow is sliding through the earth. In case (b) the objects are accelerating upward, so the net force on all three objects must be upward. To provide the same vertical acceleration for all three objects, the difference in forces must be larger for the massive helicopter than for the lighter person, and the difference in forces on the rope will be negilible. In case (c), the objects are accelerating to the right, so the net force on each object must be to the right. Again, the difference in horizontal forces must be larger for the truck than for the lighter car. I have ignored rolling and drag friction forces in this case as likely being too small to really draw (particularly if the system's speed is small), but including them would also be acceptable.

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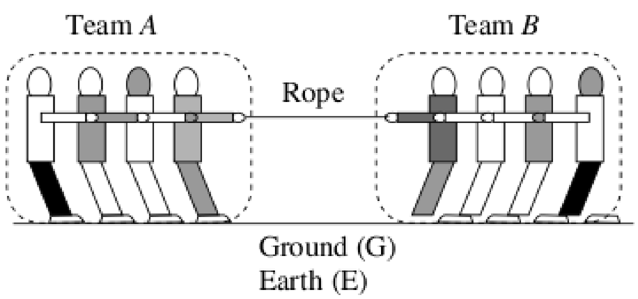
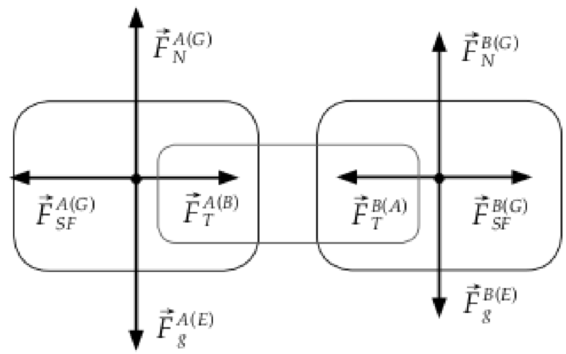
N6M.1 To see how one team can win, consider the drawings below (which assume the ideal string model for the interaction between the teams mediated by the connecting rope).

Situation diagram:

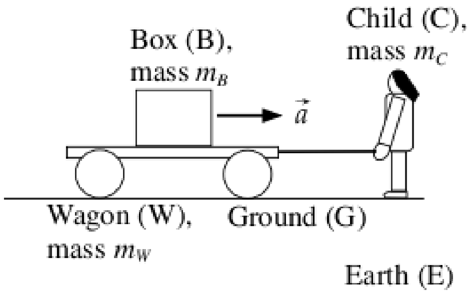


Free-body diagrams:

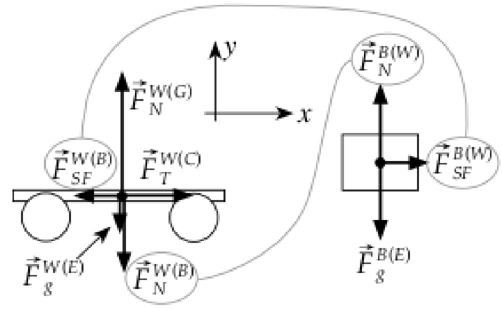


Since neither team moves vertically, the vertical forces on each individual team must cancel. In the ideal string model, we treat the rope as an interaction, so in this model, the two forces linked by the grey box are third-law partners and the teams really do exert forces of equal magnitude on each other. Even so, one team can still win if either $\vec{F}_{SF}^{A(G)}$ or $\vec{F}_{SF}^{B(G)}$ is larger than the corresponding static friction force acting on the opposing team (a team can use Newton's third law to compel the ground to exert a greater static friction force on the team by digging in its feet better and exerting more static friction force on the ground). A team might be able to achieve this by having a larger effective static friction coefficient μ_s and/or by simply being heavier (so that $\vec{F}_N$ is larger, meaning that $\vec{F}_{SF}$ can be larger even with the same μ_s). The point is that even though the force exerted by each team on the other are equal and opposite, the static friction force the *ground* exerts on each team is not necessarily the same, and this can lead to a net force on the entire system that accelerates it in one direction or the other. (The free-body diagrams above in fact show how the A team can win.)

