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N10B.2 Equation N10.9 connects an oscillator's frequency f to the spring constant k_s of its spring and the mass m of the oscillating object:

$$f = \frac{\text{cycle}}{2\pi}\sqrt{\frac{k_s}{m}} \tag{1}$$

Solving for k_s in this case yields

$$k_s = \frac{4\pi^2 f^2 m}{(\text{cycle})^2} = \frac{4\pi^2(2.2\,\cancel{\text{cycle}}/\cancel{\text{s}})^2(0.30\,\cancel{\text{kg}})}{(\cancel{\text{cycle}})^2}\left(\frac{1\,\text{N}}{1\,\cancel{\text{kg}}\cdot\text{m}/\cancel{\text{s}^2}}\right) = 57\frac{\text{N}}{\text{m}}. \tag{2}$$

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N10B.4 This situation is an example of how we can apply the SHO model to atoms! Equation N10.9 connects the atom's oscillation frequency f to the spring constant k_s (where m is the atom's mass):

$$f = \frac{\text{cycle}}{2\pi}\sqrt{\frac{k_s}{m}} \quad \Rightarrow \quad k_s = \frac{4\pi^2 f^2 m}{(\text{cycle})^2} = \frac{4\pi^2(10^{13}\ \cancel{\text{cycle}}/\cancel{\text{s}})^2(24)(1.67\times10^{-27}\ \cancel{\text{kg}})}{(\cancel{\text{cycle}})^2}\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}}\cdot\text{m}/\cancel{\text{s}^2}}\right) = 160\ \frac{\text{N}}{\text{m}}$$

where 1.67×10^{-27} kg is the mass of a proton.

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N10B.6 Equation N10.29 relates the pendulum's period T to its length L as follows: $T = 2\pi\sqrt{L/|\vec{g}|}$. Solving this expression for the pendulum's length yields

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{|\vec{g}|} \quad \Rightarrow \quad L = |\vec{g}|\left(\frac{T}{2\pi}\right)^2 = \left(9.8\,\frac{\text{m}}{\cancel{\text{s}}^{\cancel{2}}}\right)\left(\frac{2.0\,\cancel{\text{s}}}{2\pi}\right)^2 = 0.993\,\text{m}.$$

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N10B.7 If we model the child on the swing as a simple pendulum with length L, then its period T is (according to equation N10.29)

$$T = 2\pi\sqrt{\frac{L}{|\vec{g}|}} = 2\pi\sqrt{\frac{3.2\text{ m}}{9.8\text{ m/s}^2}} = 3.6\text{ s} \tag{1}$$

Note that the child's mass is irrelevant.

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N10M.3 The period will get longer. There are several ways that one might argue this. One way is to consider displacing an object at the end of a spring some distance (thus stretching the spring) and then releasing the object from rest. During the object's subsequent motion, some of the potential energy stored in the spring must go to kinetic energy in parts of the massive spring rather than being channeled entirely into kinetic energy of the object (as it would if the spring were massless). This means that the object will move more slowly in an oscillation starting at a given displacement than it would if the spring were massless, and thus take more time to cover the distance between the endpoints of the oscillation.

Another way of saying the same thing is that when the object is oscillating, part of the tension in the spring goes to accelerating parts of the spring, and so the actual tension on the end of the spring that is connected to the object is smaller during periods of maximum acceleration than we would expect it to be. Again this means that the object will move more slowly than we would expect it to from the "massless spring" approximation.

Doing the math to make a quantitative prediction of how much longer the period becomes is beyond the level of this course (it is more appropriate for a sophomore or junior-level mechanics course). It turns out that using a spring of mass m instead of a massless spring changes the system's period by the same amount that adding $\frac{1}{3}m$ to the mass of the object at the spring's end would.

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N10M.5 **(a)** An oscillation's amplitude is the distance between the system's equilibrium point and either extreme of the oscillation, which means that the amplitude is half the distance between the extreme points (where the oscillating object is at rest). In this case, the mass was at rest at its original position, so this is its upper extreme point. We are told that the lower extreme point is 12 cm lower, so the distance between extreme points is 12 cm. The amplitude is thus *half* of this, so amplitude $A = 6$ cm.

