MAT 8-236 Differential Equations
Exam 1 April 16, 2008

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Closed book and notes. Graphing calculators are allowed, but not laptop computers or calculators capable of symbolic mathematics. Do not store and use formulas, algorithms, or other information in your calculator. Write your answers on the paper provided and when you are done staple these pages to front of your exam. Show your work.

1. (10 pts.) Produce the differential equations or initial value problems that model the following physical/dynamical system. Don't solve!

a. A flu epidemic spreads through a college community of size 3000 at a rate proportional to the product of the number of members already infected and the number of those not infected. The epidemic starts with just 4 infected students. Model the number of infected students.

b. The rate at which the temperature of an object changes is proportional to the difference between the temperature T of the object and the temperature S of the objects surroundings. Model the temperature of the object

c. Money is invested at a certain interest rate k which is compounded continuously, thus the growth rate is proportional to the amount invested.

d. Two species compete for a resource. In the absence of competing species, each would exhibit logistic growth.

2. (12 pts.) a. Find the general solution of the differential equation $dy/dt = (y^2+1)t$. Then solve the initial value problem $y(0) = 1$.

b. Find the general solution of the differential equation $dy/dt = y(2-y)$.

3. (6 pts.) Use Euler's method for numerical solution of differential equations/initial value problems to solve, for $y(t)$, $0 \leq t \leq .2$ with a stepsize of 0.1 in the initial value problem $y' = y - t$, $y(0) = 1$.

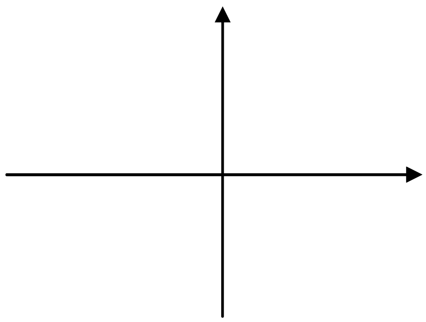
4. (12 pts.) Explain the step-by-step process of solving first-order linear differential equations using integrating factors. Illustrate on $dy/dt = -y + t^2$

5. (4 pts.) Determine if each of the following autonomous differential equations has a source at y = 1. Justify your answer in each case.

a. $\frac{dy}{dt} = \sin \pi y$

b. $\frac{dy}{dt} = |1 - y|$

6. (6 pts.) a. Sketch a phase line corresponding to the differential equation dy/dt = f(y) when the graph of f(y) is given by the graph below.



b. Sketch a possible graph of a continuous function f(y) for which the phase line of the differential equation dy/dt = f(y) is given here. What is the long-term behavior of a solution given the initial condition y(0) = 2?

7. (8 pts.) Identify all bifurcation points and near these points sketch and explain the bifurcation diagram for the family of differential equations dy/dt = y^2- 2y + μ

8. (4 pts.) Convert the second-order differential equation
$\frac{d^2y}{dt^2} + 2y = 0$ into a first-order system.

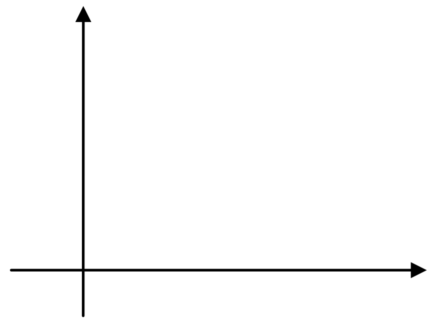
9. (12 pts.) a. Identify equilibrium point(s) of the following system.

$$\frac{dx}{dt} = 2x(1 - \frac{x}{2}) - xy$$
$$\frac{dy}{dt} = 3y(1 - \frac{y}{3}) - 2xy$$

b. sketch the vector field for the following system.

$$\frac{dx}{dt} = -x$$
$$\frac{dy}{dt} = -y$$

10. (6 pts.) For the following phase portrait, produce associated graphs of x(t) and y(t).



11. (12 pts.) True or false. If the statement is true, explain why. If it is false, give a counterexample.

a. Every solution of $dy/dt = y + e^{-t}$ tends to $+\infty$ or $-\infty$ as $t \to \infty$

b. The solution of $dy/dt = (y-2)(y\cos t + y\sin t + 1)$ with $y(0) = 1$ satisfies $y(t) < 2$ for all t.

c. Every linear differential equation is separable.

d. Suppose that f(y) is a continuous function for all y. The phase line for $dy/dt = f(y)$ must have the same number of sinks as sources.

12. (8 pts.) Matching