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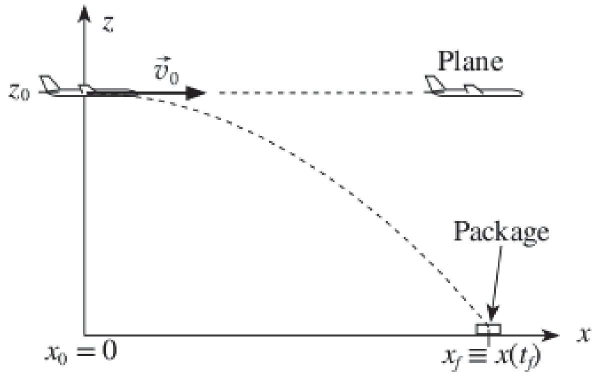
N9B.2 The drawing below shows the situation.



Let time $t = 0$ be the time that the package is dropped, and time $t = t_f$ be when it hits the ground. If we also assume that the origin of our reference frame is on the ground directly below the package when it is dropped and we orient our reference frame so that the z axis is initially upward, the package's initial position coordinates are $x_0 = 0$, $y_0 = 0$, and $z_0 = 1.0$ km. We will also assume that "dropped" means that the package is simply released while it is at rest with respect to the plane, which means that the package's initial velocity is the same as that of the plane. If we assume that the plane is flying horizontally, and define our x axis so that it points in the direction of the plane's motion at $t = 0$, then the package's initial velocity components are $v_{0x} = |\vec{v}_0|$, $v_{0y} = v_{0z} = 0$. If we assume that air resistance is unimportant, then the motion of the package will be simple projectile motion, and we have (see equation N9.7)

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} v_{0x}t + x_0 \\ v_{0y}t + y_0 \\ -\frac{1}{2}|\vec{g}\,|t^2 + v_{0z}t + z_0 \end{bmatrix} = \begin{bmatrix} |\vec{v}_0|t \\ 0 \\ -\frac{1}{2}|\vec{g}\,|t^2 + z_0 \end{bmatrix} \tag{1}$$

The package reaches the ground at time $t = t_f$, so we must have $z(t_f) = 0$. The z component of equation 1 therefore tells us that

$$0 = z(t_f) = -\tfrac{1}{2}|\vec{g}\,|t_f^2 + z_0 \quad \Rightarrow \quad \tfrac{1}{2}|\vec{g}\,|t_f^2 = z_0 \quad \Rightarrow \quad t_f = \sqrt{\frac{2z_0}{|\vec{g}\,|}} = \sqrt{\frac{2(1000\ \text{m})}{9.8\ \text{m/s}^2}} = 14.3\ \text{s} \tag{2}$$

Note that (in the absence of drag, anyway), this time is completely independent of the plane's forward speed (which is a good thing, since we are not given that speed).

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N9B.3 Define the z axis to be vertically upward. If the stone has an initial z-velocity of $v_{0z} = +22$ m/s, then equation N9.8 tells us that the stone reaches the peak of its trajectory at time

$$t_p = \frac{v_{0z}}{|\vec{g}|} = \frac{22 \text{ m/s}}{9.8 \text{ m/s}^2} = 2.24 \text{ s}$$

(This assumes that air drag is negligible.)

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N9B.6 We are given that at time $t = 0$, the object's vertical position is $z_0 = 10$ m above the ground and its z-velocity is $v_{0z} = +25$ m/s. According to equation N9.9, the time it takes for the object to return to the ground (assuming negligible air drag) is

$$t_h = \frac{v_{0z} \pm \sqrt{v_{0z}^2 + 2|\vec{g}|z_0}}{|\vec{g}|} = \frac{25\,\text{m/s} \pm \sqrt{(25\,\text{m/s})^2 + 2(9.8\,\text{m/s}^2)(10\,\text{m})}}{9.8\,\text{m/s}^2} = 5.5\text{ s or } -0.4\text{ s}$$

The second solution is not physically meaningful here: we did not in fact launch the object until $t = 0$.* The time required for the object to hit the ground after being launched as described is therefore 5.5 s.

*The negative solution actually gives the time that the object would have been rising through $z = 0$ if it had been launched below $z = 0$ at some previous time in such a way as to pass through $z = 10$ m with a z-velocity of 25 m/s at time $t = 0$.