(b) The point halfway between the extreme points (6 cm below the top point) is where the oscillating object *could* hang at rest (that is, where there is *no net force* on the object that would cause it to accelerate away from rest). In this case, two forces (an upward spring tension force and a downward gravitational force) act on the object: if we are to have no net force on the object then these forces must be equal in magnitude: $|\vec{F}_{Sp}| = m|\vec{g}|$. Because at this point the spring is stretched from its equilibrium position by a distance $|x| = 6$ cm, this expression becomes $|\vec{F}_{Sp}| = k|x| = m|\vec{g}|$. Solving this for k, we get

$$k = \frac{m|\vec{g}|}{|x|} = \frac{(0.6\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})}{0.06\ \text{m}}\left(\frac{1\ \text{N}}{1\ \cancel{\text{kg}}\cdot\cancel{\text{m/s}^2}}\right) = 98\ \frac{\text{N}}{\text{m}} \tag{1}$$

(c) We can find the object's speed in general by taking the time derivative of the expression for the object's position as a function of time, which is $x(t) = A\cos(\omega t + \theta)$ according to N11.8. So:

$$v_x(t) = \frac{dx}{dt} = \frac{d}{dt}A\cos(\omega t + \theta) = -A\omega\sin(\omega t + \theta) \tag{2}$$

(assuming the x axis is vertical). This will have its largest magnitude when $\sin(\omega t + \theta) = \pm 1$, so

$$|\vec{v}_{\max}| = \omega A = \sqrt{\frac{k_s}{m}}A = \sqrt{\frac{98\ \cancel{\text{N}}/\cancel{\text{m}}}{0.6\ \cancel{\text{kg}}}\left(\frac{1\ \cancel{\text{kg}}\cdot\cancel{\text{m}}/\text{s}^2}{1\ \cancel{\text{N}}}\right)}(0.06\ \text{m}) = 0.77\ \text{m/s}. \tag{3}$$

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N10M.6 Let's model the person on the trampoline as a mass hanging from a spring. (This is probably not a terrifically good model, because the trampoline's surface stretches in complicated ways when someone sits on it, but, as discussed in the text, it will still be a useful approximation almost no matter what the complexities are.) In this model, the person's equilibrium position 45 cm below the trampoline's surface corresponds to the equilibrium position of the mass on the spring, which we should set to $x = 0$. Equation N10.13 tells us that when $x = 0$ (since this the equilibrium point), the net force exerted on the person will be zero, and $k_s x_R = m|\vec{g}|$, where m is the person's mass and x_R is that person's position when the trampoline is relaxed (in other words, 45 cm above the trampoline's equilibrium state when the person is sitting on it). So

$$k_s = \frac{m|\vec{g}|}{x_R} = \frac{55\ \text{kg}(9.8\ \text{m/s}^2)}{0.45\ \text{m}}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 1200\ \frac{\text{N}}{\text{m}} \tag{1}$$

Now that we know k_s, we can use equation N10.8 to find the person's period of oscillation T. (Again, we're assuming that person never leaves the surface of the trampoline, so it is as if he or she remains attached to a spring). In this case, equation N10.8 tells us that

$$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{55\ \text{kg}}{1200\ \text{N/m}}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)} = 1.35\ \text{s} \tag{2}$$

Note that the units work out in both equations, and the period seems credible.

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N10M.9 Equation N10.29 comes originally from applying Newton's second law to the pendulum. The small-angle approximation enters between equations N10.26 and N10.27, where we had

$$\frac{d^2\phi}{dt^2} = -\frac{|\vec{g}|}{L}\sin\phi \quad \Rightarrow \quad \frac{d^2\phi}{dt^2} = -\frac{|\vec{g}|}{L}\phi \tag{1}$$

The approximation is necessary because equation N10.27 (the latter equation above) has the exact form of the harmonic oscillator equation, not the previous equation. In making this approximation, we are essentially saying that

$$|\vec{F}_{\text{net}}| = -m|\vec{g}|\sin\phi \approx -m|\vec{g}|\phi \tag{2}$$

Now think about the mathematical properties of ϕ and $\sin\phi$. The value of ϕ can increase without limit, whereas $\sin\phi$ can only have values between –1 and + 1. It's possible to show (by looking at the Taylor series if you've studied them in calculus, or by trying values ϕ and $\sin\phi$ on your calculator) that $\sin\phi < \phi$ for all values of ϕ. Hence the exact expression for the magnitude of the force ($m|\vec{g}|\sin\phi$) will always be smaller than the approximate value ($m|\vec{g}|\phi$). If the actual magnitude of the force is smaller than the value we're assuming by using the approximation, the pendulum bob will actually have a smaller acceleration than is implied by the approximation. If its acceleration is always smaller, its speed will also always be smaller, and the pendulum will take longer to complete one oscillation than we would predict from the approximation. Since this will be increasingly true as ϕ gets bigger, the period of a pendulum will be longer at large amplitudes (where the approximation is poor for much of the swing) than at small amplitudes.