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N7B.1 Since the car is traveling around the curve with a constant speed $|\vec{v}|$, the magnitude of Newton's second law implies that $|\vec{F}_{net}| = m|\vec{a}| = m|\vec{v}|^2/R$, where R is the radius of the curve. The effective radius of the curve is therefore

$$R = \frac{|\vec{v}|^2}{|\vec{a}|} = \frac{(50\text{ mi/h})^2}{(0.1)(22\,[\text{mi/h}]/\text{s})}\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.316\text{ mi}\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 510\text{ m}.$$

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N7B.3 Since the plane is flying in a level circle, its vertical acceleration is $a_z = 0$. However, because of its circular motion, the plane must have a nonzero acceleration toward the circle's center. At the particular instant shown in figure N7.4 in the text, we have $a_y = 0$ and $a_x = -|\vec{v}|^2/R$, where $|\vec{v}| = 320$ mi/h is the plane's speed and R is the radius of its path. Newton's second law at this instant requires that

$$\begin{bmatrix} -m|\vec{v}|^2/R \\ 0 \\ 0 \end{bmatrix} = m\vec{a} = \vec{F}_{\text{net}} = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_L|\sin\theta \\ 0 \\ +|\vec{F}_L|\cos\theta \end{bmatrix} \quad (1)$$

were m is the plane's mass. The x component of this equation tells us that $m|\vec{v}|^2/R = |\vec{F}_L|\sin\theta$ and the z component tells us that $m|\vec{g}| = |\vec{F}_L|\cos\theta$. If we divide the first of these equations by the second, we get

$$\frac{|\vec{v}|^2}{R|\vec{g}|} = \tan\theta \quad \Rightarrow \quad R = \frac{|\vec{v}|^2}{|\vec{g}|\tan\theta} = \frac{(320\ \cancel{\text{mi/h}})^2}{(9.8\ \text{m}/\cancel{\text{s}^2})\tan 11^\circ}\left(\frac{1\ \text{m}/\cancel{\text{s}}}{2.24\ \cancel{\text{mi/h}}}\right)^2 = 10{,}700\ \frac{\text{m}^2}{\text{m}} = 10.7\ \text{km} \quad (2)$$

Note that the units work out and the result is plausible.

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N7B.4 The radial part of the car's acceleration has a magnitude of $|\vec{v}|^2/R = (20\text{ m/s})^2/(200\text{ m}) = 2.0\text{ m/s}^2$. We are told that the car is slowing down at a rate of 1.0 m/s^2, so the component of the car's acceleration in the direction of its motion is $d|\vec{v}|/dt = -1.0\text{ m/s}^2$. The magnitude of the car's acceleration is thus

$$|\vec{a}| = \sqrt{\left(\frac{d|\vec{v}|}{dt}\right)^2 + \left(\frac{|\vec{v}|^2}{R}\right)^2} = \sqrt{\left(-1.0\,\frac{\text{m}}{\text{s}^2}\right)^2 + \left(2.0\,\frac{\text{m}}{\text{s}^2}\right)^2} = 2.24\,\frac{\text{m}}{\text{s}^2}$$

(Note that $\vec{a}$ points somewhat backwards in this case.)

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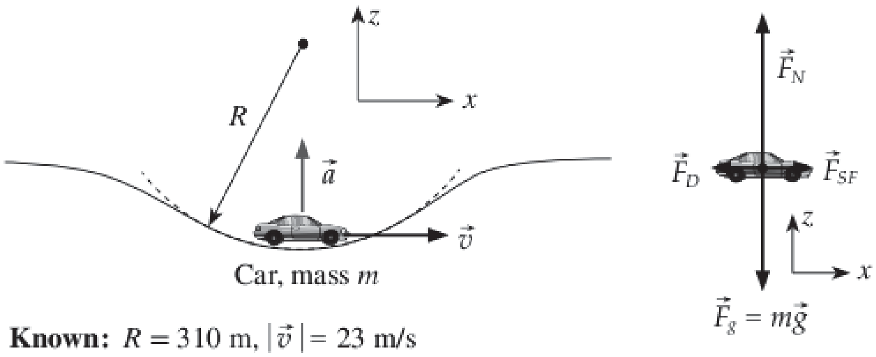
N7M.2 A picture of the situation and a free-body diagram of the car appear below.