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N9B.8 According to equation N9.16, we have

$$v_T = \sqrt{\frac{2m|\vec{g}|}{C\rho A}} \quad\Rightarrow\quad v_T^2 = \frac{2m|\vec{g}|}{C\rho A} \quad\Rightarrow\quad CA = \frac{2m|\vec{g}|}{\rho v_T^2} = \frac{2(60\,\text{kg})(9.8\,\text{m/s}^2)}{(1.2\,\text{kg/m}^3)(60\,\text{m/s})^2} = 0.27\,\frac{\text{m}^4}{\text{m}^2} = 0.27\,\text{m}^2$$

Since a person plausibly has a frontal surface area of about 0.4 m by 1.5 m = 0.60 m^2, this would correspond to a drag coefficient of about 0.5, which is plausible.

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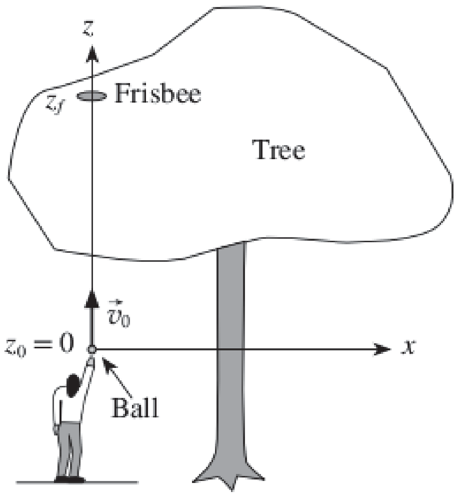
N9M.3 A diagram of the situation appears below. Note that I have defined the point where the ball leaves the thrower's hand to be the origin, and I have assumed that the point of release is approximately at the level of the thrower's head.



Known:
$v_{0x} = v_{0y} = 0$, $v_{0z} = +|\vec{v}_0|$
$x_0 = y_0 = z_0 = 0$
$z_f \approx 12$ m

Unknown:
$t_f = ?$ $|\vec{v}_0| = ?$

The effects of air friction are probably negligible for a baseball, so the ball moves under the effects of gravity alone after it leaves the thrower's hand at $t = 0$ until it hits the Frisbee (or, more likely, misses it) at $t = t_f$. I will treat the baseball as a point particle and assume that its velocity is purely vertical.

(a) Equation N9.7 tells us that in this case

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} v_{0x}t + x_0 \\ v_{0y}t + y_0 \\ -\frac{1}{2}|\vec{g}|t^2 + v_{0z}t + z_0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 0+0 \\ -\frac{1}{2}|\vec{g}|t^2 + |\vec{v}_0|t + 0 \end{bmatrix} \tag{1}$$

We have two unknowns, t_f and $|\vec{v}_0|$, so we need another equation. We also know that the ball's velocity at its peak is zero. The z component of equation N9.6 tells us, therefore, that

$$0 = v_z(t_f) = -|\vec{g}|t_f + |\vec{v}_0| \;\Rightarrow\; |\vec{g}|t_f = |\vec{v}_0| \;\Rightarrow\; t_f = \frac{|\vec{v}_0|}{|\vec{g}|} \tag{2}$$

This provides us with the extra information that we need. Substituting this into the z component of equation 1 eliminates t_f and allows us to solve for $|\vec{v}_0|$:

$$z_f \equiv z(t_f) = -\tfrac{1}{2}|\vec{g}|\left(\frac{|\vec{v}_0|}{|\vec{g}|}\right)^2 + |\vec{v}_0|\left(\frac{|\vec{v}_0|}{|\vec{g}|}\right) = +\frac{|\vec{v}_0|^2}{2|\vec{g}|} \;\Rightarrow\; |\vec{v}_0| = \sqrt{2|\vec{g}|z_f} = \sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(12\text{ m})} = 15.3\,\frac{\text{m}}{\text{s}} \tag{3}$$

Note that the units and sign are correct for a velocity, and the magnitude doesn't seem unduly unreasonable since the Frisbee is rather high up! But also note that throwing with this initial velocity means that the ball arrives at the Frisbee with zero velocity. So you had better throw the ball faster than 15.3 m/s to have a chance at dislodging the Frisbee.

