Unit: N | Chapter: N8 | Problem Number: N8B.2 | Return to User Page

N8B.2 Let's use a frame in which the baseball is moving in the $-x$ direction. We will assume that the fielder is running toward the ball in the same direction as the ball's velocity. Let $\vec{v}$ be the ball's velocity in the ground frame and $\vec{\beta}$ be the velocity of the fielder's frame relative to the ground. According to equation N8.2, the velocity of the baseball in the fielder's frame $\vec{v}'$ is

$$\begin{bmatrix} v'_x \\ v'_y \\ v'_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} - \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} = \begin{bmatrix} 15\text{ mi/h} \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -125\text{ mi/h} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 140\text{ mi/h} \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Thus the ball is moving at a speed of 140 mi/h relative to the fielder.

Unit: N Chapter: N8 Problem Number: N8B.4 Return to User Page

N8B.4 Let's choose the ground to be the S frame and the northward traveling car to be the S' frame and set up both frames in standard orientation. Let the westward traveling car be the object whose motion is examined in both frames. We are told that this car's velocity in the ground frame is $\vec{v}$ = 14 m/s west, while the northward car is traveling at a velocity of $\vec{\beta}$ = 18 m/s north. Equation N8.2 thus implies that

$$\begin{bmatrix} v'_x \\ v'_y \\ v'_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} - \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} = \begin{bmatrix} -14\text{ m/s} \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 18\text{ m/s} \\ 0 \end{bmatrix} = \begin{bmatrix} -14\text{ m/s} \\ -18\text{ m/s} \\ 0 \end{bmatrix} \tag{1}$$

The cars' speed relative to each other is therefore

$$|\vec{v}'| = \sqrt{(v'_x)^2 + (v'_y)^2 + (v'_z)^2} = \sqrt{(-14\text{ m/s})^2 + (-18\text{ m/s})^2 + 0^2} = 23\text{ m/s}. \tag{2}$$

Unit: N
Chapter: N8
Problem Number: N8B.7

Return to User Page

N8B.7 The ball's acceleration relative to the bus is the same as its acceleration relative to the ground because the bus's velocity is constant. To see this mathematically, let the ground frame be frame S and the bus be frame S'. Thus, the ball's acceleration relative to the ground is $\vec{a} = \vec{g}$ and the bus's acceleration relative to the ground is $\vec{A} = 0$ since the bus's velocity is constant. According to equation N8.7, the ball's acceleration relative to the bus is therefore $\vec{a}' = \vec{a} - \vec{A} = \vec{a} - 0 = \vec{a} = \vec{g}$, just as it is relative to the ground.

Unit: N | Chapter: N8 | Problem Number: N8B.8 | Return to User Page

N8B.8 **(a)** If we ignore modest friction forces, the unbelted passenger, as viewed from the inertial reference frame of the ground, experiences no significant horizontal forces and so would keep moving forward in the direction of the car's original velocity. So relative to the ground, the passenger has essentially zero acceleration.

(b) The windshield (which is attached to the car) is not an inertial reference frame, so it appears that the unbelted person accelerates through it (as the car comes to a rest). We would expect, therefore, that the passenger's acceleration relative to the windshield would be equal in magnitude and opposite in direction to the windshield's acceleration relative to the ground. Indeed, equation N8.7 implies that

$$\vec{a}' = \vec{a} - \vec{A} = 0 - \vec{A} = -\vec{A} \tag{1}$$

where $\vec{a}'$ is the person's acceleration relative to the windshield, $\vec{a}$ is the person's acceleration relative to the ground, and $\vec{A}$ is the windshield's (= the car's) acceleration relative to the ground. If we assume that the car accelerates at a constant rate, then its acceleration is equal to its change in velocity $\Delta\vec{\beta}$ divided by the time interval Δt required to come to rest. If we define our reference frames so that all motion is in the $+x$ direction, then the values given imply that

$$\vec{A} = \frac{\Delta\vec{\beta}}{\Delta t} = \frac{1}{0.12\text{ s}}\left(\begin{bmatrix}0\\0\\0\end{bmatrix} - \begin{bmatrix}25\text{ m/s}\\0\\0\end{bmatrix}\right) = \begin{bmatrix}(-25\text{ m/s})/(0.12\text{ s})\\0\\0\end{bmatrix} \quad\Rightarrow\quad \vec{a}' = -\vec{A} = \begin{bmatrix}+208\text{ m/s}^2\\0\\0\end{bmatrix} \tag{2}$$

according to equation 1. The person's acceleration relative to the car's windshield is thus 208 m/s^2 forward. (The direction is what we would intuitively expect.)