Known: $R = 310$ m, $|\vec{v}| = 23$ m/s
Unknown: $|\vec{a}| = ?$ $m = ?$ $|\vec{F}_N| = ?$ $|\vec{F}_{SF}| = ?$ $|\vec{F}_D| = ?$

The car touches the road and the air, and interacts gravitationally with the earth. The contact interaction with the road exerts a normal force $\vec{F}_N$ and a static friction force $\vec{F}_{SF}$ on the car, while the car's interaction with the air exerts a drag force $\vec{F}_D$ and a buoyant force that I will ignore. The constraint on the car's motion is that it moves over a hill that can be approximated by a circle of radius R. We are also told that the car's speed is constant, so it is undergoing uniform circular motion, at least at the instant shown. Its acceleration is therefore upward (in the $+z$ direction) with a magnitude of $|\vec{a}| = |\vec{v}|^2/R$. Newton's second law applied to the car then implies that

$$m\begin{bmatrix} 0 \\ 0 \\ +|\vec{v}|^2/R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ +|\vec{F}_N| \end{bmatrix} + \begin{bmatrix} +|\vec{F}_{SF}| \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -|\vec{F}_D| \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

Now, the x component of this equation implies that $|\vec{F}_{SF}| = |\vec{F}_D|$, but we really don't care about the magnitude of these forces. The z component of the equation implies that

$$m\frac{|\vec{v}|^2}{R} = |\vec{F}_N| - m|\vec{g}| \quad \text{or} \quad |\vec{F}_N| = m\frac{|\vec{v}|^2}{R} + m|\vec{g}| \quad \Rightarrow \quad \frac{|\vec{F}_N|}{m|\vec{g}|} = \frac{|\vec{v}|^2}{R|\vec{g}|} + 1 \quad (2)$$

Even though we don't know $|\vec{F}_N|$ or m separately, equation 2 gives us what we were asked to find (the ratio of the normal force to the car's weight) in terms of the known quantities $|\vec{v}|, |\vec{g}|$, and R, so we do not need to look for any more equations to solve the problem. Substituting in the numbers yields

$$\frac{|\vec{v}|^2}{R|\vec{g}|} = \frac{(23\,\cancel{\text{m}}/\cancel{\text{s}})^2}{(310\,\cancel{\text{m}})(9.8\,\text{m}/\cancel{\text{s}^2})} + 1 = 0.17 + 1 = 1.17 \quad (3)$$

Note that the ratio of these forces should be unitless, consistent with the unit analysis in equation 3. This ratio is also greater than 1, which makes sense because $|\vec{F}_N|$ will need to be larger than $|\vec{F}_g|$ if the net force on the car is to be upward, as required for the car's acceleration is to be upward. Therefore, this result seems quite plausible.

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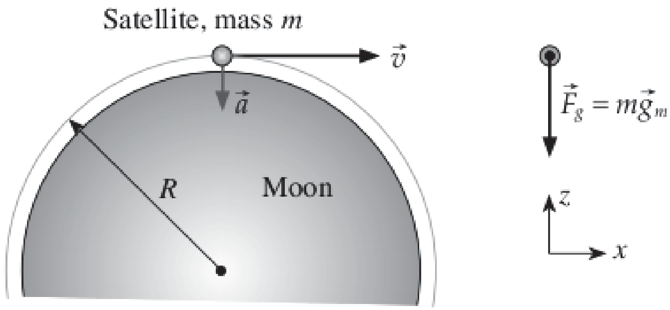
N7M.4 A drawing of the situation and a free-body diagram of the satellite appear below.