(b) We can also do this problem using conservation of energy. Take the ball and the earth to be the system. Note that we can keep track of the only significant internal interaction using the near-earth potential energy formula $V_g = m|\vec{g}|z$, and define K_{ei} and K_{ef} to be the negligible initial and final kinetic energies of the earth and $U_i = U_f$ to be the unchanging initial and final system internal energies. The conservation of energy master equation tells us that

$$\tfrac{1}{2}m|\vec{v}_0|^2 + \overbrace{K_{ei}}^{0} + m|\vec{g}|\overbrace{z_0}^{0} + \cancel{U_i} = \tfrac{1}{2}m\overbrace{|\vec{v}_f|^2}^{0} + \overbrace{K_{ef}}^{\approx 0} + m|\vec{g}|z_f + \cancel{U_f}$$
$$\Rightarrow\; \tfrac{1}{2}m|\vec{v}_0|^2 = m|\vec{g}|z_f \;\Rightarrow\; |\vec{v}_0| = \sqrt{2|\vec{g}|z_f} \tag{4}$$

which is the same result as before.

(c) The energy calculation is significantly easier, in that we can do the calculation in a single step (we don't have to separately solve for t_f). On the other hand, if we had wanted t_f for some reason, the calculation in part (a) would be much faster.

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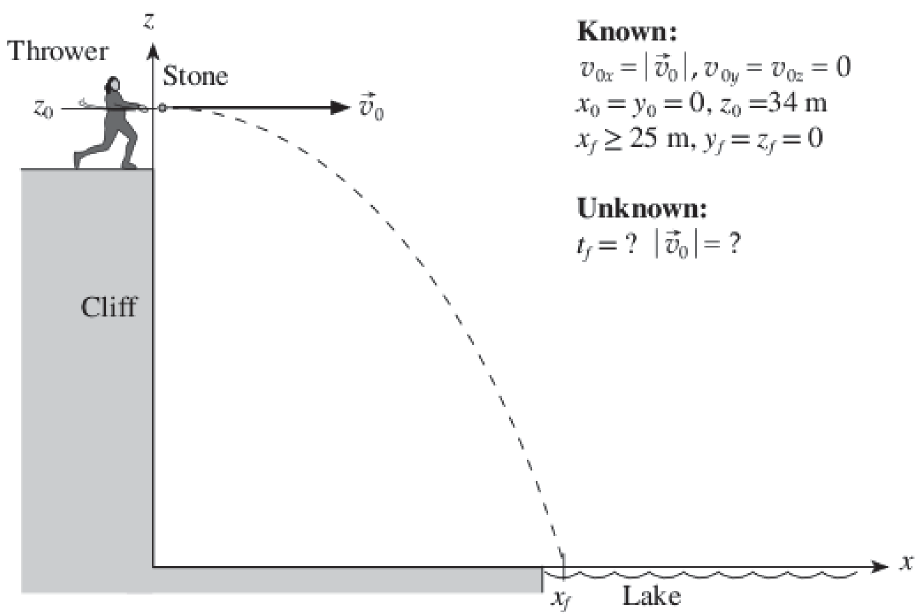
N9M.5 A drawing of the situation appears below. Note that I am defining the origin to be at the base of the cliff (other choices are reasonable). The thrower throws the rock horizontally at time $t = 0$, and the rock hits the lake at $t = t_f$. I am also assuming that the thrower releases the rock about 2 m above the cliff top, so that the rock's initial height above the lake is 34 m instead of 32 m.



Known:
$v_{0x} = |\vec{v}_0|$, $v_{0y} = v_{0z} = 0$
$x_0 = y_0 = 0$, $z_0 = 34$ m
$x_f \geq 25$ m, $y_f = z_f = 0$

Unknown:
$t_f = ?$ $|\vec{v}_0| = ?$

If we assume that air friction is negligible (probably a pretty good assumption for a rock if its speed is not too large), then the rock is a simple projectile. Equation N9.7 tells us that

$$\begin{bmatrix} x_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x(t_f) \\ y(t_f) \\ z(t_f) \end{bmatrix} = \begin{bmatrix} v_{0x}t_f + x_0 \\ v_{0y}t_f + y_0 \\ -\frac{1}{2}|\vec{g}|t_f^2 + v_{0z}t_f + z_0 \end{bmatrix} = \begin{bmatrix} |\vec{v}_0|t_f + 0 \\ 0+0 \\ -\frac{1}{2}|\vec{g}|t_f^2 + 0 + z_0 \end{bmatrix} = \begin{bmatrix} |\vec{v}_0|t_f \\ 0 \\ -\frac{1}{2}|\vec{g}|t_f^2 + z_0 \end{bmatrix} \quad (1)$$