N6M.4 A diagram of the situation and free-body diagrams for the box and wagon appear below:



Known:
$m_B = 15$ kg
$m_W = 12$ kg
$m_C = 32$ kg
$|\vec{F}_T^{W(C)}| = 65$ N



In order to draw the free-body diagrams, we must consider the implications of Newton's third law. The third-law pairs in this situation arise from the contact interaction of the box and the wagon. The wagon exerts an upward normal force on the box (supporting it): therefore by the third law, the box must exert a downward normal force on the wagon. Similarly, the wagon must exert a forward static friction force on the box to accelerate it forward along with the wagon, so by the third law, the box must exert a backward static friction force on the wagon. I have circled and connected these third-law partners in the free-body diagrams above. If we ignore drag on both the box and wagon and other forms of friction on the wagon, then Newton's second law for the wagon and the box implies that

$$m_W \begin{bmatrix} +|\vec{a}| \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m_W|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -|\vec{F}_N^{W(B)}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N^{W(G)}| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_{SF}^{W(B)}| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} |\vec{F}_T^{W(C)}| \\ 0 \\ 0 \end{bmatrix} \tag{1a}$$

$$m_B \begin{bmatrix} +|\vec{a}| \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m_B|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ +|\vec{F}_N^{B(W)}| \end{bmatrix} + \begin{bmatrix} |\vec{F}_{SF}^{B(W)}| \\ 0 \\ 0 \end{bmatrix} \tag{1b}$$

Note that if the box and wagon accelerate together, their acceleration $\vec{a}$ is the same. The x and z components of the two equations above plus the third-law relations $|\vec{F}_{SF}^{W(B)}| = |\vec{F}_{SF}^{B(W)}|$ and $|\vec{F}_N^{W(B)}| = |\vec{F}_N^{B(W)}|$ provide the six equations we need to solve for the six unknown quantities $|\vec{a}|, |\vec{F}_N^{W(B)}|, |\vec{F}_N^{B(W)}|, |\vec{F}_{SF}^{W(B)}|, |\vec{F}_{SF}^{B(W)}|$ and $|\vec{F}_N^{W(G)}|$, so we should be able to find everything, but we are actually not much interested in the vertical forces. The x components of equations $1a$ and $1b$ tell us that

$$m_W|\vec{a}| = |\vec{F}_T^{W(C)}| - |\vec{F}_{SF}^{W(B)}| \quad \text{and} \quad m_B|\vec{a}| = |\vec{F}_{SF}^{B(W)}| \tag{2}$$

Because $|\vec{F}_{SF}^{W(B)}| = |\vec{F}_{SF}^{B(W)}|$, if we add these equations, the static friction forces cancel out, leaving

$$m_W|\vec{a}| + m_B|\vec{a}| = |\vec{F}_T^{W(C)}| \quad \Rightarrow \quad |\vec{a}| = \frac{|\vec{F}_T^{W(C)}|}{m_W + m_B} = \frac{65\ \text{N}}{12\ \text{kg} + 15\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 2.4\ \frac{\text{m}}{\text{s}^2} \tag{3}$$

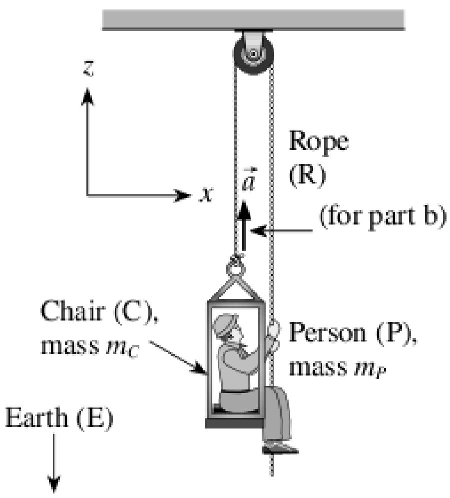
Substituting this into equation 2 yields

$$|\vec{F}_{SF}^{B(W)}| = m_B|\vec{a}| = (12\ \text{kg})(2.4\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 29\ \text{N} \tag{4}$$

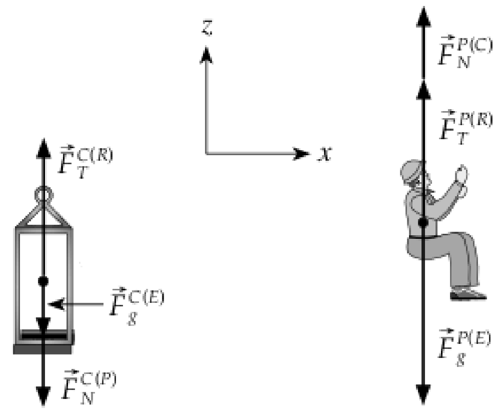
Note that the signs come out right for magnitudes and the units work out. The answer also seems plausible (a result larger than 65 N, for example, would be suspect).

N6M.10 A situation diagram and free-body diagrams for the person and chair appear below.