Known:
$R \approx 1740$ km
$|\vec{g}_m| \approx \frac{1}{6}|\vec{g}|$

Unknown:
$|\vec{v}| = ?$ $m = ?$
period of orbit $T = ?$

Since the moon has no very little atmosphere, there no air friction force exerted on the satellite. Thus, the only significant force acting on the satellite is the force of gravity due to its interaction with the moon. The satellite is confined to a circular orbit around the moon. I am assuming that its orbital speed $|\vec{v}|$ is constant and that it orbits just barely above the moon's surface, so that its orbital radius R is about equal to the moon's radius = 1740 km. If all this is so, the satellite's acceleration will point toward the moon's center and have a magnitude of $|\vec{a}| = |\vec{v}|^2/R$. At the instant shown in the diagrams, both the net force and the satellite's acceleration have components only in the $-z$ direction (according to the reference frame axes). At that instant, Newton's second law in this case is given by

$$m\begin{bmatrix} 0 \\ 0 \\ -|\vec{v}|^2/R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}_m| \end{bmatrix} \quad \Rightarrow \quad |\vec{v}|^2 = |\vec{g}_m|R = \tfrac{1}{6}|\vec{g}|R \quad \Rightarrow \quad |\vec{v}| = \sqrt{\tfrac{1}{6}|\vec{g}|R} \qquad (1)$$

As shown, we can solve the z component of Newton's second law for $|\vec{v}|$ in terms of known quantities, and the unknown satellite mass m cancels out. But we need another equation to determine the remaining unknown T. If we assume that the speed is constant, we can get the orbit's period T from the relation $|\vec{v}| = 2\pi R/T$. Solving the latter for T and substituting $|\vec{v}|$ as found in equation 1 yields

$$T = \frac{2\pi R}{|\vec{v}|} = \frac{2\pi R}{\sqrt{\frac{1}{6}|\vec{g}|R}} = 2\pi\sqrt{6}\sqrt{\frac{R}{|\vec{g}|}} = 2\pi\sqrt{6}\sqrt{\frac{1740\ \cancel{\text{km}}}{9.8\ \cancel{\text{m}}/\text{s}^2}\left(\frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}}\right)} = 6490\ \cancel{\text{s}}\left(\frac{1\ \text{h}}{3600\ \cancel{\text{s}}}\right) = 1.8\ \text{h} \qquad (2)$$

Note that the units, sign and magnitude all seem reasonable (an object orbits the earth in about 1.5 h, so this result has a credible magnitude).

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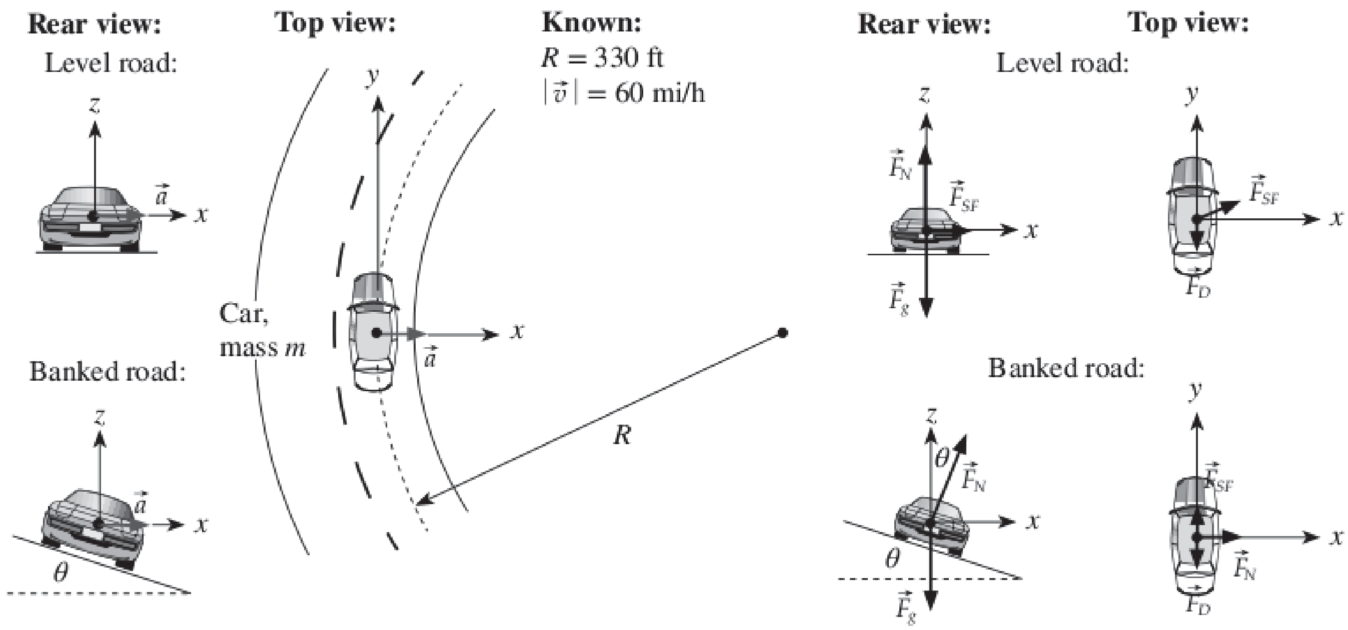
N7M.5 Rear and top views of the situation and rear and top-view free-body diagrams of a car on the highway appear below for both the level-highway and banked-highway cases:



The car touches the road and the air. The contact interaction between the car and road will exert a normal force and a static friction force on the car (assuming that its wheels don't skid). The air will exert a rearward drag force on the car. Assume the car is moving at a constant speed. Then the component of the car's acceleration in the forward direction is zero, and (as the free-body diagrams show), the static friction force must at least cancel the drag force, but in the level-highway scenario, that static friction force must also supply the required inward acceleration.

(a) If the highway is not banked, the x component of Newton's second law at the instant shown implies that the static friction force must *at least* provide the inward acceleration:

$$|\vec{F}_{SF}| \geq F_{SF,x} = ma_x = m\frac{|\vec{v}|^2}{R} \tag{1}$$

The z component of Newton's second law implies that $|\vec{F}_N| = m|\vec{g}|$, and the definition of the coefficient of static friction implies that $\mu_s|\vec{F}_N| \geq |\vec{F}_{SF}|$. Substituting these things into the equation above yields

$$\mu_s m|\vec{g}| \geq |\vec{F}_{SF}| \geq m\frac{|\vec{v}|^2}{R} \quad \Rightarrow \quad \mu_s \geq \frac{|\vec{v}|^2}{R|\vec{g}|} = \frac{(60\ \text{mi/h})^2}{(330\ \text{ft})(9.8\ \text{m/s}^2)}\left(\frac{1\ \text{m/s}}{2.24\ \text{mi/h}}\right)^2\left(\frac{3.28\ \text{ft}}{1\ \text{m}}\right) = 0.73 \tag{2}$$

(b) This coefficient comes out unitless, as it must, but also is very high for the tire-road interface to manage (Table N5.1 says that $\mu_s = 0.6$ for tires and asphalt). So it is *not* reasonable to expect that static friction will be able to supply sufficient sideward force to keep the car on the road.

(c) The optimal banking angle for the road will be such that the x component of the tilted normal force provides all of the inward force required to keep the car moving in a circle: $|\vec{F}_N|\sin\theta = m|\vec{v}|^2/R$. The z component of Newton's second law in this case requires that $|\vec{F}_N|\cos\theta = m|\vec{g}|$. We can eliminate the unknown normal force by taking the ratio of these two equations:

$$\frac{|\vec{F}_N|\sin\theta}{|\vec{F}_N|\cos\theta} = \tan\theta = \frac{m|\vec{v}|^2/R}{m|\vec{g}|} = 0.73\ (\text{as before}) \quad \Rightarrow \quad \theta = \tan^{-1}0.73 = 36° \tag{3}$$

The units work out and this is credible (though pretty steep, actually!).