The x and z components of this equation provide two equations in our two unknowns t_f and $|\vec{v}_0|$, so we can solve. We can do this by solving the z component of this equation for t_f and substituting the result into the x component to eliminate t_f:

$$0 = z(t_f) = -\tfrac{1}{2}|\vec{g}|t_f^2 + z_0 \quad \Rightarrow \quad \tfrac{1}{2}|\vec{g}|t_f^2 = z_0 \quad \Rightarrow \quad t_f = \sqrt{\frac{2z_0}{|\vec{g}|}} \quad (2)$$

$$x_f \equiv x(t_f) = |\vec{v}_0|t_f \quad \Rightarrow \quad |\vec{v}_0| = \frac{x_f}{t_f} = \frac{x_f}{\sqrt{2z_0/|\vec{g}|}} \geq 25\,\text{m}\sqrt{\frac{9.8\,\text{m}/\text{s}^2}{2(34\,\text{m})}} = 9.5\,\frac{\text{m}}{\text{s}} \quad (3)$$

So one must throw the rock with a horizontal speed of at least 9.5 m/s to reach the lake. Note that the units work out nicely and the magnitude of the speed seems plausible.

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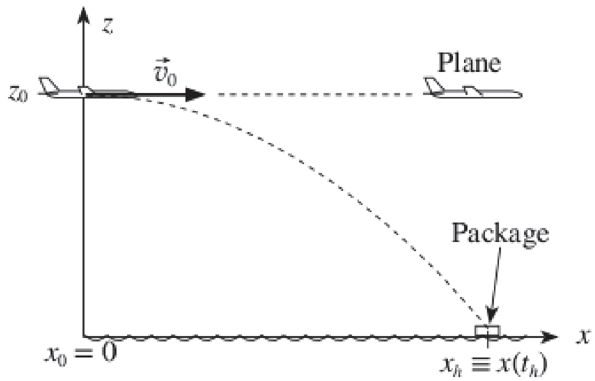
N9M.7 A drawing of the situation appears below. Note that $t = 0$ is when the package leaves the plane and $t = t_h$ is the time the package hits the water (which we can hope is close to the person!). Because we are dropping the package from rest in the plane, the package's initial velocity will be the same as the plane's horizontal velocity. I have set the origin on the ocean below the plane at $t = 0$ (though another perfectly reasonable choice would have been to put the origin at the person).



Known:
$v_{0x} = 85\ \text{m/s},\ v_{0y} = v_{0z} = 0$
$x_0 = 0,\ y_0 = 0,\ z_0 = 520\ \text{m}$
$z_h \equiv z(t_h) = 0,\ y_h = 0$

Unknown:
$t_h = ?\ \ x_h \equiv x(t_h) = ?$

If we assume that air resistance is negligible, then the package is a simple projectile. (This may not be a very reasonable model, depending on how light the package is, but we really have no other simple option.) Equation N9.7 then tells us that

$$\begin{bmatrix} x_h \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x(t_h) \\ y(t_h) \\ z(t_h) \end{bmatrix} = \begin{bmatrix} v_{0x}t_h + x_0 \\ v_{0y}t_h + y_0 \\ -\frac{1}{2}|\vec{g}|t_h^2 + v_{0z}t + z_0 \end{bmatrix} = \begin{bmatrix} |\vec{v}_0|t_h + 0 \\ 0 + 0 \\ -\frac{1}{2}|\vec{g}|t_h^2 + 0 + z_0 \end{bmatrix} = \begin{bmatrix} |\vec{v}_0|t_h \\ 0 \\ -\frac{1}{2}|\vec{g}|t_h^2 + z_0 \end{bmatrix} \tag{1}$$