Situation diagram:



Free-body diagrams:



Known:
$M = m_P + m_C$
$a_z = |\vec{a}| = 0$ (part a)
$= 0.1|\vec{g}|$ (part b)

Unknown:
$m_P = ?$ $m_C = ?$
$|\vec{F}_T^{C(R)}|, |\vec{F}_T^{P(R)}| = ?$
$|\vec{F}_N^{C(P)}|, |\vec{F}_N^{P(C)}| = ?$

The chair and the person only touch each other, the rope, and the air. The free-body diagrams above are drawn assuming the interactions with the air (buoyancy, drag) are negligible (which should be an excellent approximation). Since the person is seated in the chair, both share a common acceleration. If we assume that the acceleration is entirely in the $+x$ direction, then Newton's second law for each object is as follows (note that I am going to solve the case of general acceleration and then set the acceleration to be zero to handle the constant velocity case):

$$\begin{bmatrix} 0 \\ 0 \\ m_C|\vec{a}| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m_C|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -|\vec{F}_N^{C(P)}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_T^{C(R)}| \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ m_P|\vec{a}| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m_P|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_N^{P(C)}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ |\vec{F}_T^{P(R)}| \end{bmatrix} \quad (1)$$

for the chair and person respectively. These equations only provide two non-trivial equations the six unknowns listed above, so we need more equations! Note, however, that $\vec{F}_N^{C(P)}$ and $\vec{F}_N^{P(C)}$ are third-law partners, so $|\vec{F}_N^{C(P)}| = |\vec{F}_N^{P(C)}|$. The ideal pulley and string approximation implies that $|\vec{F}_T^{C(R)}| = |\vec{F}_T^{P(R)}|$, and we also know that $M = m_P + m_C$, and we are "given" M. Now we have five equations in our six unknowns, but note that we are not really interested in m_P and m_C but only their sum M, so let's press ahead in the hope that we won't have to determine them separately. Indeed, simply adding the two bottom rows in equation (1) yields

$$(m_C + m_A)|\vec{a}| = -(m_C + m_A)|\vec{g}| - \cancel{|\vec{F}_N^{C(P)}|} + \cancel{|\vec{F}_N^{P(C)}|} + |\vec{F}_T^{C(R)}| + |\vec{F}_T^{P(R)}| \quad (2)$$

where the normal force magnitudes cancel by Newton's third law. Substituting $M = m_C + m_A$ into the above and noting that $|\vec{F}_T^{C(R)}| = |\vec{F}_T^{P(R)}|$ by the ideal string/pulley approximation, this becomes

$$M|\vec{a}| = -M|\vec{g}| + 2|\vec{F}_T^{P(R)}| \quad \Rightarrow \quad |\vec{F}_T^{P(R)}| = \tfrac{1}{2}M(|\vec{a}| + |\vec{g}|) \quad (3)$$

(a) So in the case where $|\vec{a}| = 0$, we have $|\vec{F}_T^{P(R)}| = \frac{1}{2}M|\vec{g}|$, meaning that the person must pull on the rope with a force equal in magnitude to half the combined weight of the person and chair.

(b) In the case where $|\vec{a}| = 0.1|\vec{g}|$, we have $|\vec{F}_T^{P(R)}| = \frac{1}{2}M(\frac{1}{10}|\vec{g}| + |\vec{g}|) = \frac{11}{20}M|\vec{g}| = 0.55\,M|\vec{g}|$, a bit more. This makes sense.

In both cases, the result has the same units as weight, which is correct for a force, and the answers seem plausible, if a bit surprising. Basically, we can understand the first case as two strands of rope supporting the weight of the person + chair, so each strand needs to exert only half that weight.

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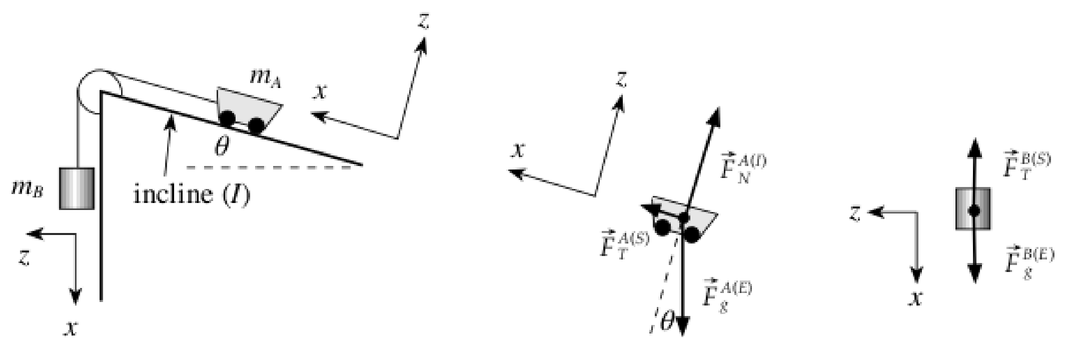
N6M.11 **(a)** Mass m_B is accelerating downward. But the only two forces acting on it are its weight and the tension force exerted by the rope. Since the net force on this object must be downward, the tension force exerted by the rope must be *less than* that object's weight.