Unit: N
Chapter: N8
Problem Number: N8M.2

Return to User Page

N8M.2 This is similar to example N8.3 in the text, except that we are given the plane's velocity relative to the ground, not the air. The plane will be pointed in the direction of its velocity relative to the air, since the pilot will orient the plane so that the air slips by the plane's body most easily. Therefore, the question essentially asks us to find the direction and magnitude of the plane's velocity in the air frame given its velocity in the ground frame. Let us take frame S to be the ground frame and frame S' to be the air frame. We are told that the air is moving at a speed of 25 mi/h east, so if the frames are oriented in the standard way, the velocity of the air frame relative to the ground frame is $\vec{\beta} = [+25 \text{ mi/h}, 0, 0]$. The velocity of the plane relative to the ground frame is $\vec{v} = [0, +250 \text{ mi/h}, 0]$. According to equation N8.2, the plane's velocity relative to the air frame is:

$$\vec{v}\,' = \vec{v} - \vec{\beta} = \begin{bmatrix} 0 \\ 250 \text{ mi/h} \\ 0 \end{bmatrix} - \begin{bmatrix} 25 \text{ mi/h} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -25 \text{ mi/h} \\ 250 \text{ mi/h} \\ 0 \end{bmatrix} \tag{1}$$

The angle that this plane makes with the y axis in the air frame is therefore

$$\theta = \tan^{-1}\left|\frac{v'_y}{v'_x}\right| = \tan^{-1}\left|\frac{-25 \,\cancel{\text{mi/h}}}{250 \,\cancel{\text{mi/h}}}\right| = \tan^{-1}(0.10) = 5.7° \tag{2}$$

Since v'_x is negative, the plane must point 5.7° west of north. The plane's speed relative to the air is

$$|\vec{v}\,'| = \sqrt{(v'_x)^2 + (v'_y)^2 + (v'_z)^2} = \sqrt{(-25 \text{ mi/h})^2 + (250 \text{ mi/h})^2 + 0^2} = 251 \text{ mi/h} \tag{3}$$

(Note that the units all make sense here and the magnitudes are plausible.)

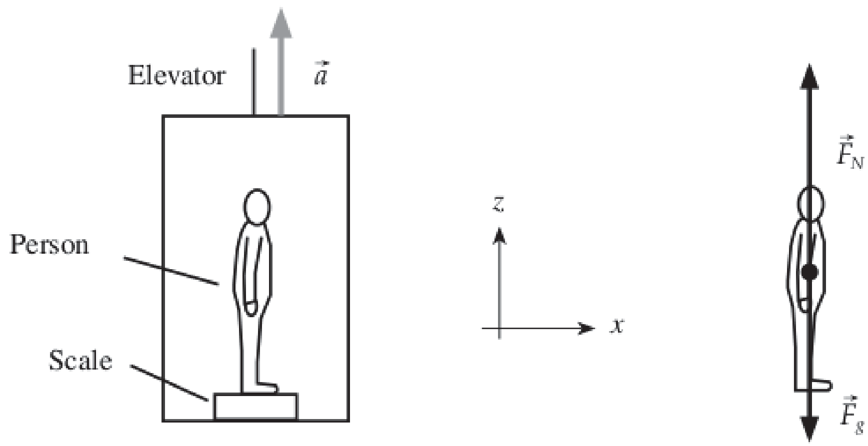
Unit: N

Chapter: N8

Problem Number: N8M.5

Return to User Page

N8M.5 The drawings below show the situation and the free-body diagram for the person.



