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N11B.3 According to equation N11.13, the period T of a circular orbit is given by $T^2 = (4\pi^2 / GM)R^3$, where where M is the mass of the primary (the earth in this case), R is the circular orbit's radius, and G is the universal gravitational constant. Solving this for R and substituting in the numbers yields

$$R = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\ \not{\text{N}}\cdot\text{m}^2/\not{\text{kg}}^2)(5.98\times10^{24}\ \not{\text{kg}})(24\ \not{\text{h}})^2}{4\pi^2}\left(\frac{1\ \not{\text{kg}}\cdot\text{m}/\not{\text{s}}^2}{1\ \not{\text{N}}}\right)\left(\frac{3600\ \not{\text{s}}}{1\ \not{\text{h}}}\right)^2\right]^{1/3}$$

$$= 4.23\times10^7\,(\text{m}^3)^{1/3} = 42{,}300\ \text{km}.$$

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N11B.5 Assuming that the earth's orbit is circular, equation N11.11 tells us that its orbital speed must be $|\vec{v}| = \sqrt{GM/R}$, where M is the mass of the primary (the sun in this case), R is the circular orbit's radius, and G is the universal gravitational constant. Substituting in the numbers yields

$$|\vec{v}| = \sqrt{\frac{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times 10^{30}\ \text{kg})}{1.50\times 10^{11}\ \text{m}}\left(\frac{1\ \text{kg}\cdot\text{m}/\text{s}^2}{1\ \text{N}}\right)} = 29{,}700\ \frac{\text{m}}{\text{s}} = 29.7\ \frac{\text{km}}{\text{s}}.$$

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N11B.7 The radius of Neptune's orbit R_N is about 30 times larger than the radius R of the earth's orbit. Let T_N be the period of Neptune's orbit, and $T = 1$ y be the period of the earth's orbit. Since both orbit the same primary (the sun) with mass M, equation N11.13 tells us that

$$\frac{T_N^2}{T^2} = \frac{4\pi^2 R_N^3 / GM}{4\pi^2 R^3 / GM} = \left(\frac{R_N}{R}\right)^3 \Rightarrow \frac{T_N}{T} = \left(\frac{R_N}{R}\right)^{3/2} = 30^{3/2} = 164 \Rightarrow T_N = 164T = 164 \text{ y}$$

This is very close to the observed period listed inside the front cover (the slight difference is due to the ratio of R_N to R being a bit larger than 30).

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N11B.9 In this case I am in a circular orbit of radius $R = 320$ km $= 320{,}000$ m around a neutron star with a mass $M = 2.8 \times 10^{30}$ kg. We can use equation N11.13 to calculate the period T of such an orbit:

$$T^2 = \frac{4\pi^2 R^3}{GM} \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{(320{,}000\ \text{m})^3}{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.8\times 10^{30}\ \text{kg})}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m}/\text{s}^2}\right)} = 0.083\ \text{s}$$

So I go around the neutron star at a dizzying rate of about 12 times a second!

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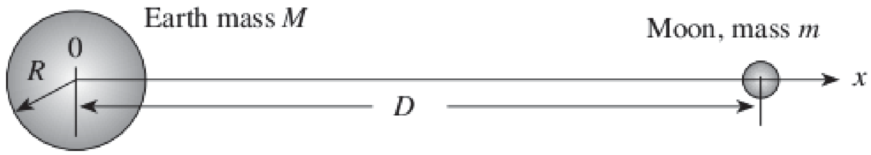
N11M.1 Let's define our reference frame sot that the x axis goes through the centers of both the earth and the moon, with the earth at the origin, as shown below.



We'll assume that the center of mass of each object is located at its physical centers, and that the distance between the center of masses is the same as the moon's mean orbital radius given on the inside front cover of the text. In chapter C4, we learned that the position of the system's center of mass when we use a reference frame like that in the diagram is

$$\vec{r}_{CM} = \frac{m\vec{r}_M + M\vec{r}_E}{m+M} = \frac{1}{m+M}\left(m\begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} + M\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = \frac{1}{m+M}\begin{bmatrix} mD \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad |\vec{r}_{CM}| = \frac{mD}{m+M} = \frac{(7.36\times 10^{22}\ \cancel{\text{kg}})(384{,}000\ \text{km})}{(7.36\times 10^{22}\ \cancel{\text{kg}} + 5.98\times 10^{24}\ \cancel{\text{kg}})} = 4670\ \text{km} \tag{1}$$

Since the earth's radius is $R = 6380$ km, the system's center of mass lies 6380 km – 4670 km = 1710 km below the earth's surface.