The x and z components of this equation provide two equations in our two unknowns t_h and x_h, so we can solve. We can do this by solving the z component of this equation for t_h and substituting the result into the x component to eliminate t_h:

$$0 = z(t_h) = -\frac{1}{2}|\vec{g}|t_h^2 + z_0 \quad \Rightarrow \quad \frac{1}{2}|\vec{g}|t_h^2 = z_0 \quad \Rightarrow \quad t_h = \sqrt{\frac{2z_0}{|\vec{g}|}} \tag{2}$$

$$x_h \equiv x(t_h) = |\vec{v}_0|t_h = |\vec{v}_0|\sqrt{\frac{2z_0}{|\vec{g}|}} = (85\ \text{m}/\cancel{\text{s}})\sqrt{\frac{2(520\ \cancel{\text{m}})}{9.8\ \cancel{\text{m}}/\cancel{\text{s}^2}}} = 880\ \text{m} \tag{3}$$

So we should drop the package about 880 m ahead of the person if it is to land near the person. Note that the sign and units are correct for a displacement that we expect to be in the positive direction relative to where we drop the package and the magnitude is reasonable, given the plane's high speed. But we should probably drop the package a bit closer than the calculation would suggest, because friction is likely to be pretty significant for something light enough to float. A calculation using the Newton app might yield a better result.

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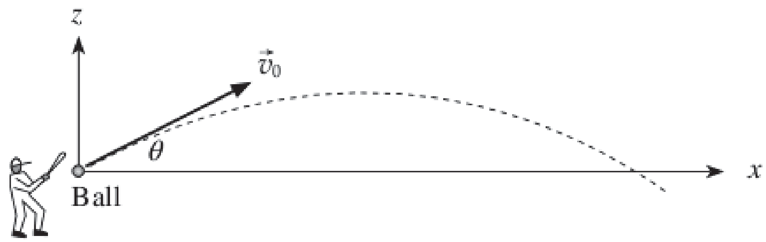
N8M.8 A drawing of the situation appears below. I have defined $t = 0$ to be the time when the ball leaves the bat and $t = t_f$ to be the time that the ball returns to its initial height a few feet above the ground, so that $z_0 = z_f$. This is approximately the vertical position where the fielder is likely to catch the ball anyway, and making this choice makes the problem significantly simpler (the result will be only a tiny fraction of a second different if the fielder catches the ball at a different vertical position). The problem becomes even easier if we define the origin so that $z_0 = z_f = 0$.



Known:
$|\vec{v}_0| = 37$ m/s, $\theta = 32°$
$z_0 = z_f \equiv z(t_f) = 0$
$y_0 = y_f = 0$
$x_0 = 0$

Unknown:
$t_f = ?$ $x_f = x(t_f) = ?$

If the effects of air resistance are negligible, then the ball will be a simple projectile after $t = 0$ until it is caught. (Neglecting air resistance is probably not a good approximation here, but it is the best we can do without using the Newton app.) Equation N9.7 therefore tells us that in this case,

$$\begin{bmatrix} x_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x(t_f) \\ y(t_f) \\ z(t_f) \end{bmatrix} = \begin{bmatrix} v_{0x} t_f + x_0 \\ v_{0y} t_f + y_0 \\ -\frac{1}{2}|\vec{g}|t_f^2 + v_{0z} t_f + z_0 \end{bmatrix} = \begin{bmatrix} (|\vec{v}_0|\cos\theta)t_f + 0 \\ 0+0 \\ -\frac{1}{2}|\vec{g}|t_f^2 + (|\vec{v}_0|\sin\theta)t_f + 0 \end{bmatrix} = \begin{bmatrix} (|\vec{v}_0|\cos\theta)t_f \\ 0 \\ -\frac{1}{2}|\vec{g}|t_f^2 + (|\vec{v}_0|\sin\theta)t_f \end{bmatrix} \quad (1)$$

Because we are only interested in calculating t_f, solving the z component of this equation for t_f is all that we need to do, because everything else in that equation is known.

$$0 = -\tfrac{1}{2}|\vec{g}|t_f^2 + |\vec{v}_0|\sin\theta\, t_f \;\Rightarrow\; \tfrac{1}{2}|\vec{g}|t_f = |\vec{v}_0|\sin\theta \;\Rightarrow\; t_f = \frac{2|\vec{v}_0|\sin\theta}{|\vec{g}|} = \frac{2(37\text{ m/s})\sin 32°}{9.8\text{ m/s}^2} = 4.0\text{ s} \quad (2)$$

The outfielder therefore has about 4.0 s to run to where the ball will pass through the $z = 0$ plane. Note that the units work out, and the magnitude seems plausible.