(b) The diagram to the left shows suitable coordinate systems for this problem:



The basic point is to align the x-axes for each of the object's so that they acceleration in the $+x$ direction. This will make working the problem easier.

(c) Free-body diagrams for each of the objects appear above and to the right. Newton's second law in column-vector form for each object tells us that

$$m_A\begin{bmatrix}a_x\\0\\0\end{bmatrix}=\begin{bmatrix}-m_A|\vec{g}|\sin\theta\\0\\-m_A|\vec{g}|\cos\theta\end{bmatrix}+\begin{bmatrix}0\\0\\|\vec{F}_N^{A(I)}|\end{bmatrix}+\begin{bmatrix}|\vec{F}_T^{A(S)}|\\0\\0\end{bmatrix}\quad\text{and}\quad m_B\begin{bmatrix}a_x\\0\\0\end{bmatrix}=\begin{bmatrix}+m_B|\vec{g}|\\0\\0\end{bmatrix}+\begin{bmatrix}-|\vec{F}_T^{B(S)}|\\0\\0\end{bmatrix}\tag{1}$$

The first line of the vector equation for m_A tells us that

$$m_A a_x=|\vec{F}_T^{A(S)}|-m_A|\vec{g}|\sin\theta\tag{2}$$

The first line of the vector equation for m_B tells us that

$$m_B a_x=m_B|\vec{g}|-|\vec{F}_T^{B(S)}|\quad\Rightarrow\quad|\vec{F}_T^{B(S)}|=m_B(|\vec{g}|-a_x)\tag{3}$$

[Note that the last means that $m_B|\vec{g}|>|\vec{F}_T^{B(S)}|$ if $a_x>0$, consistent with the answer found above in part (a).] To calculate $|\vec{F}_T^{B(S)}|$, we need to know a_x. If the string and pulley are ideal, then $|\vec{F}_T^{B(S)}|=|\vec{F}_T^{A(S)}|\equiv|\vec{F}_T^{S}|$. Therefore, equation (2) becomes

$$m_A a_x=|\vec{F}_T^{S}|-m_A|\vec{g}|\sin\theta=m_B|\vec{g}|-m_B a_x-m_A|\vec{g}|\sin\theta\quad\Rightarrow\quad(m_A+m_B)a_x=m_B|\vec{g}|-m_A|\vec{g}|\sin\theta$$

$$\Rightarrow\quad a_x=\frac{m_B|\vec{g}|-m_A|\vec{g}|\sin\theta}{m_A+m_B}=|\vec{g}|\frac{m_B-m_A\sin\theta}{m_A+m_B}\tag{4}$$

Substituting this into equation 3 yields

$$|\vec{F}_T^{S}|=m_B|\vec{g}|-m_B|\vec{g}|\frac{m_B-m_A\sin\theta}{m_A+m_B}=m_B|\vec{g}|\frac{m_A+\cancel{m_B}-\cancel{m_B}+m_A\sin\theta}{m_A+m_B}=m_B|\vec{g}|\frac{m_A}{m_A+m_B}(1+\sin\theta)\tag{5}$$

Note that this is always positive and has units of $m|\vec{g}|$, which are the units of force. The condition that the tension force is greater than $m_B|\vec{g}|$ is basically that $(m_A+m_B)/m_A>1+\sin\theta$, so this puts some constraints on the range of angles θ that will work for a given pair of masses m_A and m_B. But note that whenever a_x is positive equation 4 implies that

$$m_B>m_A\sin\theta\quad\Rightarrow\quad\sin\theta>\frac{m_B}{m_A}\quad\Rightarrow\quad1+\sin\theta<1+\frac{m_B}{m_A}=\frac{m_A+m_B}{m_A}$$

$$\Rightarrow\quad|\vec{F}_T^{S}|=m_B|\vec{g}|\frac{m_A}{m_A+m_B}(1+\sin\theta)<m_B|\vec{g}|\frac{m_A}{m_A+m_B}\frac{m_A+m_B}{m_A}=m_B|\vec{g}|\tag{6}$$

So as long as the acceleration is downward, then $|\vec{F}_T^S| < m_B|\vec{g}|$, consistent with the argument given above in part (a).