As discussed in example N5.5, the scale reads the magnitude of the normal force that it exerts on the person. Since the person is accelerating (along with the elevator) relative to the building frame (which we will take to be inertial) in such a way that $a_z = 1.8\ \mathrm{m/s^2}$, the z component of Newton's second law implies that $|\vec{F}_N| - |\vec{F}_g| = ma_z$, which we can solve for $|\vec{F}_g|$. Doing this yields

$$|\vec{F}_N| - |\vec{F}_g| = ma_z \quad \Rightarrow \quad |\vec{F}_N| = |\vec{F}_g|\left(\frac{a_z}{|\vec{g}|} + 1\right) \quad \Rightarrow \quad |\vec{F}_g| = \frac{|\vec{F}_N|}{\left(\frac{a_z}{|\vec{g}|} + 1\right)} = \frac{180\ \mathrm{lbs}}{\left(\frac{1.8\ \cancel{\mathrm{m/s^2}}}{9.8\ \cancel{\mathrm{m/s^2}}} + 1\right)} = 152\ \mathrm{lbs}$$

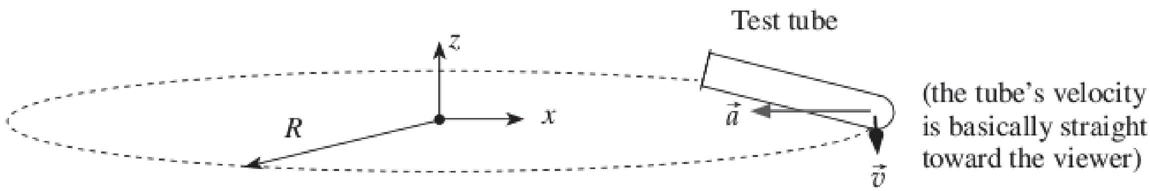
Alternatively, we could have calculated all this in the elevator's frame. In that frame, as discussed in section N8.6, we can make Newton's second law work in an accelerating frame if we add a frame correction force $-m\vec{A}$, where $\vec{A}$ is the frame's acceleration relative to an inertial frame. In our particular case, $\vec{A} = \vec{a} = 1.8\ \mathrm{m/s^2}$ upward. In the elevator's frame, the person is at rest, so Newton's second law in this NONINERTIAL frame says that $0 = \vec{F}_g + \vec{F}_N - m\vec{A}$. The z component of this equation reads $0 = -|\vec{F}_g| + |\vec{F}_N| - m|\vec{A}|$. Solving for the person's true weight, $|\vec{F}_g| = m|\vec{g}|$, we get

$$|\vec{F}_N| = m|\vec{g}| + m|\vec{A}| = m|\vec{g}| + m|\vec{g}|\left(\frac{|\vec{A}|}{|\vec{g}|}\right) \quad \Rightarrow \quad |\vec{F}_g| = \frac{|\vec{F}_N|}{\left(1 + \frac{|\vec{A}|}{|\vec{g}|}\right)} = \frac{180\ \mathrm{lbs}}{\left(1 + \frac{1.8\ \cancel{\mathrm{m/s^2}}}{9.8\ \cancel{\mathrm{m/s^2}}}\right)} = 152\ \mathrm{lbs}$$

Both approaches yield the same result, which is an encouraging sign. The units are all self-consistent, and the magnitude seems reasonable. In particular, we would expect intuitively the person's true weight to be somewhat less than the scale value while the elevator is accelerating upward.

Unit: N Chapter: N8 Problem Number: N8M.6 Return to User Page

N8M.6 The drawing below illustrates the situation.