(d) If we are designing this road with a curve this tight, then we *must* bank the road at something like this angle if cars are going to stay with the curve. We saw in part (b) that a level road is completely impractical in this case. If we don't back the road at nearly this angle, cars will not be able to stay on the road if it is slippery. (On the other hand, the banking angle is so steep that cars at rest might slide sideways down it when road conditions are bad, so we really should double or triple the radius of the curve if at all possible.)

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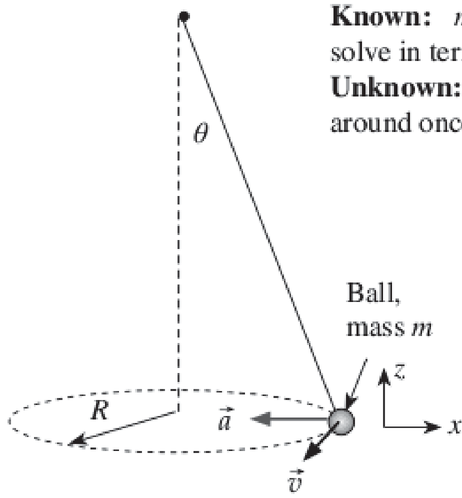
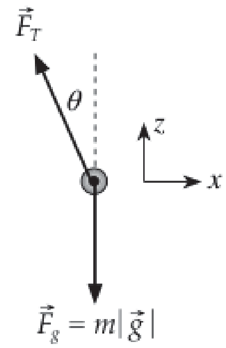
N7M.7 A drawing of the situation and a free-body diagram of the ball appear below. Though the ball touches both the string and the air, I am going to ignore any drag and buoyancy forces exerted by the air.



Known: m, $|\vec{g}|$, L, and θ (we are asked to solve in terms of these).
Unknown: The time T it takes the ball to go around once, also $|\vec{a}|, |\vec{v}|, |\vec{F}_T|$, and R.



Because the ball is in uniform circular motion, the magnitude of its acceleration is $|\vec{a}| = |\vec{v}|^2/R$. Newton's second law for the ball at the instant shown is therefore

$$m\begin{bmatrix} -|\vec{a}| \\ 0 \\ 0 \end{bmatrix} = m\begin{bmatrix} -|\vec{v}|^2/R \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m|\vec{g}| \end{bmatrix} + \begin{bmatrix} -|\vec{F}_T|\sin\theta \\ 0 \\ +|\vec{F}_T|\cos\theta \end{bmatrix} \tag{1}$$

Since we have already eliminate $|\vec{a}|$, the x and z components of this equation provide two equations in our four remaining unknowns $|\vec{v}|, |\vec{F}_T|$, R, and T. But since the ball's speed is constant, $|\vec{v}| = 2\pi R/T$, and trigonometry implies that $R = L\sin\theta$. So we have enough equations to solve. We can easily eliminate the unknown $|\vec{F}_T|$ by dividing the x component of equation 1 by the z component of equation 1:

$$\frac{\cancel{m}|\vec{v}|^2/R}{\cancel{m}|\vec{g}|} = \frac{\cancel{|\vec{F}_T|}\sin\theta}{\cancel{|\vec{F}_T|}\cos\theta} \quad \Rightarrow \quad \frac{|\vec{v}|^2}{R|\vec{g}|} = \tan\theta \tag{2}$$

Substituting in $|\vec{v}| = 2\pi R/T$ and $R = L\sin\theta$ yields

$$\tan\theta = \frac{1}{R|\vec{g}|}\left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R}{|\vec{g}|T^2} \quad \Rightarrow \quad T^2 = \frac{4\pi^2 (L\sin\theta)}{|\vec{g}|\tan\theta} = 4\pi^2 \frac{L\cancel{\sin\theta}}{|\vec{g}|}\frac{\cos\theta}{\cancel{\sin\theta}} = 4\pi^2 \frac{L\cos\theta}{|\vec{g}|}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L\cos\theta}{|\vec{g}|}} \tag{3}$$

Note that $L/|\vec{g}|$ has units of m/(m/s^2) = s^2, so the units of T will be correct. Note also that the period gets longer as L gets longer or θ gets smaller or as the gravitational field strength $|\vec{g}|$ gets, weaker, which all seems plausible.