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N11M.2 The sun is $R_E = 1$ AU from the earth by definition and has mass $M = 1.99 \times 10^{30}$ kg. According to data on the back of book's first leaf, Jupiter has a mass of $m = 318 M_E = 318\,(5.98 \times 10^{24}\text{ kg}) = 1.90 \times 10^{27}$ kg, where M_E is the mass of the earth. Jupiter is an average distance of 5.204 AU from the sun and therefore $R_J = 4.204$ AU from the earth when the three are in line (assuming the earth is between the sun and Jupiter). According to the law of universal gravitation, the ratio of the magnitudes of the gravitational forces $\vec{F}_{gJ}$ and $\vec{F}_{gS}$ that the Jupiter and the sun exert on the earth, respectively, is

$$\frac{|\vec{F}_{gJ}|}{|\vec{F}_{gS}|} = \frac{GmM_E / R_J^2}{GMM_E / R_S^2} = \left(\frac{m}{M}\right)\left(\frac{R_S}{R_J}\right)^2 = \frac{1.90 \times 10^{27}\ \cancel{\text{kg}}}{1.99 \times 10^{30}\ \cancel{\text{kg}}}\left(\frac{1.0\ \cancel{\text{AU}}}{4.204\ \cancel{\text{AU}}}\right)^2 = 5.40 \times 10^{-5}.$$

This means that the sun's influence on the earth is about 18,500 times larger than that exerted by Jupiter. Note that the magnitude seems reasonable, since the sun is much more massive than Jupiter and also quite a bit closer. So we can safely ignore Jupiter's gravitational effects when computing the earth's orbit unless we need to do so very precisely.

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N11M.6 Note that the proton's charge is $e = 1.602 \times 10^{-19}$ C, and the electrons' charge is $-e$. The only force acting on the electron is the electrostatic force $\vec{F}_e$ exerted by its interaction with the proton, which has a magnitude $|\vec{F}_e| = ke^2/R^2$, where $k = 8.99 \times 10^9$ N·m²/C² is the Coulomb constant. The proton's mass is very massive compared to the electron's mass m, so we can consider the proton to be at rest. If the electron orbits the stationary proton at a speed $|\vec{v}|$ in a circular orbit of radius $R = 5.29 \times 10^{-11}$ m, then its acceleration is $|\vec{v}|^2/R$, and the magnitude of Newton's second law implies that we must have

$$\frac{ke^2}{R^2} = m\frac{|\vec{v}|^2}{R} \quad \Rightarrow \quad |\vec{v}|^2 = \frac{ke^2}{mR} \quad \Rightarrow \quad \frac{|\vec{v}|}{c} = \sqrt{\frac{ke^2}{mc^2R}} \tag{1}$$

Substituting in the numbers yields

$$\frac{|\vec{v}|}{c} = \sqrt{\frac{(8.99\times 10^9\ \cancel{\text{N}}\cdot\cancel{\text{m}}^2/\cancel{\text{C}}^2)(1.602\times 10^{-19}\ \cancel{\text{C}})^2}{(9.11\times 10^{-31}\ \cancel{\text{kg}})(3.0\times 10^8\ \cancel{\text{m}}/\cancel{\text{s}})^2(5.29\times 10^{-11}\ \cancel{\text{m}})}\left(\frac{1\ \cancel{\text{kg}}\cdot\cancel{\text{m}}/\cancel{\text{s}}^2}{1\ \cancel{\text{N}}}\right)} = \frac{1}{137} \tag{2}$$

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N11M.7 **(a)** According to Newton's law of gravitation, the gravitational force that the earth (of mass M_E) exerts on an object of mass m at a point P a distance R from the earth's center is given by

$$|\vec{F}_g| = \frac{GM_E m}{R^2} \tag{1}$$

The gravitational field strength $|\vec{g}_P|$ at any location P is defined to be such that $|\vec{F}_g| = m|\vec{g}_P|$, so

$$|\vec{g}_P| = \frac{GM_E}{R^2} \tag{2}$$

We see that the gravitational field strength decreases with the inverse square of the distance between the objects. So if the moon is 60 times farther from the earth's center than the earth's surface is, then the earth's gravitational field strength at the distance of the moon is therefore $|\vec{g}_M| = |\vec{g}|/60^2 = |\vec{g}|/3600$, where $|\vec{g}|$ is the gravitational field strength at the earth's surface.

(b) Since $|\vec{g}_M|$ gives the moon's acceleration towards the earth in its circular orbit of radius R_M about the earth, we can calculate the moon's orbital speed $|\vec{v}|$ as follows:

$$|\vec{g}_M| = \frac{|\vec{v}|^2}{R_M} \quad \Rightarrow \quad |\vec{v}|^2 = |\vec{g}_M| R_M = \frac{|\vec{g}|}{3600} 60 R_E = \frac{|\vec{g}| R_E}{60} \tag{3}$$

where R_E is the earth's radius (which was well-known during Newton's life). The time that it takes the moon to go around the earth is therefore

$$|\vec{v}| = \frac{2\pi R_M}{T} \quad \Rightarrow \quad T = \frac{2\pi R_M}{|\vec{v}|} = \frac{2\pi \cdot 60 R_E}{\sqrt{|\vec{g}| R_E / 60}} = 2\pi \sqrt{\frac{60^3 R_E}{|\vec{g}|}} \tag{4}$$

Substituting in the numbers yields

$$T = 2\pi \sqrt{\frac{60^3 (6.38 \times 10^6 \text{ m})}{9.8 \text{ m/s}^2}} \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1 \text{ d}}{24 \text{ h}}\right) = 27.3 \text{ d} \tag{5}$$

(c) This result corresponds to the observed orbital period of the moon to the accuracy of our calculation, giving Newton a very compelling argument that the same gravitational force that causes an apple to fall with acceleration 9.8 m/s^2 is the force that holds the moon in its circular orbit.