If the test tube's bottom follows an approximately circular path with radius $R = 38$ cm, its acceleration toward the center of its circular path (as measured in the frame of the ground) at the instant shown is

$$\vec{A} = \begin{bmatrix} -|\vec{v}|^2/R \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where $|\vec{v}|$ is the speed of the test tube's bottom. Let the test tube's bottom define a noninertial reference frame. According to equation N8.12, we can make Newton's second law work in this frame if we add a fictitious frame-correction force $-m\vec{A}$ to the physical forces that act on a given object in that frame. Since this force, like the gravitational force, is proportional to the object's mass, an object of mass m in this frame behaves exactly as if it was experiencing an effective gravitational force of $m\vec{g}_{\text{eff}} = m\vec{g} - m\vec{A}$. Therefore, the effective gravitational field strength in this frame is given by

$$\vec{g}_{\text{eff}} = \vec{g} - \vec{A} = \begin{bmatrix} 0 \\ 0 \\ -|\vec{g}| \end{bmatrix} - \begin{bmatrix} |\vec{v}|^2/R \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} |\vec{v}|^2/R \\ 0 \\ |\vec{g}| \end{bmatrix} \Rightarrow |\vec{g}_{\text{eff}}| = \sqrt{\left(\frac{|\vec{v}|^2}{R}\right)^2 + |\vec{g}|^2} = |\vec{g}|\sqrt{\left(\frac{|\vec{v}|^2}{|\vec{g}|R}\right)^2 + 1} \tag{2}$$

The problem statement tells us that the effective gravitational field strength in this frame is 18 times normal; that is, $|\vec{g}_{\text{eff}}| = 18|\vec{g}|$. Since we know $|\vec{g}_{\text{eff}}|, |\vec{g}|$, and R, we can solve equation 1 for $|\vec{v}|$ and then determine the time T required for a complete rotation using the fact $|\vec{v}| = 2\pi R/T$ (assuming that the centrifuge is rotating at a constant rate):

$$18 = \frac{|\vec{g}_{\text{eff}}|}{|\vec{g}|} = \sqrt{\left(\frac{|\vec{v}|^2}{|\vec{g}|R}\right)^2 + 1} \Rightarrow 18^2 - 1 = \left(\frac{|\vec{v}|^2}{|\vec{g}|R}\right)^2 \Rightarrow |\vec{v}|^2 = |\vec{g}|R\sqrt{18^2 - 1} \Rightarrow |\vec{v}| = \sqrt{|\vec{g}|R\sqrt{18^2-1}} \tag{3}$$

$$\Rightarrow T = \frac{2\pi R}{|\vec{v}|} = \frac{2\pi R}{\sqrt{|\vec{g}|R\sqrt{18^2-1}}} = 2\pi\sqrt{\frac{R}{|\vec{g}|\sqrt{18^2-1}}} = 2\pi\sqrt{\frac{0.38\ \text{m}}{9.8\ \text{m/s}^2\sqrt{18^2-1}}} = 0.292\ \text{s} \tag{4}$$

The centrifuge's rotation rate in revolutions per second is therefore

$$\frac{1\ \text{rev}}{T} = \frac{1\ \text{rev}}{0.292\ \text{s}} = 3.43\ \frac{\text{rev}}{\text{s}} \tag{5}$$

Note that the units all work out and the answer seems large but not implausibly so. Note also that one gets an answer that is only insignificantly different if one ignores the actual force of gravity. But one cannot know this *a priori*, so it is a mistake to do so unless one argues the validity of that approximation.

Unit: N
Chapter: N8
Problem Number: N8M.8

Return to User Page

N8M.8 We need the capsule's acceleration in its rotating reference frame to have a magnitude $|\vec{A}| \approx \frac{1}{2}|\vec{g}|$. If the tether's length is L and the capsule and booster have about the same mass, then the distance between the capsule and the system's center of mass (and thus the radius of rotation) will be about $R = \frac{1}{2}L$. We need the period of rotation T to be larger than 30 s. Therefore, we must have

$$\frac{1}{2}|\vec{g}| = |\vec{A}| = \frac{|\vec{v}|^2}{\frac{1}{2}L} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2[\frac{1}{2}L]}{T^2} \quad \Rightarrow \quad L = \frac{|\vec{g}|T^2}{4\pi^2} > \frac{(9.8\text{ m}/\cancel{\text{s}}^2)(30\,\cancel{\text{s}})^2}{4\pi^2} = 220\text{ m}.$$